# Animating Deformable Models: Different Approaches

Uğur Güdükbay and Bülent Özgüç

Bilkent University
Department of Computer Engineering and Information Science
Bilkent, 06533 Ankara, Turkey
E-mail: {gudukbay, ozguc}@bilkent.edu.tr

## Abstract

*Physically-based modeling remedies the problem of producing realistic animation by including forces, masses, strain energies, and other physical quantities. The behavior of physically-based models is governed by the laws of rigid and nonrigid dynamics expressed through a set of equations of motion. This paper discusses various formulations for animating deformable models. The formulations based on elasticity theory express the interactions between discrete deformable model points using the stiffness matrices. These matrices store the elastic properties of the models and they should be evolved in time according to changing elastic properties of the models. An alternative to these formulations seems to be external force formulations of different types. In these types of formulations, elastic properties of the materials are represented as external spring or other tensile forces as opposed to forming complicated stiffness matrices.*

## 1 Introduction

Modeling the behavior of *deformable objects* is an important aspect in realistic animation. To simulate the behavior of deformable objects, we should approximate a continuous model by using discretization methods, such as finite difference and finite element methods. For finite difference discretization, a deformable object could be approximated by using a grid of control points where the points are allowed to move in relation to one another. The manner in which the points are allowed to move determines the properties of the deformable object. Simulating the physical properties (such as tension and rigidity), static shapes exhibited by a wide range of deformable objects (including string, rubber, cloth, paper, and flexible metals) can be modeled. For example, to obtain the effect of an elastic surface, the grid points are connected by springs. The physical quantities, such as forces, torques, velocities, accelerations, kinetic and potential energies, should be used to simulate the dynamics of these objects.

There are some formulations which employ continuous elasticity theory to model the shapes and motions of deformable models. The *primal* [1] and *hybrid* [2] formulations are in this category. In these formulations, elastic properties of the materials are represented using potential energy functionals and stored in stiffness matrices. Potential energies of deformation are defined using the concepts from differential geometry and spline energies. However, forming the stiffness matrices at any step of an animation is very difficult and sometimes the differential equations that should be solved to produce animation become ill-conditioned.

Instead of modeling the elasticities using stiffness matrices, the interactions between model points could be expressed using external spring or other tensile forces. Such formulations [3] represent the elastic properties of the materials as external forces, as opposed to forming complicated stiffness matrices. Although handling the elasticities using the stiffness matrix approach is an elegant and a very suitable way, external force approaches are usually more effective and very fast.

### 1.1 Organization of the Paper

In section 2, methods for animating deformable models using elasticity theory, namely the primal formulation and the hybrid formulation, are explained. In section 3, other methods for the animation of deformable models are explained. These methods do not employ continuous elasticity theory for deformable models. In section 4, external force formulations for animating deformable models are briefly explained. Section 5 gives animation examples based on external force approaches. Section 6 gives conclusions and suggestions for future research.

## 2 Methods for Deformable Models using Elasticity Theory

In this section, two different formulations, namely the *primal formulation* and the *hybrid formulation*, are explained. These methods use elasticity theory and differential geometry to model the behavior of deformable models. Elasticity theory provides methods to construct the differential equations that model the behavior of nonrigid curves, surfaces, and solids as a function of time.

There is also a good deal of research for deformable models based on these formulations.



0-8186-7062-2/95 $04.00 © 1995 IEEE

Metaxas and Terzopoulos [4] propose an approach for creating dynamic solid models capable of realistic physical behaviors starting from common solid primitives such as spheres, cylinders, cones, and superquadrics. Such primitives can "deform" kinematically in simple ways. For example, a cylinder deforms as its radius (or height) is changed. To gain additional modeling power they allow the primitives to undergo parameterized global deformations (bends, tapers, twists, shears, etc.). Even though their models' kinematic behavior is stylized by the particular solid primitives used, the models behave in a physically correct way with prescribed mass distributions and elasticities. Metaxas and Terzopoulos also proposed efficient constraint methods for connecting the dynamic primitives together to make articulated models.

