Chapter 4
Informed search and Exploration

CS 461 – Artificial Intelligence

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Bilkent University, Spring 2008

Slides are mostly adapted from AIMA and MIT Open Courseware

Outline

Informed search strategies use problem specific knowledge beyond the definition of the problem itself

- Best-first search
- Greedy best-first search
- A^* search
- Heuristics
- Local search algorithms
- Hill-climbing search
- Simulated annealing search
- Local beam search
- Genetic algorithms

Best-first search

- Idea: use an evaluation function *f(n)* to select the node for expansion
  - estimate of "desirability"

  → Expand most desirable unexpanded node

- Implementation:

  Order the nodes in fringe in decreasing order of desirability

- A key component in best-first algorithms is a heuristic function, *h(n)*, which is the estimated cost of the cheapest path from n to a goal node

Best-first search

Best-first:

Pick "best" (measured by heuristic value of state) element of Q

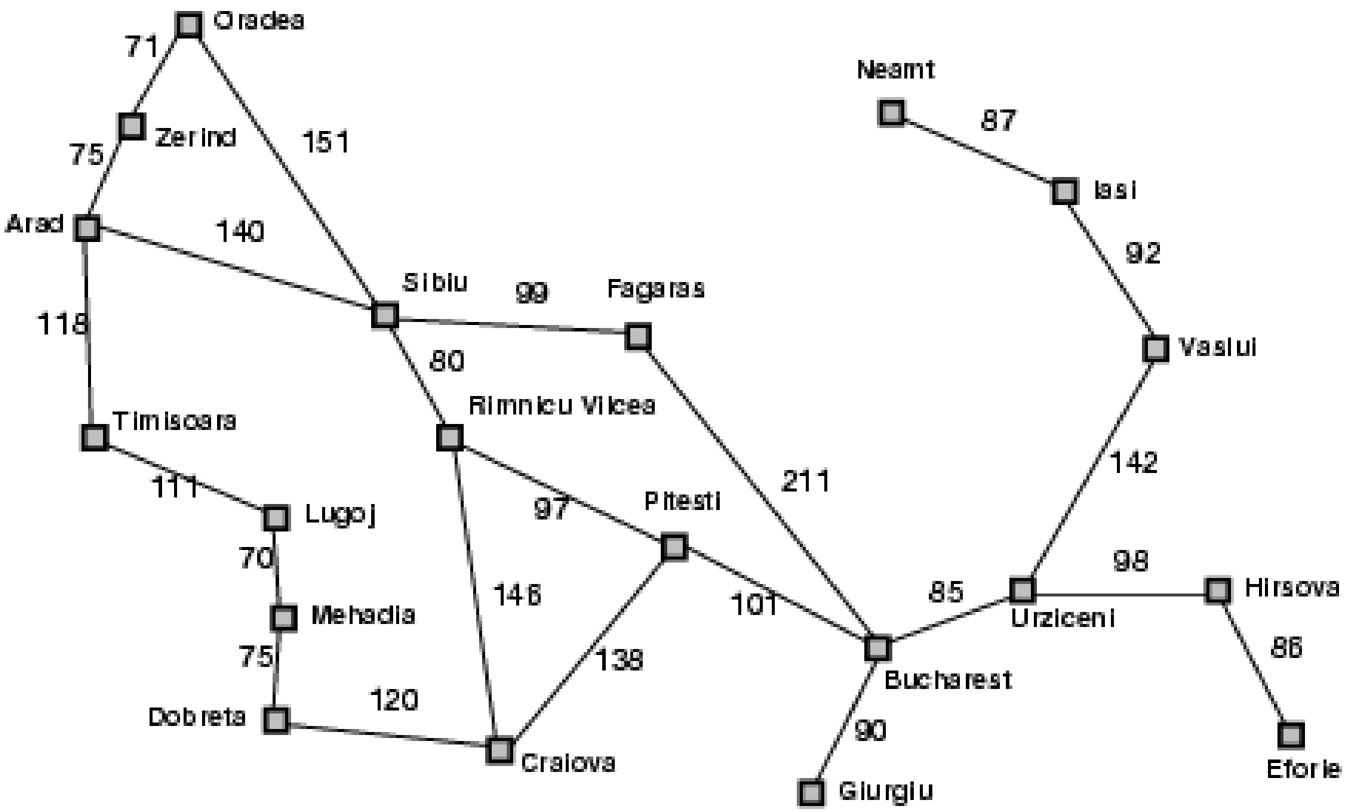
Add path extensions anywhere in Q (it may be more efficient tc keep the Q ordered in some way so as to make it easier to find the "best" element).

There are many possible approaches to finding the best node in Q.

- **Scanning Q to find lowest value**
- **Sorting Q and picking the first element**
- **Keeping the Q sorted by doing "sorted" insertions**
- **Keeping Q as a priority queue**

Romania with step costs in km

e.g. For Romania, cost of the cheapest path from Arad to Bucharest can be estimated via the straight line distance



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
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Hirsova	151
Iasi	226
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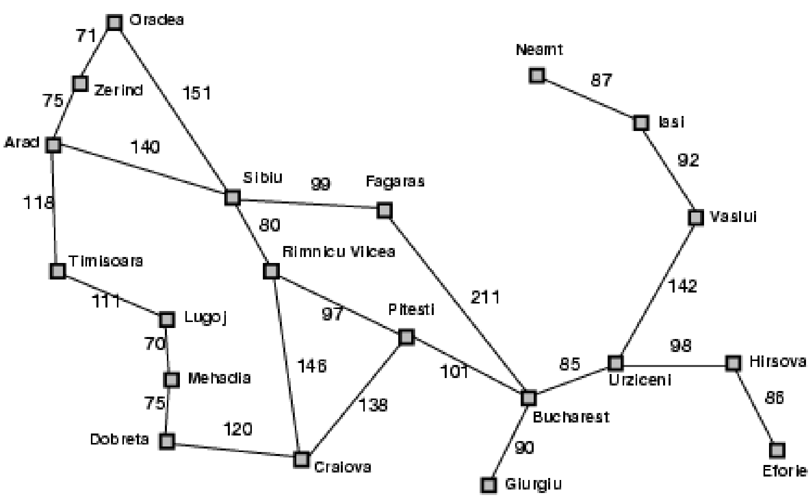
Greedy best-first search

- Greedy best-first search expands the node that appears to be closest to goal
- Evaluation function $f(n) = h(n)$ (heuristic)
- = estimate of cost from n to *goal*
- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest
- Note that, h_{SLD} cannot be computed from the problem description itself. It takes a certain amount of experience to know that it is correlated with actual road distances, and therefore it is a useful heuristic

Greedy best-first search example



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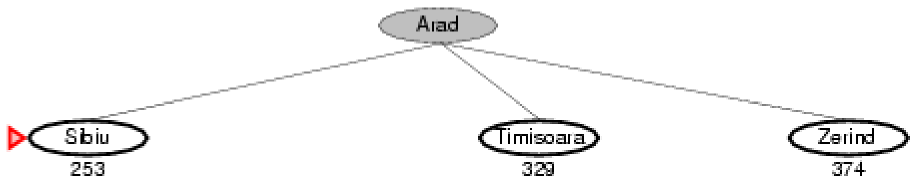
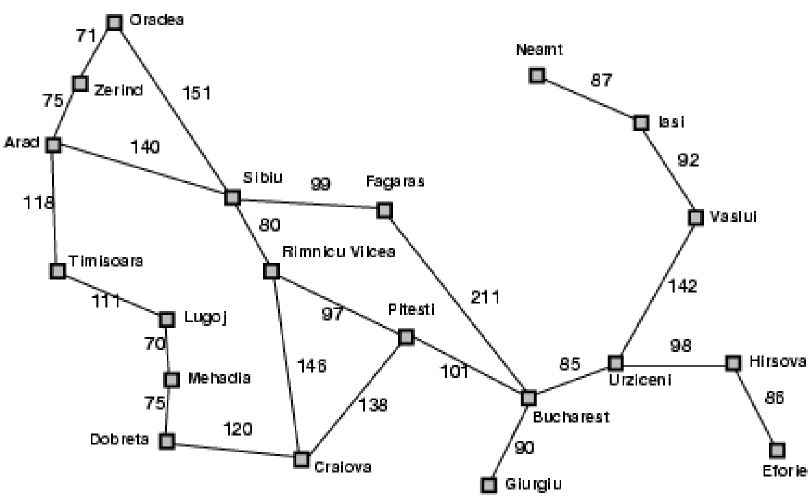
Greedy best-first search example





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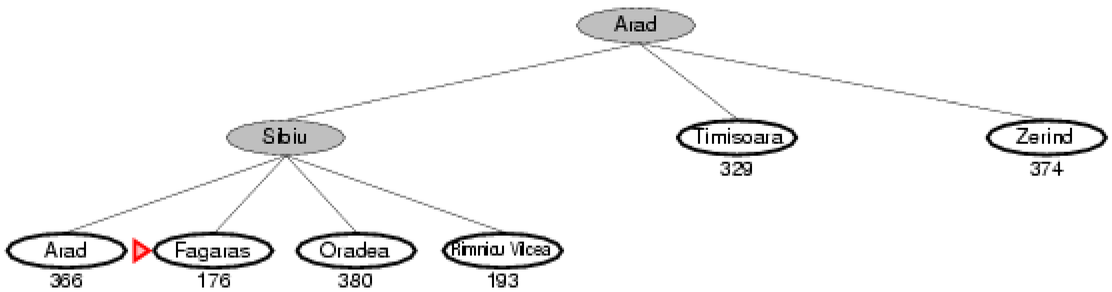
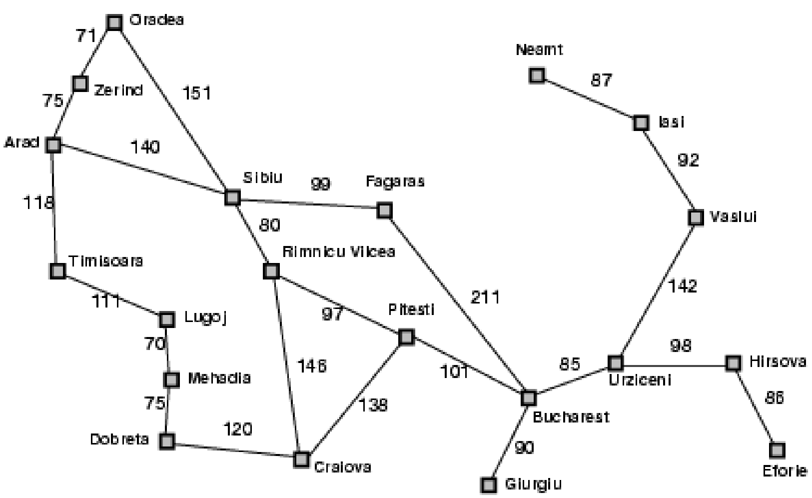
Greedy best-first search example





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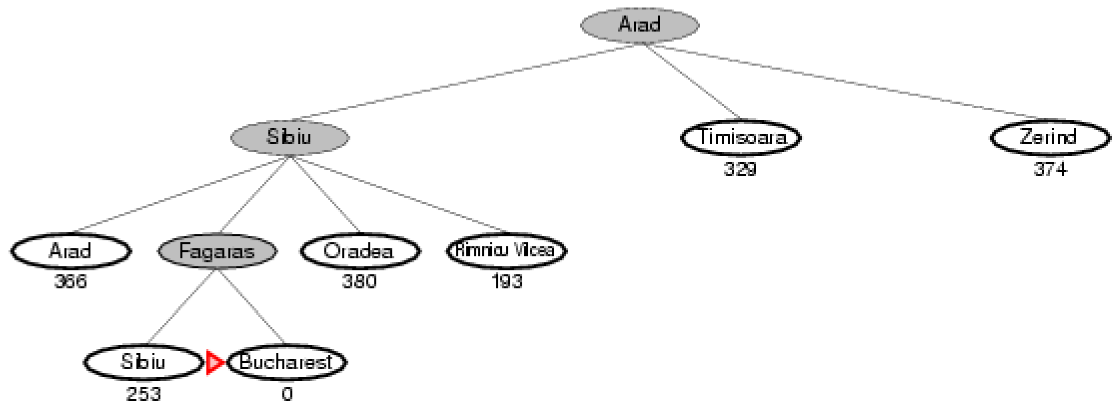
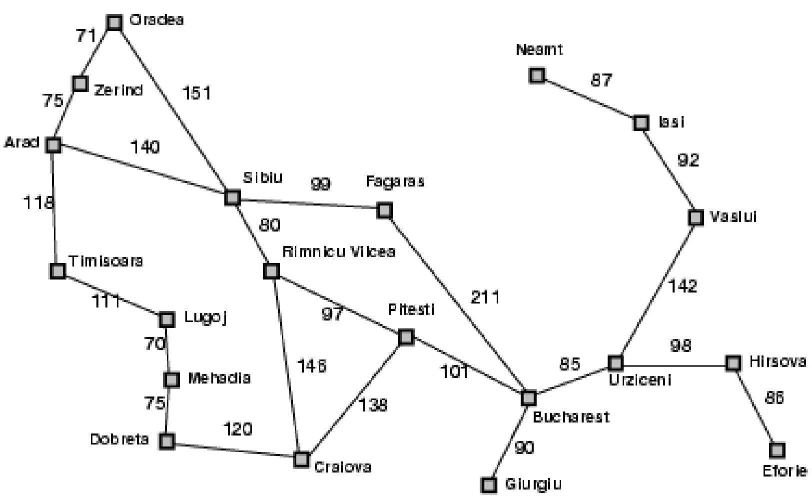
Greedy best-first search example





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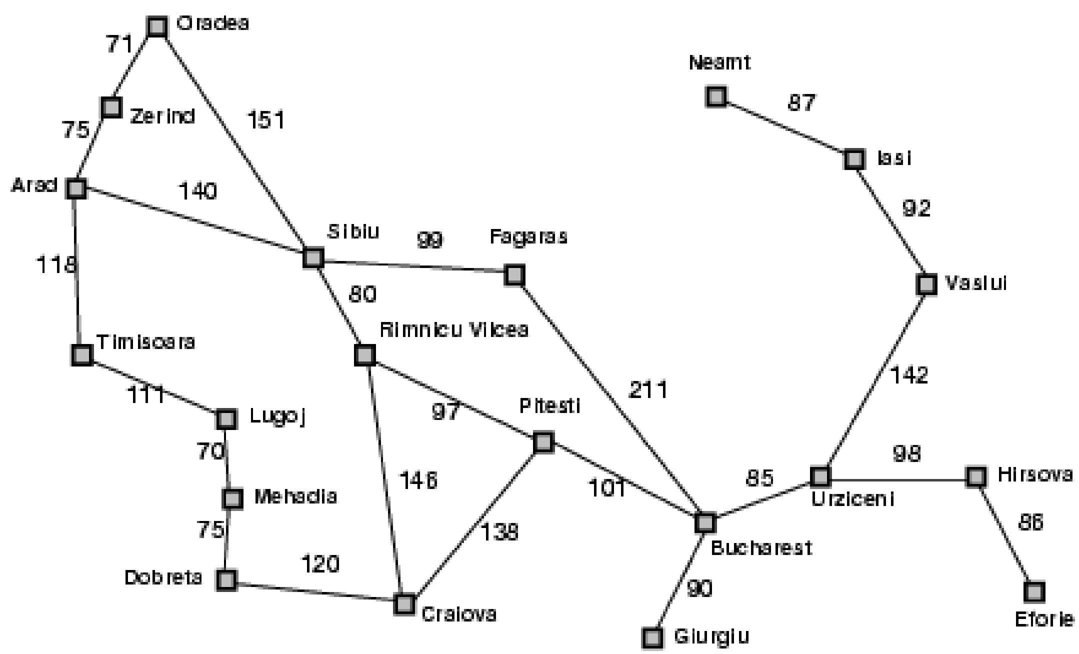
Greedy best-first search example

Problems:

Path through Faragas is not the optimal

In getting Iasi to Faragas, it will expand Neamt first but it is a dead end



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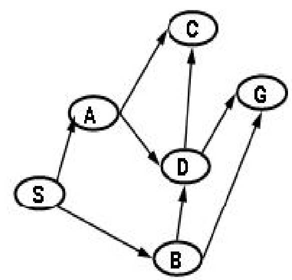
Greedy best-first search – Another example

Pick "best" (by heuristic value) element of Q; Add path extensions anywhere in Q

	Q	Visited
1	(10 S)	S
2		
3		
4		
5		



Heuristic Values

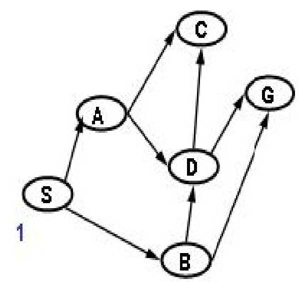
A=2	C=1	S=10
B=3	D=4	G=0

Added paths in blue; heuristic value of node's state is in front.

We show the paths in reversed order; the node's state is the first entry.