In [5], a physically-based model for animating clothes on synthetic actors in motion is described by Carignan et al. This work is based on the Lagrange equations of motion described by Terzopoulos et al. [1] with the damping term replaced by a more accurate one proposed by Platt and Barr [6].

In [7], Tu and Terzopoulos propose a framework for animation that can achieve the intricacy of motion evident in certain natural ecosystems with minimal input from the animator. This work uses a fish model using spring-mass dynamics where the skeleton of the fish is a discrete model composed of point masses and springs between these point masses.

### 2.1 Primal Formulation

In this formulation, a deformable model is formulated by using the material coordinates of points in the body (denoted by $\Omega$). For a solid body $\mathbf{u} = (u_1, u_2, u_3)$, for a surface $\mathbf{u} = (u_1, u_2)$ and for a curve $\mathbf{u} = (u_1)$ denotes the material coordinates. The Euclidean 3-space positions of points in the body are given by time-varying vector-valued function $\mathbf{x}(\mathbf{u}, t) = [x_1(\mathbf{u}, t), x_2(\mathbf{u}, t), x_3(\mathbf{u}, t)]$. The body in its natural rest state is given by $\mathbf{x}^0(\mathbf{u}) = [x_1^0(\mathbf{u}), x_2^0(\mathbf{u}), x_3^0(\mathbf{u})]$ (Fig. 1). The equations of motion for a deformable model can be written in Lagrange's form as (which should hold for all $\mathbf{u}$ in the material domain $\Omega$)

$$\frac{\partial}{\partial t}(\mu\frac{\partial \mathbf{x}}{\partial t}) + \gamma\frac{\partial \mathbf{x}}{\partial t} + \frac{\delta\varepsilon(\mathbf{x})}{\delta \mathbf{x}} = \mathbf{f}(\mathbf{x}, t), \qquad (1)$$

where $\mu(\mathbf{u})$ is the mass density of the body at $\mathbf{u}$, $\gamma(\mathbf{u})$ is the damping density of the body at $\mathbf{u}$, $\mathbf{f}(\mathbf{x}, t)$ is the net externally applied force, and $\varepsilon(\mathbf{x})$ is the energy functional which measures the net instantaneous potential energy of the elastic deformation of the body.

The shape of a body is determined by the Euclidean distances between nearby points. As the body deforms, these distances change. Let $\mathbf{u}$ and $\mathbf{u} + d\mathbf{u}$ denote the material coordinates of two nearby points in the body. The distance between these points in the deformed body in Euclidean 3-space is given by:

$$dl = \sum_{i,j} G_{ij} du_i du_j, \qquad (2)$$

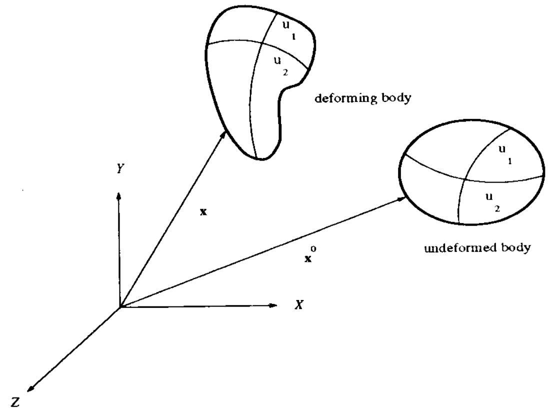



**Figure 1: Geometric representation of a deformable body for primal formulation. Reproduced from [1]. Copyright 1987, Association for Computing Machinery, Inc.**

where the symmetric matrix:

$$G_{ij}(\mathbf{x}(\mathbf{u})) = \frac{\partial \mathbf{x}}{\partial u_i} \cdot \frac{\partial \mathbf{x}}{\partial u_j} \qquad (3)$$

is the metric tensor, which is a measure of deformation (the dot indicates the scalar product of two vectors).