Greedy best-first search – Another example

	Q	Visited
1	(10 S)	S
2	(2 A S) (3 B S)	A,B,S
3		
4		
5		



Heuristic Values

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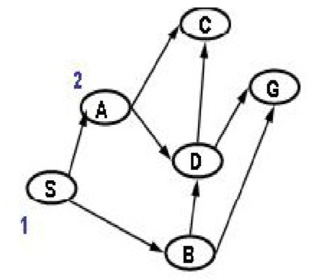
Greedy best-first search – Another example

	Q	Visited
1	(10 S)	S
2	(2 A S) (3 B S)	A,B,S
3	(1 C A S) (3 B S) (4 D A S)	C,D,B,A,S
4		
5		



Heuristic Values

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Greedy best-first search – Another example

	Q	Visited
1	(10 S)	S
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3	(1 C A S) (3 B S) (4 D A S)	C,D,B,A,S
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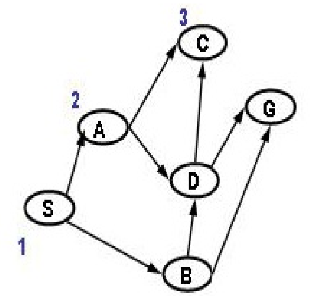
5		



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Greedy best-first search – Another example

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2	(2 A S) (3 B S)	A,B,S
3	(1 C A S) (3 B S) (4 D A S)	C,D,B,A,S
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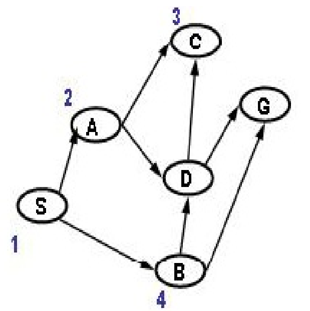
5	(0 G B S) (4 D A S)	G,C,D,B,A,S



Heuristic Values

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Greedy best-first search – Another example

	Q	Visited
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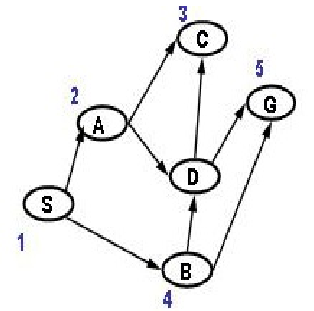

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Greedy best-first search – Another example

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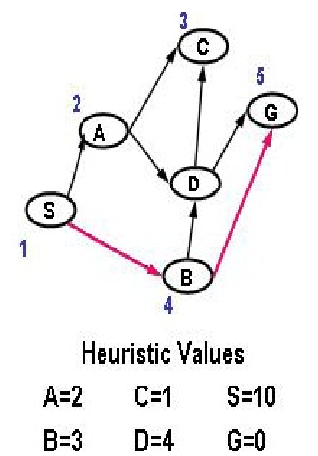


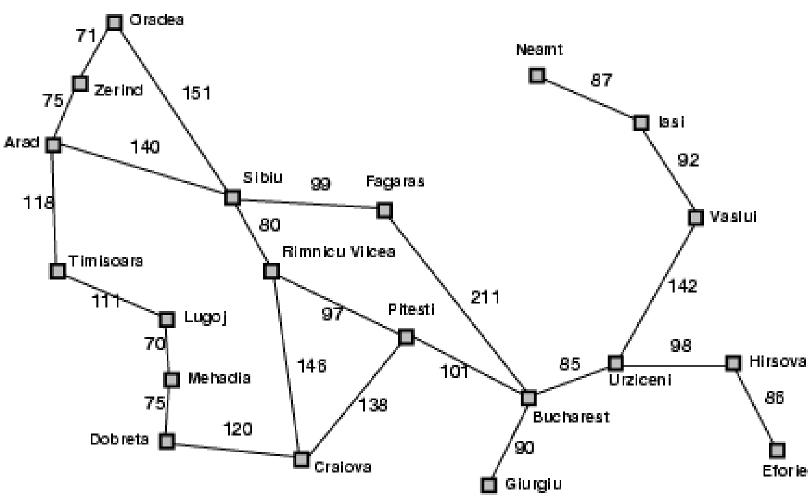
Properties of greedy best-first search

- Complete? No – can get stuck in loops,
  - e.g., Iasi → Neamt → Iasi → Neamt →
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

A^* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal

A* search example



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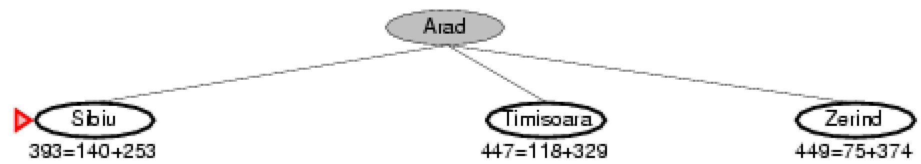
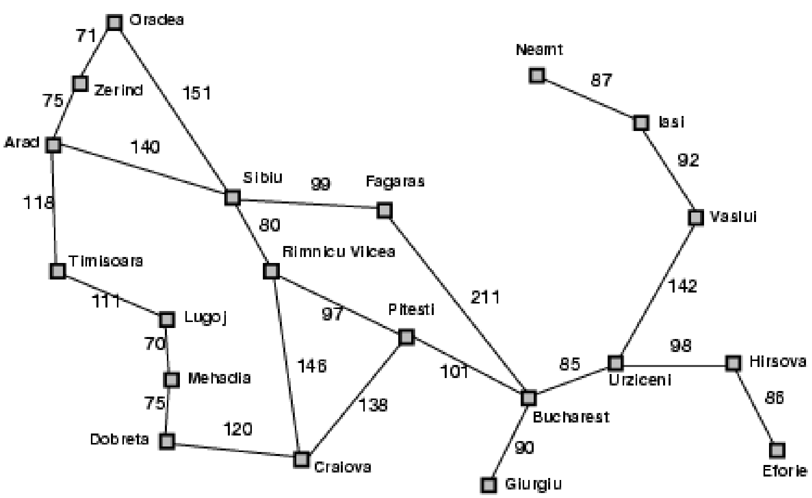
A* search example





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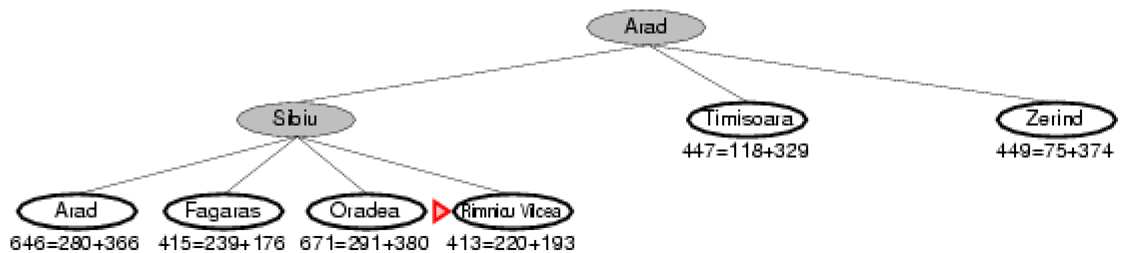
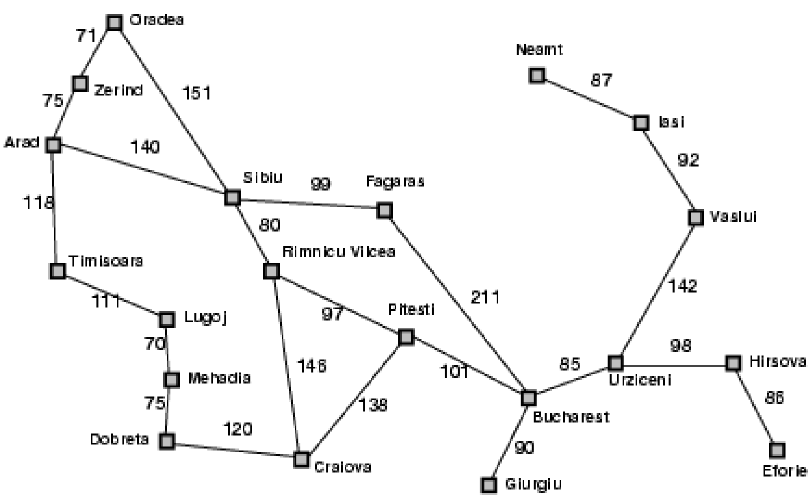
A* search example





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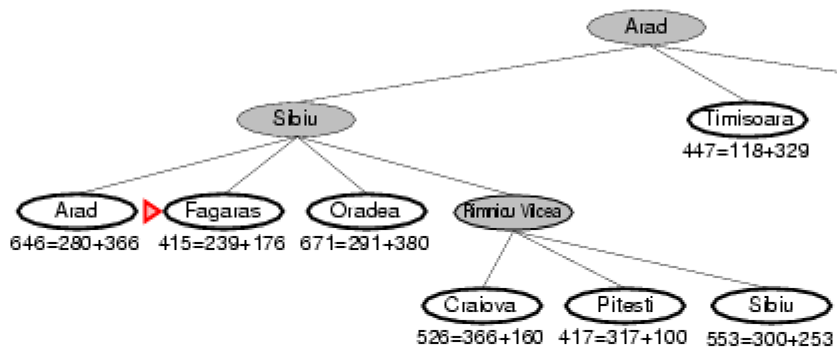

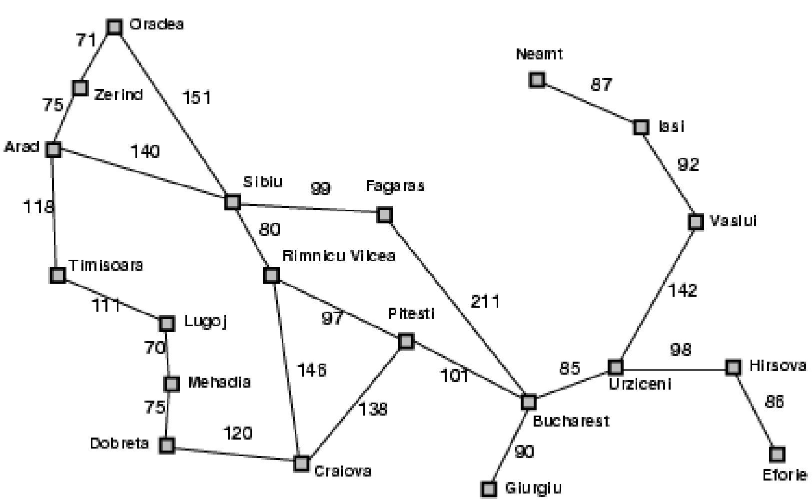

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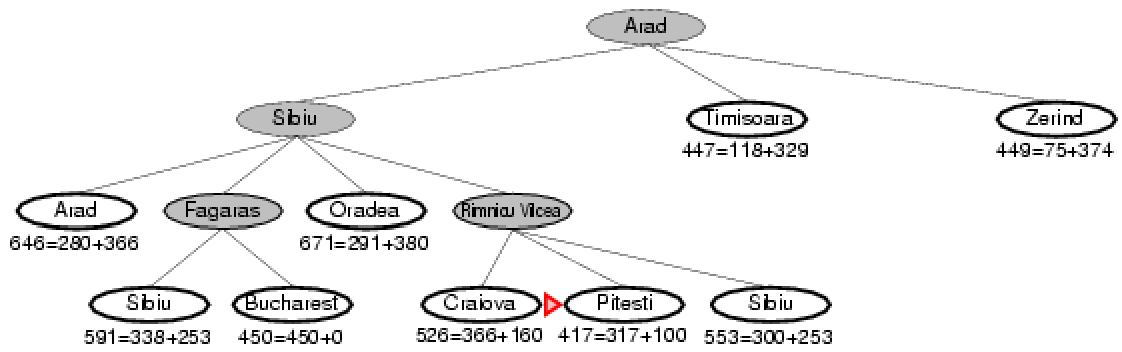

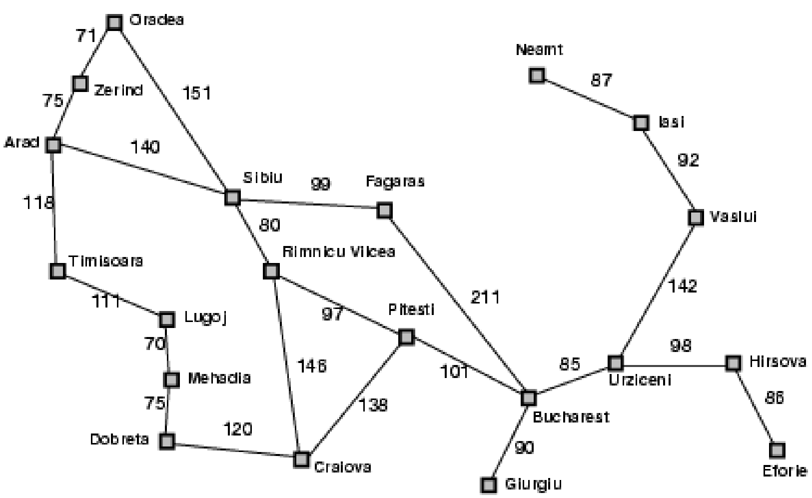

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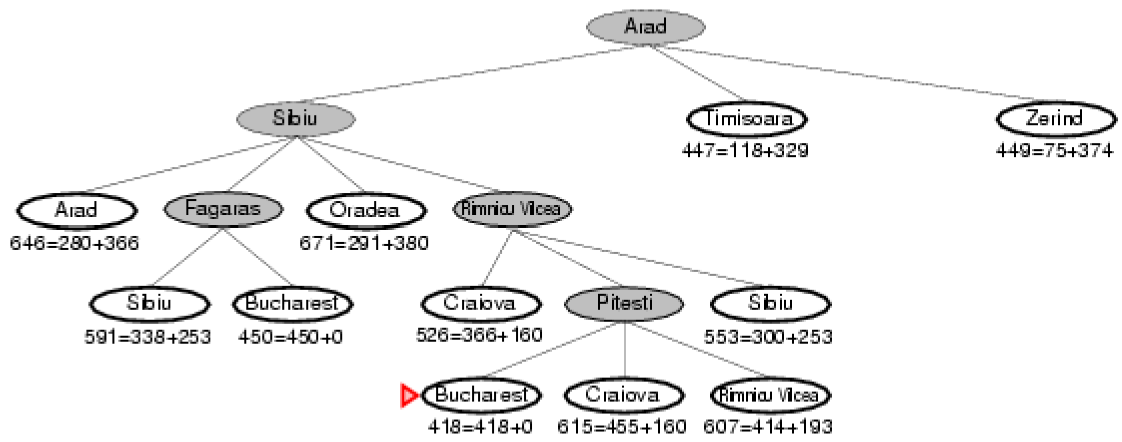

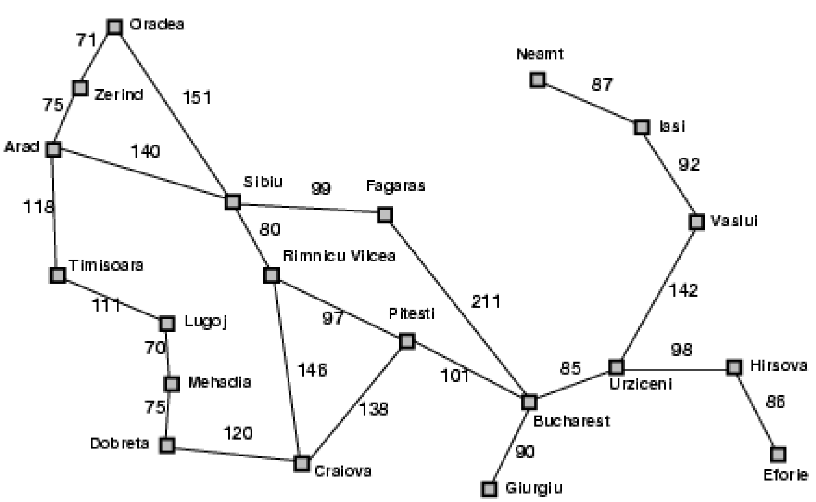

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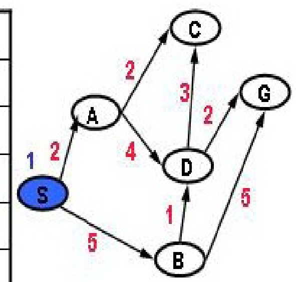
A* search – Another example

Pick best (by path length+heuristic) element of Q; Add path extensions anywhere in Q

	Q
1	(0 S)



Heuristic Values

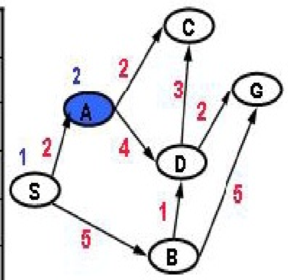
A=2	C=1	S=0
B=3	D=1	G=0

Added paths in blue; underlined paths are chosen for extension.

We show the paths in reversed order; the node's state is the first entry.

A* search – Another example

	Q
1	(0 S)
2	(4 A S) (8 B S)

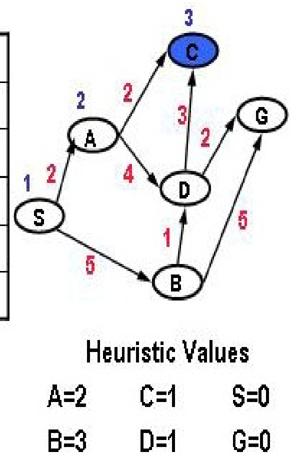


Heuristic Values

A=2	C=1	S=0
B=3	D=1	G=0

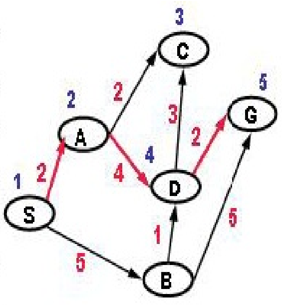
A* search – Another example

	Q
1	(0 S)
2	(4 A S) (8 B S)
3	(5 C A S) (7 D A S) (8 B S)



A* search – Another example

	Q
1	(0 S)
2	(4 A S) (8 B S)
3	(5 C A S) (7 D A S) (8 B S)
4	(7 D A S) (8 B S)
5	(8 G D A S) (10 C D A S) (8 B S)