Two 3D solids have the same shape (differ only by a rigid body motion) if their $3 \times 3$ metric tensors are identical forms of $\mathbf{u} = [u_1, u_2, u_3]$, for all $\mathbf{u}$ in the material domain $\Omega$. Two surfaces have the same shape if their metric tensors $\mathbf{G}$ as well as their curvature tensors $\mathbf{B}$ are identical forms of $\mathbf{u} = [u_1, u_2]$, for all $\mathbf{u}$ in the material domain $\Omega$. The components of the curvature tensor are:

$$B_{ij}(\mathbf{x}(\mathbf{u})) = \mathbf{n} \cdot \frac{\partial^2 \mathbf{x}}{\partial u_i \partial u_j}, \qquad (4)$$

where $\mathbf{n} = [n_1, n_2, n_3]$ is the unit surface normal. See [8] for a detailed discussion of these formulations.

Using the above differential quantities, potential energies of deformation for use in Lagrange equations can be defined as the norm of the difference between the fundamental forms of the deformed body and those of the undeformed body. This norm measures the amount of deformation away from the natural shape so that the potential energy is zero when the body is in its natural shape and increases as the model gets increasingly deformed away from its natural shape.

If the fundamental forms associated with the natural shape are denoted by the superscript 0, then the strain energy for a surface can be defined as

$$\varepsilon(\mathbf{x}) = \int_\Omega ||\mathbf{G} - \mathbf{G}^0||^2_{\mathbf{W}^1} + ||\mathbf{B} - \mathbf{B}^0||^2_{\mathbf{W}^2} du_1 du_2, \quad (5)$$

where the weighted matrix norms $|| \cdot ||_{\mathbf{W}^1}$ and $|| \cdot ||_{\mathbf{W}^2}$ involve the weighting functions $w^1_{ij}(u_1, u_2)$ and

$w_{ij}^2(u_1, u_2)$. These weighting functions determine the properties of the simulated deformable material. The weighting function $w_{ij}^1$ shows the amount of resistance to stretching deformation, and the weighting function $w_{ij}^2$ shows the amount of resistance to bending deformation.

This energy denotes the amount of energy to restore the deformed objects to their natural shapes. The net external force in Lagrange's equations is the sum of various types of external forces, such as gravitational force, constraint forces, etc.

To create animation with deformable models using primal formulation, the differential equations of motion should be discretized by applying finite difference approximation methods and solving the system of linked ordinary differential equations of motion obtained in this way. At every step of an animation, a new stiffness matrix that stores the elastic properties of the deformable model is constructed.

To simulate the dynamics of a deformable surface, the elastic force expression, which is an approximation of the derivative of the expression for potential energy, should be discretized. This discretization produces a sparse and banded stiffness matrix. Besides this, the mass and damping matrices for the deformable model should be produced as diagonal matrices.

Then, the total external force for each point of the model should be calculated by adding the forces effecting a point, which are gravitational, collision, constraint forces etc.

### 2.2 Hybrid Formulation

In this formulation, a deformable body is represented as the sum of a reference component $\mathbf{r}(\mathbf{u}, t)$ and a deformation component $\mathbf{e}(\mathbf{u}, t)$ (Fig. 2). The positions of mass elements in the body relative to a body frame $\phi$ (whose origin coincides with the body's center of mass and which should be evolved over time according to the rigid body dynamics to have a rigid body motion besides its elastic motion) is given by

$$\mathbf{q}(\mathbf{u}, t) = \mathbf{r}(\mathbf{u}, t) + \mathbf{e}(\mathbf{u}, t). \tag{6}$$

Deformations are measured with respect to the reference shape $\mathbf{r}$. Elastic deformations are represented by an energy $\varepsilon(\mathbf{e})$, which depends on the position of the body frame $\phi$.

Implementation of the hybrid formulation follows the same steps described for the primal formulation. The only difference is that the sparse, banded stiffness matrix K is constant. The equations of motion can be expressed in semidiscrete form by a system of coupled ordinary differential equations. The system contains two ordinary differential equations for the translational and rotational motion of the model as if all of its mass is concentrated at its center of mass, and a system of ordinary differential equations whose size is proportional to the size of the discrete model. These equations are solved in tandem for each time step with respect to the initial conditions given.