Heuristic Values

A=2 C=1 S=0

B=3 D=1 G=0

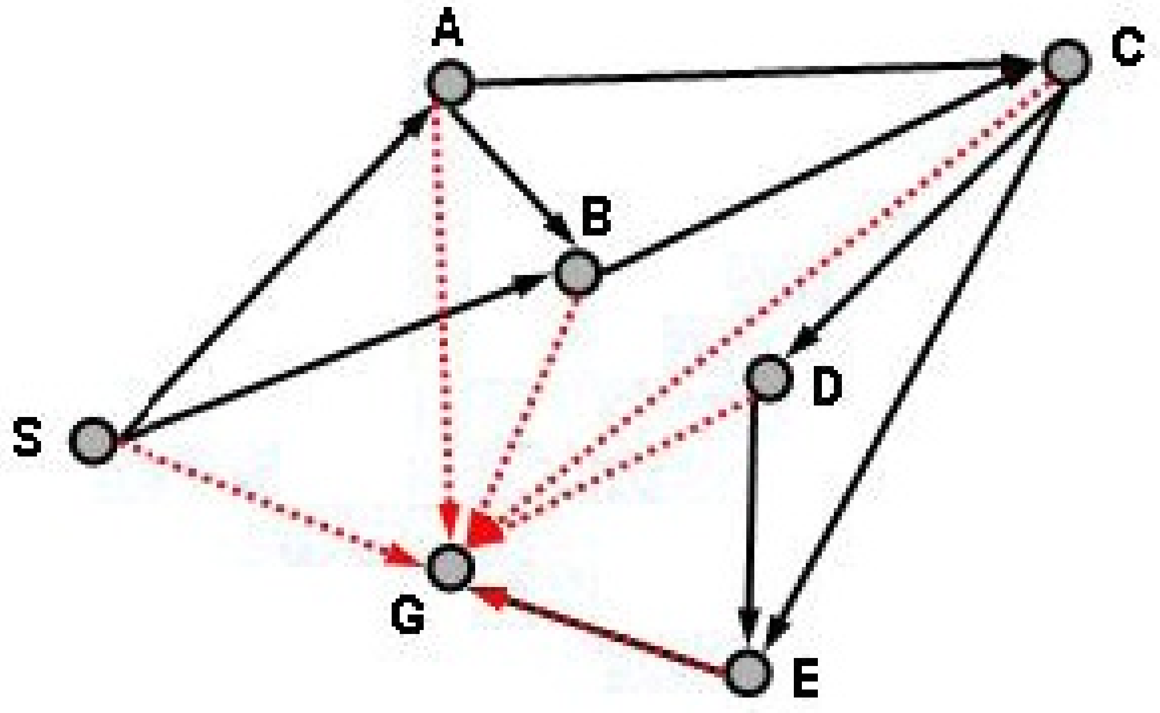
Classes of search

Class	Name	Operation
Any Path Uninformed	Depth-First Breadth-First	Systematic exploration of whole tree until a goal node is found.
Any Path Informed	Best-First	Uses heuristic measure of goodness of a node, e.g. estimated distance to goal.
Optimal Uninformed	Uniform-Cost	Uses path "length" measure. Finds "shortest" path.
Optimal Informed	A*	Uses path "length" measure and heuristic Finds "shortest" path

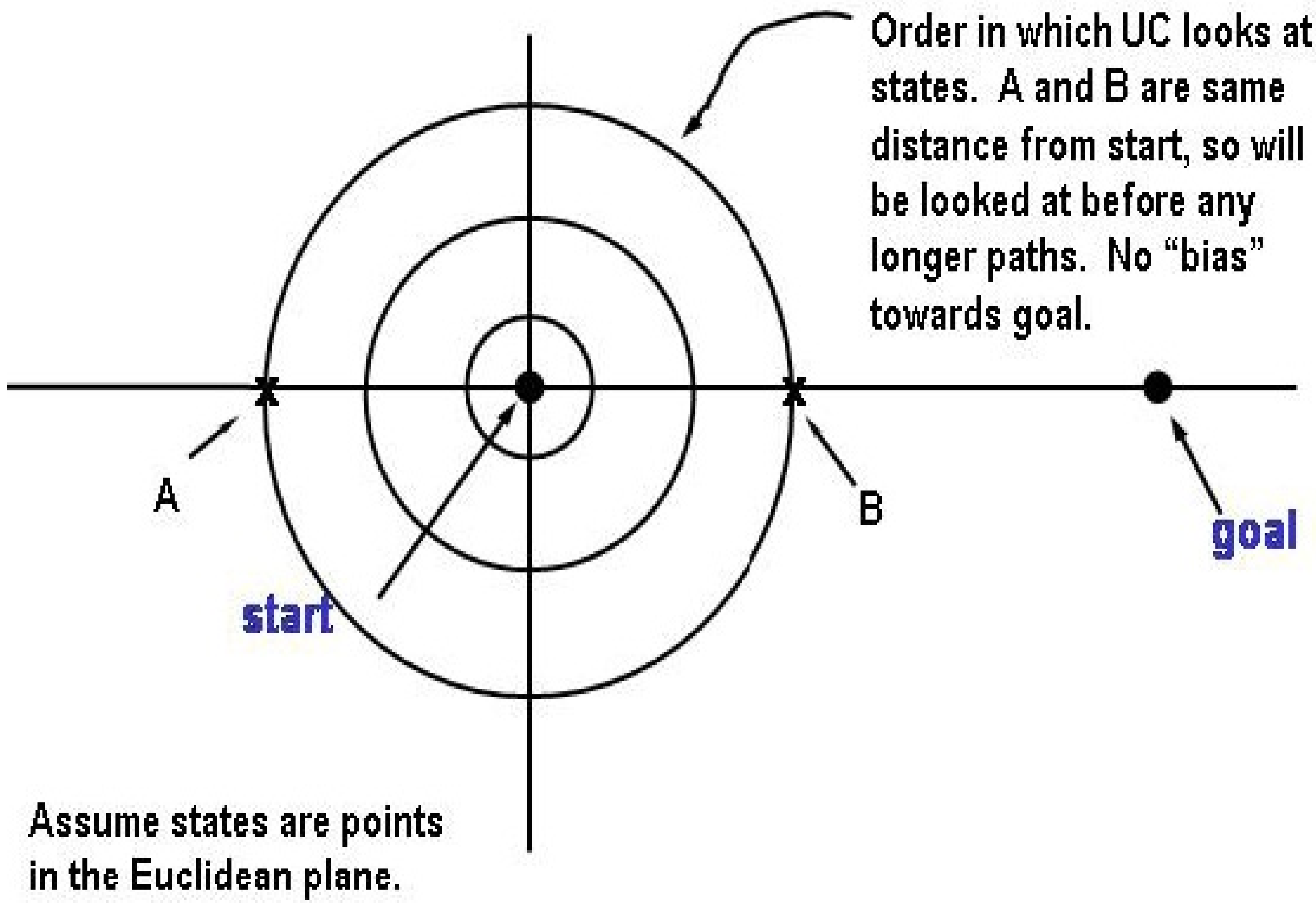
Uniform Cost (UC) versus A*

- UC is really trying to identify the shortest path to every state in the graph in order. It has no particular bias to finding a path to a goal early in the search.
- We can introduce such a bias by means of heuristic function h(N), which is an estimate (h) of the distance from a state n to a goal.
- Instead of enumerating paths in order of just length (g), enumerate paths in terms of f = estimated total path length = g + h.
- An estimate that always underestimates the real path length to the goal is called admissible. For example, an estimate of 0 is admissible (but useless). Straight line distance is admissible estimate for path length in Euclidean space.
- Use of an admissible estimate guarantees that UC will still find the shortest path.
- UC with an admissible estimate is known as A* (pronounced "A star") search.

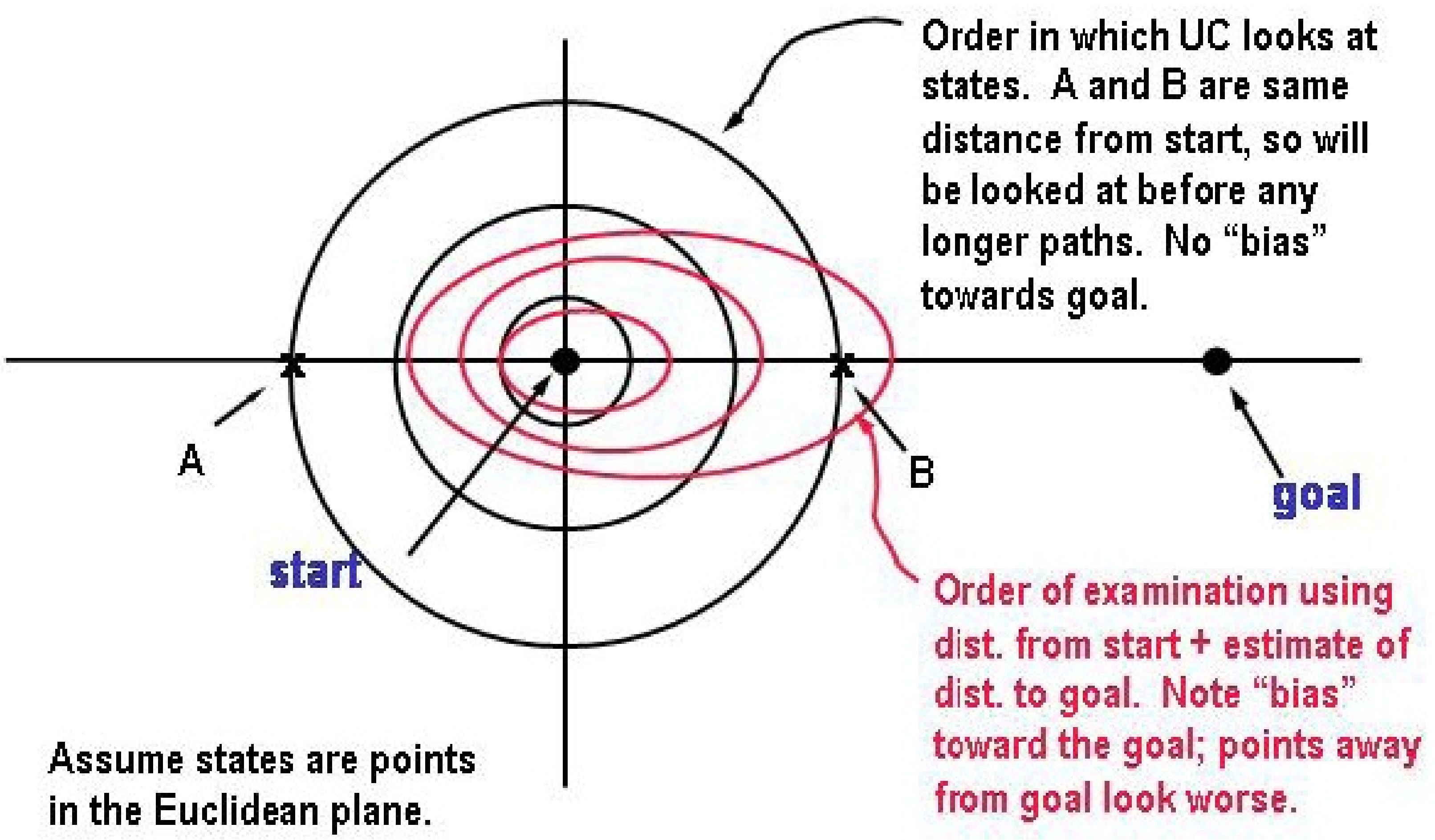
Straight line estimate



Why use estimate of goal distance



Why use estimate of goal distance



Not all heuristics are addmissible

Given the link lengths in the figure, is the table of heuristic values that we used in our earlier best-first example an admissible heuristic?

No!

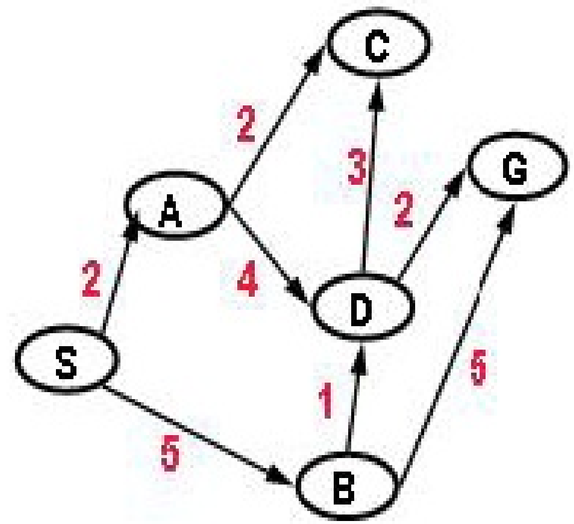
A is ok

B is ok

C is ok

D is too big, needs to be <= 2

S is too big, can always use 0 for start



Heuristic Values

A=2	C=1	S=10
B=3	D=4	G=0

Admissible heuristics

- A heuristic *h(n)* is admissible if for every node *n*,

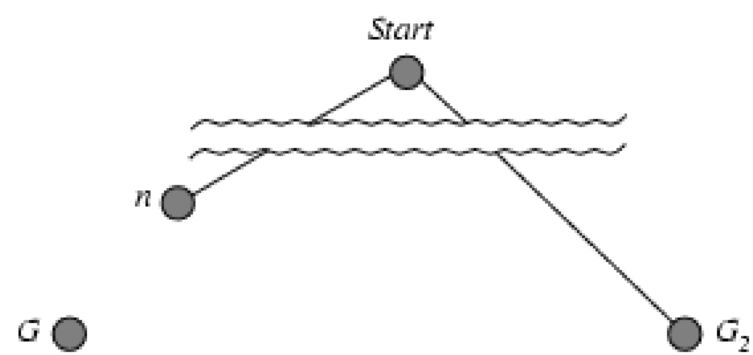
  $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from *n*.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic – thinks that the cost of solving the problem is less than it actually is
- Consequence: f(n) never over estimates the the true cost of a solution through n since g(n) is the exact cost to reach n
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance) since the shortest path between any two points is a straight line

Optimality of A^* (proof)

- Theorem: If *h(n)* is admissible, A^* using `TREE-SEARCH` is optimal
-
- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let the cost of the optimal solution to goal *G is C**

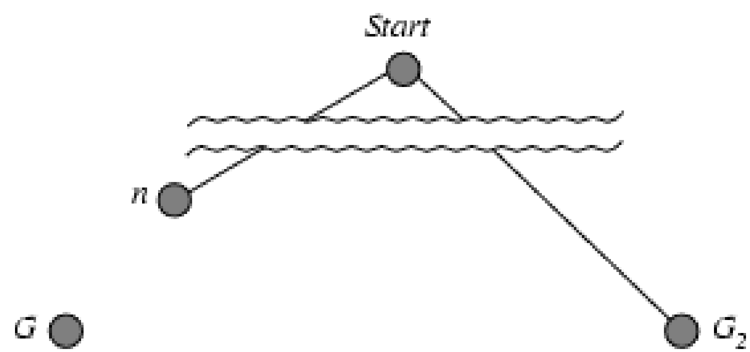
  f = g + h

- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > C^*$ since G_2 is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above



Optimality of A* (proof)

- Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G (e.g. Pitesti)*.



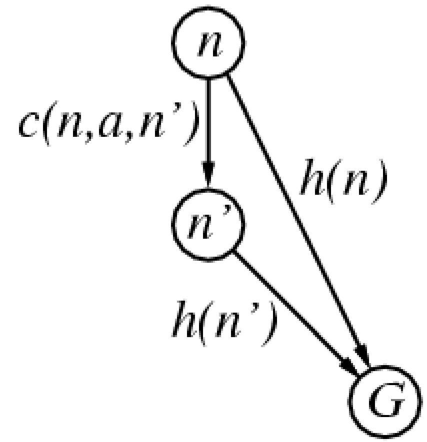
- If h(n) does not overestimate the cost of completing the solution path, then
- $f(n) = g(n) + h(n) \leq C^*$
- $f(n) \leq f(G)$
- $f(G_2) > f(G)$ from above
- Hence $f(G_2) > f(G) >= f(n)$, and A* will never select G_2 for expansion

Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

  $h(n) \leq c(n,a,n') + h(n')$

  n' = successor of n generated by action a



- The estimated cost of reaching the goal from n is no greater than the step cost of getting to n' plus the estimated cost of reaching the goal from n'
-
- If *h* is consistent, we have

  $f(n') = g(n') + h(n')$

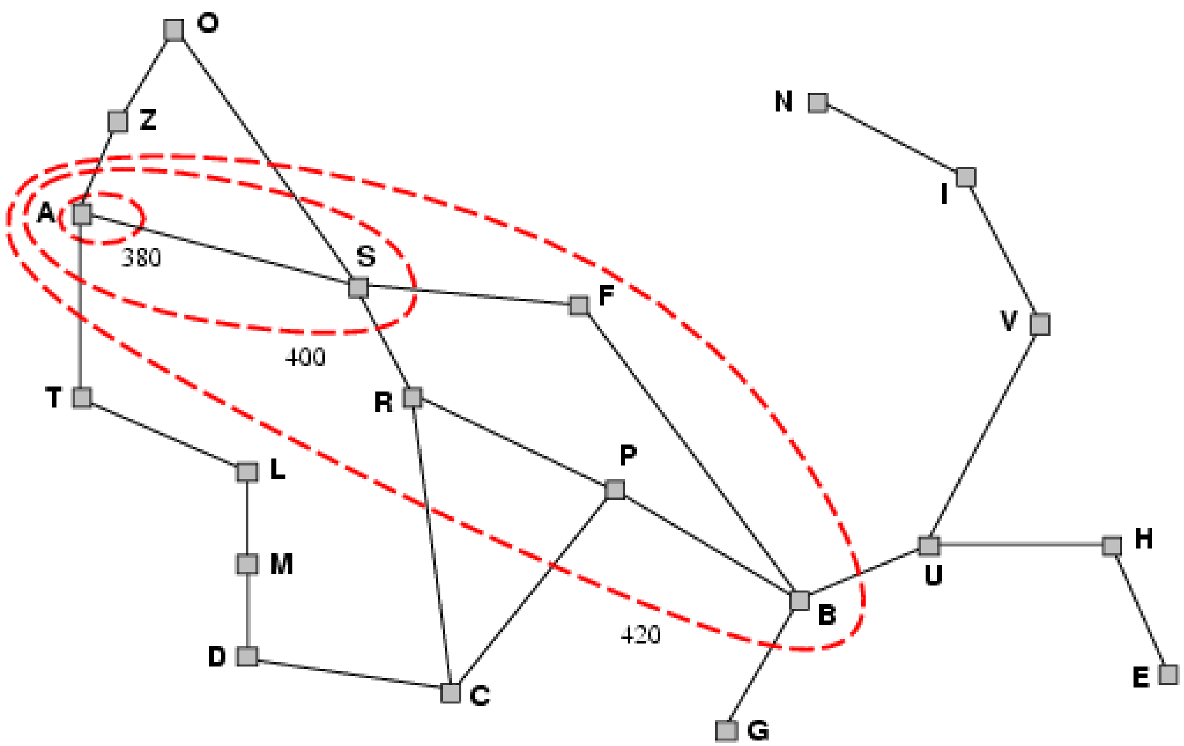
  $= g(n) + c(n,a,n') + h(n')$

  $\geq g(n) + h(n)$

  $= f(n)$
- if *h(n) is consistent then the values of f(n)* along any path are non-decreasing
- Theorem: If *h(n)* is consistent, A* using `GRAPH-SEARCH` is optimal