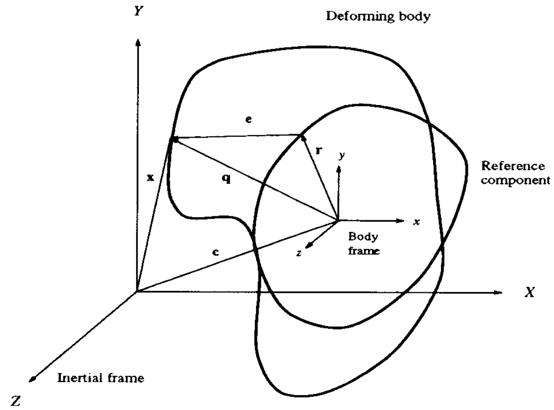



**Figure 2: Geometric representation of deformable models for hybrid formulation. Reproduced from [2]. Copyright 1988, Association for Computing Machinery, Inc.**

## 3 Other Methods for the Animation of Nonrigid Models

The formulations mentioned above employ continuous elasticity theory to model the shapes and motions of deformable models. There are other approaches to model and animate deformable models. In this section, some of these approaches are explained*.

Witkin et al. formulate a model for nonrigid dynamics based on global deformations with relatively few degrees of freedom [10]. This model is restricted to simple linear deformations that can be formulated by affine transformations. The use of deformations that are linear in the state of the system causes the constraint matrices in equations of motion to be constant. Hence, pre-inverting these matrices yields an enormous benefit in performance.

In [11], Pentland and Williams describe the use of ***modal analysis*** to create simplified dynamic models of nonrigid objects. This approach breaks nonrigid dynamics down into the sum of independent vibration modes. This allows Pentland and Williams to achieve a level of control not possible with the massed equations normally used in dynamic simulation. This approach reduces the dimensionality and stiffness of the models by discarding high-frequency modes. High-frequency modes have no effect on linear deformations and rigid body dynamics. Both of these methods achieve large computational savings at the expense of limited deformations.

Another method, based on physics and optimization theory, uses mathematical constraint methods to create realistic animation of flexible models [6]. This method of Platt and Barr uses reaction constraints for fast computation of collisions of flexible models with polygonal models, and augmented Lagrangian constraints for creating animation effects, such as vol-

*A useful bibliography of computer animation can be found in [9].

ume preserving squashing, and the molding of taffy-like substances. To model the flexible objects, the finite element method is used in Platt and Barr's work.

Thingvold and Cohen [12] define a model of elastic and plastic B-spline surfaces which supports both animation and design operations. They develop "refinement" operations for spring and hinge B-spline models which are compatible with the physics and the mathematics of B-spline models. Their model can be viewed as a continuous physical representation of a physical model rather than the more standard discretized geometry point mass models. The motion of their models is controlled by assigning different physical properties and kinematic constraints on various portions of the surface.

In [13], an approach to imposing and solving geometric constraints on parameterized models is given. This approach is applicable to animation as well as model construction. Constraints are expressed as energy functions, and constraint satisfaction is achieved by solving energy minimization problems. Although this approach is not as realistic as the above three approaches because of the lack of physics, it is simple and general.

Breen et al. [14] propose a physically-based model and a simulation methodology, which when used together are able to reproduce many of the attributes of the characteristic behavior of cloth. Their model utilizes a microscopic particle representation that directly treats the mechanical constraints between the threads in a woven material rather than a macroscopic continuum approximation. Their simulation technique is hybrid, employing force methods for gross movement of the cloth and energy methods to enforce constraints within the material. Although limited only to cloth object behavior in scope, their approach is very realistic since a microscopic particle representation is utilized.