Optimality of A*

- A* expands nodes in order of increasing *f* value
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Properties of A*

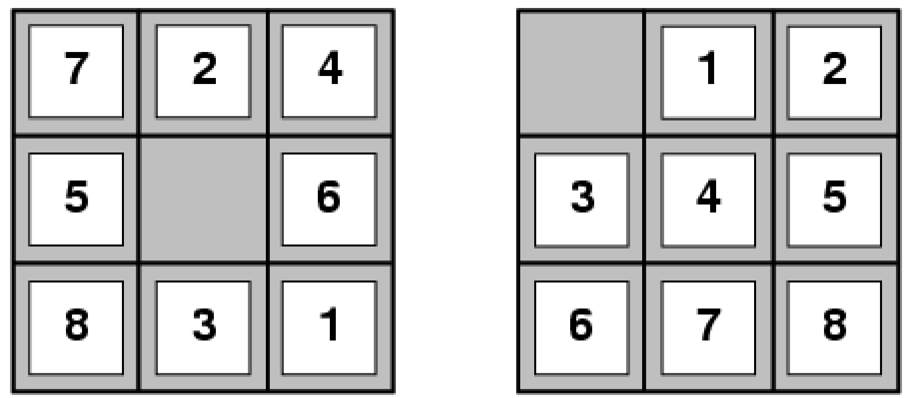
- Complete? Yes (unless there are infinitely many nodes with f $\leq f(G)$)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance – the sum of the distances of the tiles from their goal positions

- $h_1(S) = ?$
- $h_2(S) = ?$

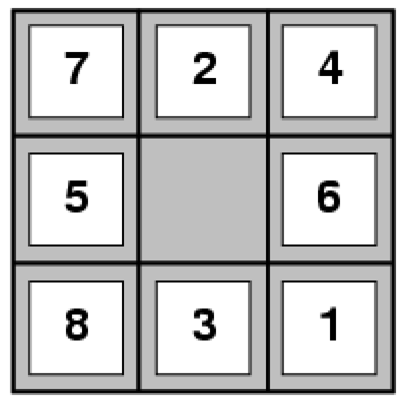


Start State Goal State

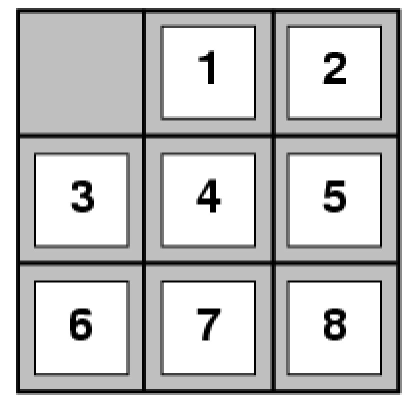
Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance



Start State



Goal State

- $h_1(S) = ?$ 8
- $h_2(S) = ?$ 3+1+2+2+2+3+3+2 = 18

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 dominates h_1
- h_2 is better for search
- It is always better to use a heuristic function with higher values, provided it does not overestimate and that the computation time for the heuristic is not too large

- Typical search costs (average number of nodes expanded):

- $d=12$ IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- $d=24$ IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- The heuristic is admissible because the optimal solution in the original problem is also a solution in the relaxed problem and therefore must be at least as expensive as the optimal solution in the relaxed problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Inventing admissible heuristic functions

- If a problem definition is written down in a formal language, it is possible to construct relaxed problems automatically (ABSOLVER)
  - If 8-puzzle is described as
    - A tile can move from square A to square B if
    - A is horizontally or vertically adjacent to B and B is blank
  - A relaxed problem can be generated by removing one or both of the conditions
    - (a) A tile can move from square A to square B if A is adjacent to B
    - (b) A tile can move from square A to square B if B is blank
    - (c) A tile can move from square A to square B
  - h2 can be derived from (a) – h2 is the proper score if we move each tile into its destination
  - h1 can be derived from (c) – it is the proper score if tiles could move to their intended destination in one step
- Admissible heuristics can also be derived from the solution cost of a subproblem of a given problem

Local search algorithms

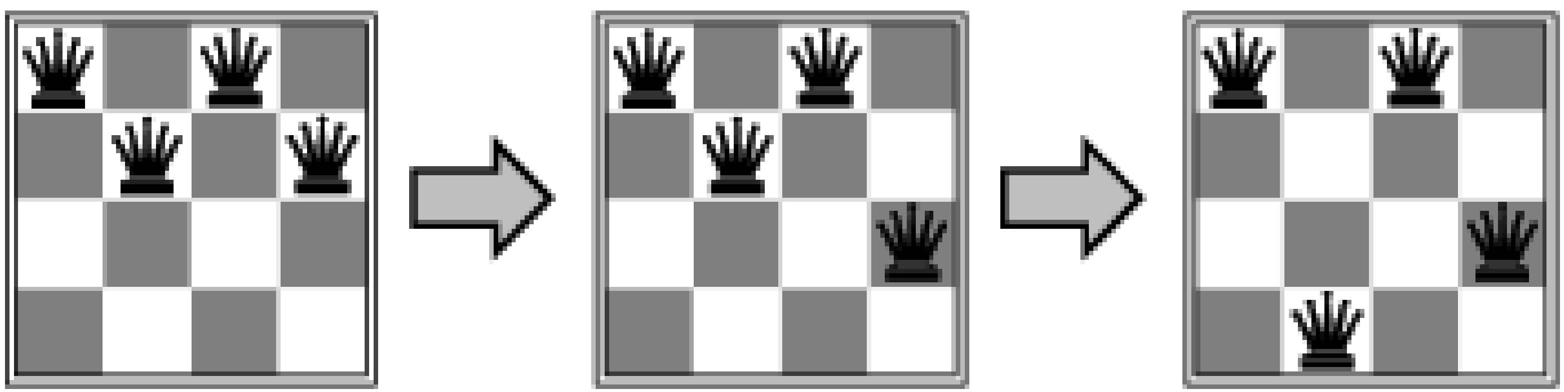
- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution

- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens

- In such cases, we can use local search algorithms
- keep a single "current" state, try to improve it

Example: *n*-queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



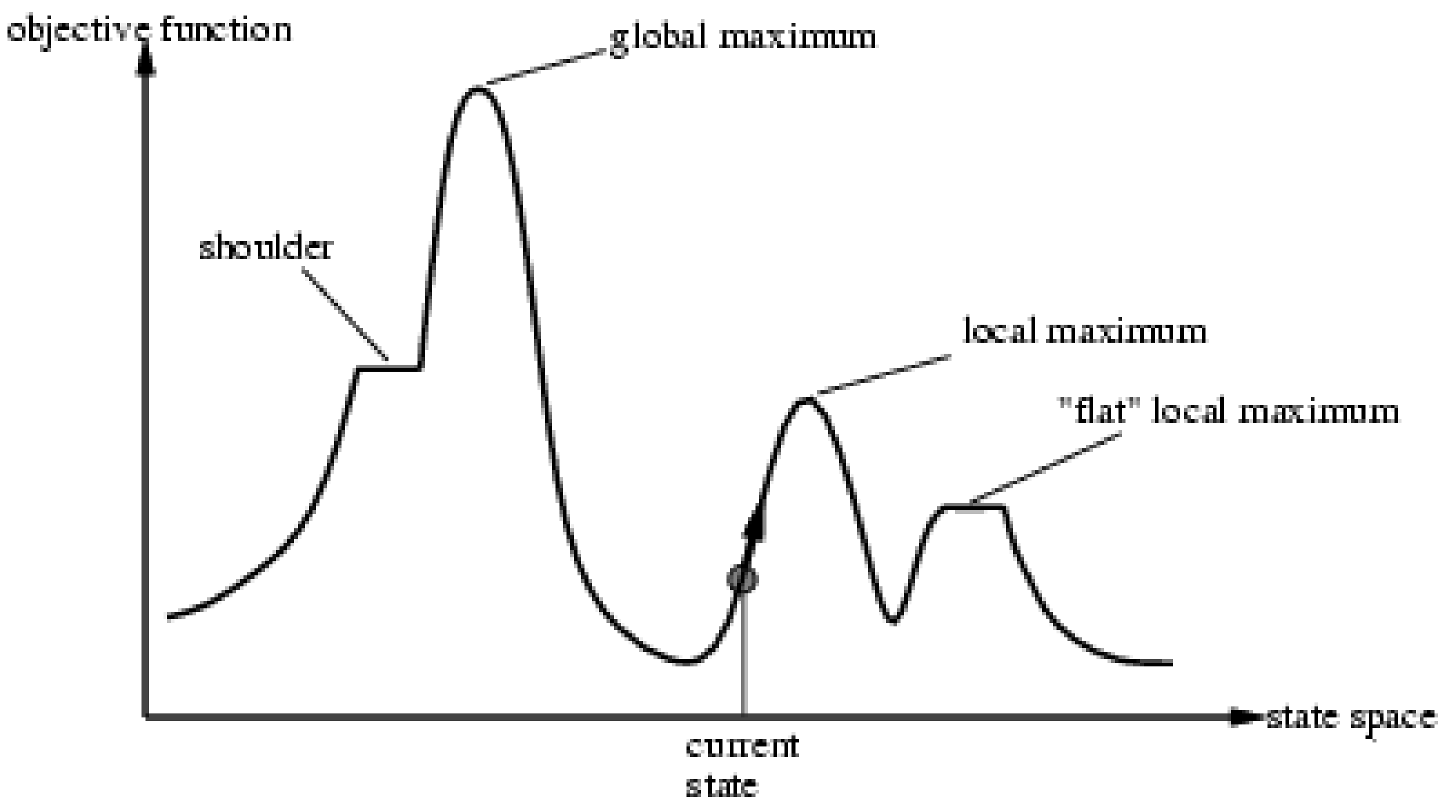
Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"