There are other physically-based models of flexible objects which are concerned only with the static shape. Weil [15] propose a geometric approach for interpolating surfaces to produce draped "cloth" effects. The clothes synthesized with his model contain folds and appear very realistic. The cloth is assumed to be rectangular, and is represented as a grid of three-dimensional coordinates. Weil uses the catenary curves to define the positioning of the points along a given thread.

Feynman [16] described a technique for modeling the appearance of cloth. His computational framework minimizes energy functions defined over a grid of points. Feynman derives his functions from the theory of elasticity and from the assumption that cloth is a flexible shell.

# 4 External Force Formulations for Deformable Models

In other formulations based on elasticity theory (*primal* [1] and *hybrid* [2] formulations), the elastic properties of the materials are stored in the stiffness matrix. However, the formation of the stiffness matrix automatically is very difficult and sometimes it becomes impossible to solve the differential equations for animating the models because of the numerical ill-conditioning problems. An alternative to these formulations is using external force formulations. In this section, one such type of formulation, namely the external spring-force formulation [3], is explained. In these types of formulations, instead of forming the stiffness matrix automatically, elastic properties are represented as external forces. Although handling the elasticities using the stiffness matrix approach is an elegant and a very suitable way, external force approaches are more effective and very fast.

## 4.1 Spring-Force Formulation for Deformable Models

In this formulation, deformable models are discretized as a grid of control points. The inter-node spacings on the grid are $h_1 = L_h/N$, $h_2 = L_v/M$ in the horizontal and vertical directions, respectively. Initially, we take $h_1 = h_2 = h$, for simplicity.

External forces may be applied to many of the grid points at the same time. One type of such external force can be the gravitational force. These external forces are known. Besides, if some of the grid points are constrained to the fixed positions in space, then there will be some unknown constraint forces at these points.

The line segments in the grid (Fig. 3) will correspond to the spring elements. According to the initial positions of the grid points, there will be some spring forces on the model.

The equations of motion for a deformable model can be written in Lagrange's form as follows (this should hold for all grid points):

$$\mathbf{M}\frac{d^2}{dt^2}\mathbf{x} + \mathbf{C}\frac{d}{dt}\mathbf{x} + \mathbf{K}(\mathbf{x})\,\mathbf{x} = \mathbf{f}(\mathbf{x}) \qquad (7)$$

We can take the elastic force expression as an external force $\mathbf{f}_K = \mathbf{K}(\mathbf{x})\,\mathbf{x}$ , and take $\mathbf{f}_K$ to the right hand side of the Eq. (7). This new form of the equation will simplify the formulation procedure.

The position vector $\mathbf{x}$ for the model points is as follows ($T$ denotes the transpose of a matrix):

$$\mathbf{x}^T = [\, \mathbf{x}_0^T \ \mathbf{x}_1^T \ \cdots \ \mathbf{x}_M^T \,] \qquad (8)$$

where $\mathbf{x}_i$ represents all the position vectors of the grid points on the $i$-th row, and

$$\mathbf{x}_i^T = [\, \mathbf{x}_{i,0}^T \, \mathbf{x}_{i,1}^T \ \cdots \ \mathbf{x}_{i,N}^T \,] \qquad (9)$$

where $\mathbf{x}_{i,j}$ is the position vector of the grid point $(i, j)$ $(i = 0, 1, \cdots, M\,;\, j = 0, 1, \cdots, N)$.

In Eq. (7), $\mathbf{M}$ is the *mass matrix*, an $(M+1)(N+1) \times (M+1)(N+1)$ diagonal matrix which contains masses of the grid points as diagonal elements, and $\mathbf{C}$ is the *damping matrix*, an $(M+1)(N+1) \times (M+1)(N+1)$ diagonal matrix which contains dampers of the grid points as diagonal elements.

Note that Eq. (7) can be rewritten as

$$\mathbf{M}\frac{d^2}{dt^2}\mathbf{x} + \mathbf{C}\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x}) - \mathbf{f}_K(\mathbf{x}) \qquad (10)$$

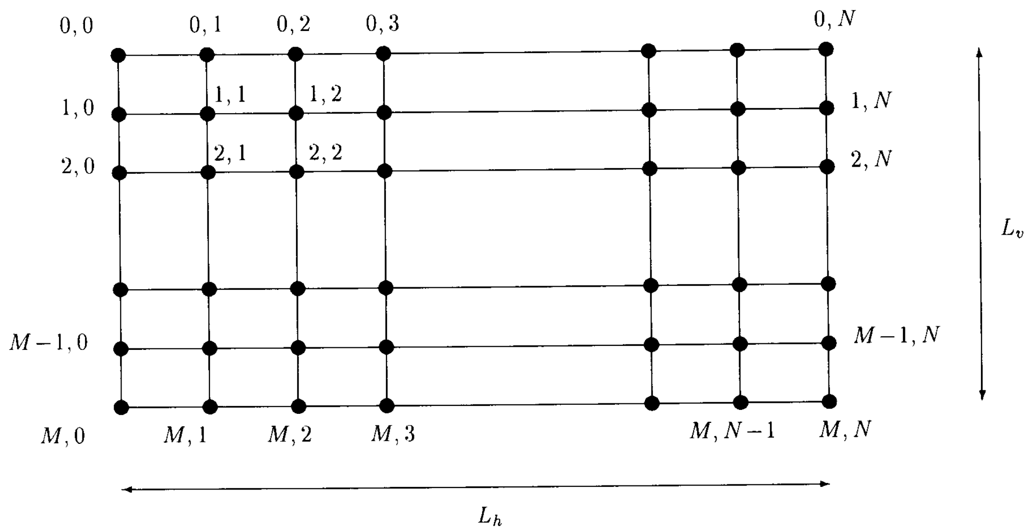



**Figure 3: Numbering of the grid.**

In this way, there will be no need for calculating the entries of the stiffness matrix. Instead of this, it is necessary to find the expressions for the column matrix $\mathbf{f}_K^T$ (external spring forces representing elasticities). The spring force vector can also be partitioned as

$$\mathbf{f}_K^T = [\mathbf{f}_0^T \; \mathbf{f}_1^T \cdots \mathbf{f}_M^T] \qquad (11)$$

where the entries in the vector $\mathbf{f}_i^T = [\mathbf{f}_{i,0}^T \; \mathbf{f}_{i,1}^T \cdots \mathbf{f}_{i,N}^T]$ correspond to the spring forces acting at the grid points.

Using the discussion in [17] (pp. 359–362), the terminal equation of a two-terminal spring component of free length $\ell$ in three-dimensional space is given as

$$\mathbf{f}_K = k \left[ (\mathbf{x}_1 - \mathbf{x}_2) - \ell \frac{\mathbf{x}_1 - \mathbf{x}_2}{||\mathbf{x}_1 - \mathbf{x}_2||} \right] \qquad (12)$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the position vectors of its terminal points. Note that calculation of the vector $(\mathbf{x}_1 - \mathbf{x}_2)$ is essential; it also appears in the second term of this expression. Eq. (12) can be used to obtain expressions for the entries of $\mathbf{f}_K$ in (11).

For the grid points not on the boundaries, the elastic force is calculated by adding the spring forces applied to the grid point by its four neighbors. For the grid points on the boundaries, three neighbors have an effect on the elastic force. For the grid points on the corners, only two neighbors have an effect on the elastic force. See [3] for the details of calculating external spring forces.

## 4.2 Implementation of the Spring Force Formulation

- Since the initial position vectors of the grid points are known, the external force vector $\mathbf{f}_K$ can be calculated from the external force equations.
- Then by solving the differential equation in Eq. (10) at the first step, next values of the position vectors of the grid points are determined.
- The next value of the external force vector $\mathbf{f}_K$ is calculated and the process is repeated.