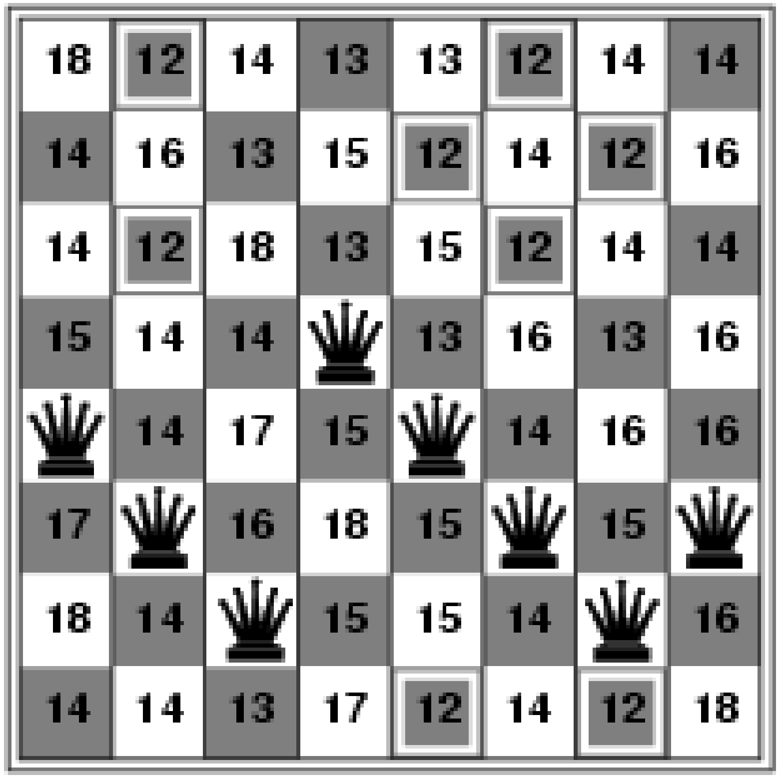
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
       neighbor ← a highest-valued successor of current
       if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
       current ← neighbor
```

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima

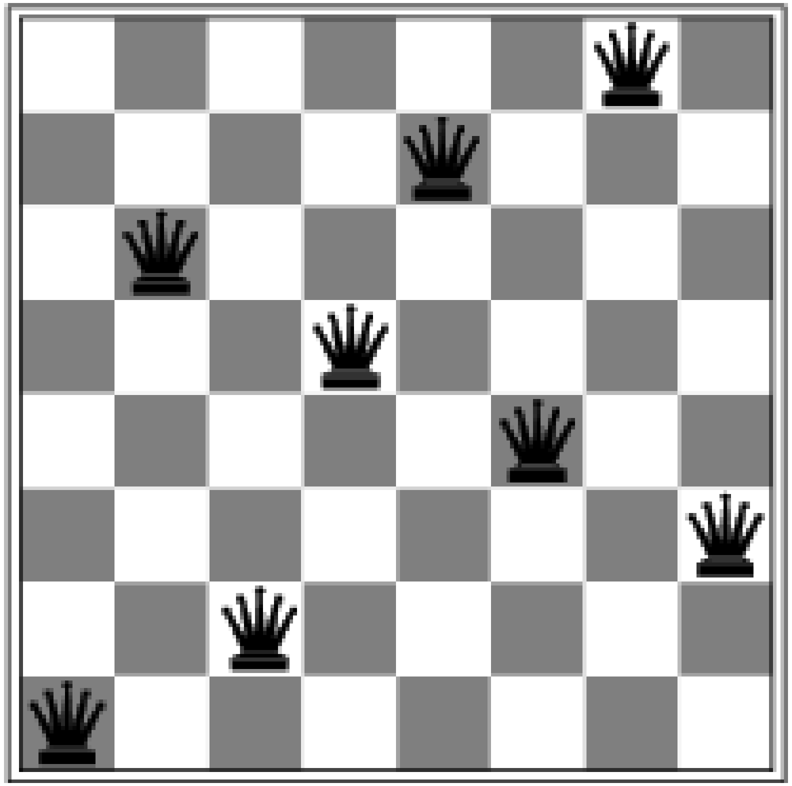


Hill-climbing search: 8-queens problem



- *h* = number of pairs of queens that are attacking each other, either directly or indirectly
- *h = 17* for the above state

Hill-climbing search: 8-queens problem



- A local minimum with $h = 1$

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  local variables: *current*, a node
    next, a node
    T, a "temperature" controlling prob. of downward steps

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for t ← 1 **to** ∞ **do**
    T ← *schedule*[t]
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    ΔE ← VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

Properties of simulated annealing search

- One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1
- Widely used in VLSI layout, airline scheduling, etc

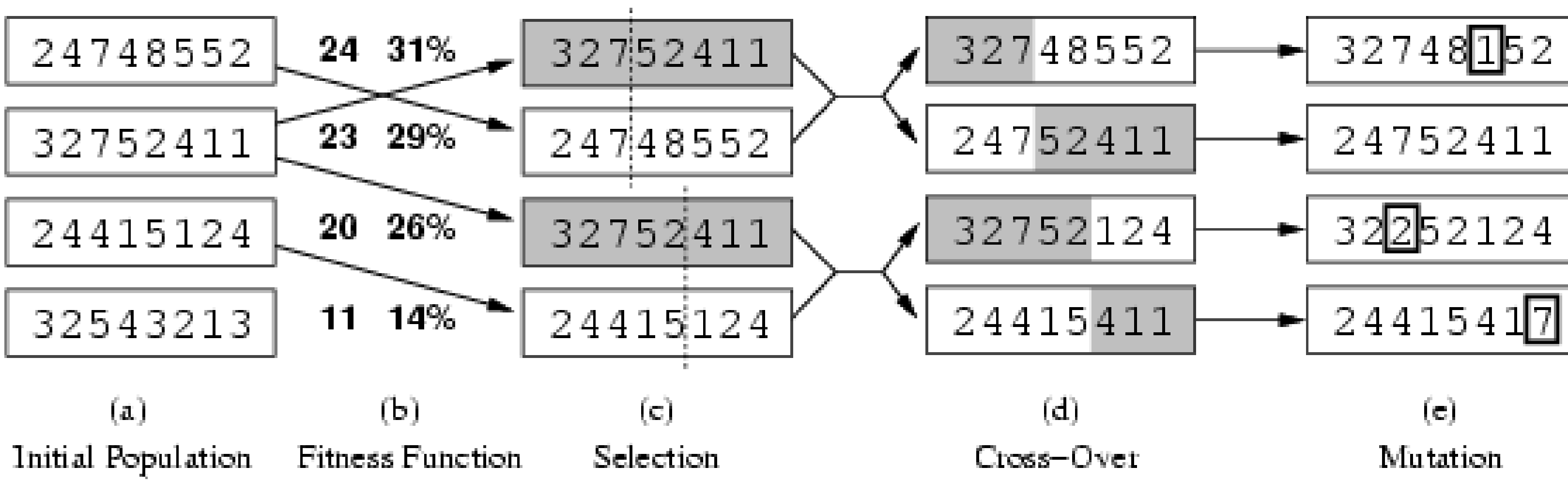
Local beam search

- Keep track of *k* states rather than just one
- Start with *k* randomly generated states
- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat.

Genetic algorithms

- A successor state is generated by combining two parent states
- Start with k randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation

Genetic algorithms



Genetic algorithms