As initial positions, we have $h \neq \ell$ in general. Therefore $\mathbf{f}_K \neq \underline{0}$. In other words, there will be some internal stresses in the system. If $h = \ell$, then $\mathbf{f}_K \equiv \underline{0}$. On the other hand, if $h_1 \neq h_2$, then $\mathbf{f}_K \neq \underline{0}$ initially (assuming that all the springs have the same lengths). We may select the lengths of the horizontal springs as $\ell_1 = h_1$ and the lengths of the vertical springs as $\ell_2 = h_2$. In this case, $\mathbf{f}_K = \underline{0}$ initially, and some of the $\ell$ factors will change to $\ell_1$ and the remaining ones to $\ell_2$ in the external spring force equations. Other modifications are also possible; e.g., on the spring coefficients ($k$).

# 5 Animation Examples Using External Force Formulations

In the external force formulations, by setting the stiffness constants to different values it is possible to obtain different elastic properties. In Fig. 4, a surface is assigned different elastic properties and constrained from its center of mass. Each part of the figure shows the form of the surface after a specific number of animation frames. Initially, the surface is flat.

In Fig. 5, a strechy sheet constrained from its four corners falls with the effect of gravity. In Fig. 6, an elastic surface drops over a toroid with a very small hole. These examples are produced by the animation system described in [3, 18].

Any point on a model could be constrained to a fixed location in space so that when the model is animated, the constrained points remain in their initial positions. The constraint forces are taken into account in the following way. When a constrained point tends to move, an opposite force for bringing it back to its original position is calculated and added to the total external force for that point. Each constrained point has an effect on the total external force for all points in the model depending on the difference between the body coordinates of the points. This coupling effect is taken into account automatically according to the elastic properties of the models.

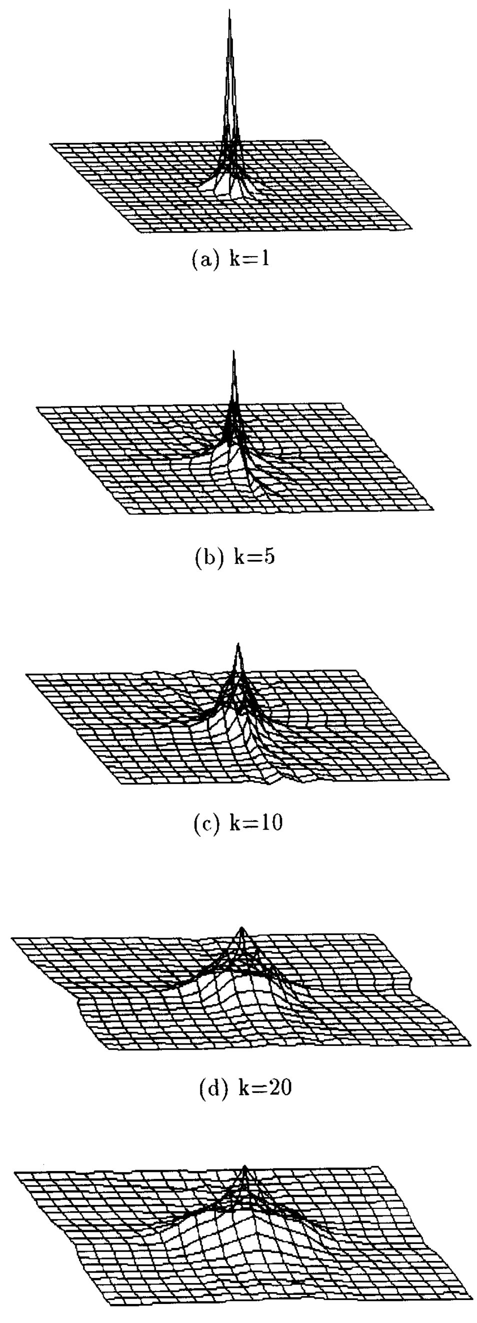
The forces due to the collision of deformable models with impenetrable obstacles are calculated using the obstacle's implicit (inside-outside) function. The obstacle exerts a repulsive force on the deformable model which can be calculated as a function of the obstacle's implicit function such that the force grows quickly if the model attempts to penetrate the obstacle. This is achieved by creating a potential energy function around each obstacle. Then, the repulsive force due to an impenetrable obstacle could be defined using the surface normal vector and the implicit function of of the deformable body's surface.

## 6 Conclusion

This paper discusses different approaches for animating deformable models. Two formulations, namely the primal and the hybrid formulations, employ elasticity theory to model the behavior of deformable models. The nonquadric energy functional in primal formulation causes a nonlinear elastic force associated with the deformable body to appear in the partial differential equations of motion. Nonlinearity results because the elastic force attempts to restore the shape of the deformed body to a rest shape. The advantage of nonlinear elasticity is that it is in principle the most accurate way to characterize the behavior of certain elastic phenomena. Because of this, the primal formulation is the most suitable formulation for highly nonrigid models. However, it can lead to serious practical difficulties in the numerical implementation of deformable models for animation since it is very difficult to form the stiffness matrix automatically.

The hybrid formulation offers a practical advantage for fairly rigid models (the stiffness matrix is almost constant), whereas primal formulation becomes unpractical due to the nonquadric energy functional with increasing rigidity and complexity of the models.

An important advantage of the primal formulation over other formulations is that it is easier to establish an intuitive link between the weighting functions of the deformable models and the resulting elastic behavior.



(a) k=1

(b) k=5

(c) k=10

(d) k=20

(e) k=30

**Figure 4: Different elastic surfaces, constrained from the center of mass, fall.**

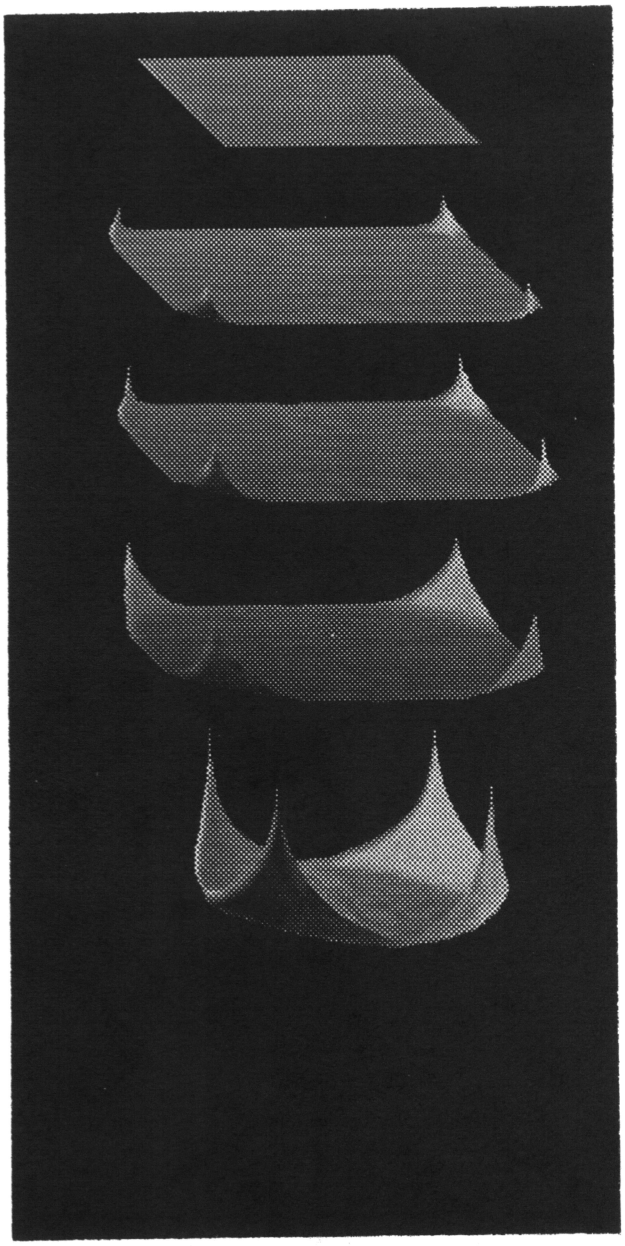


Figure 5: A strechy sheet, constrained from its four corners, falls.

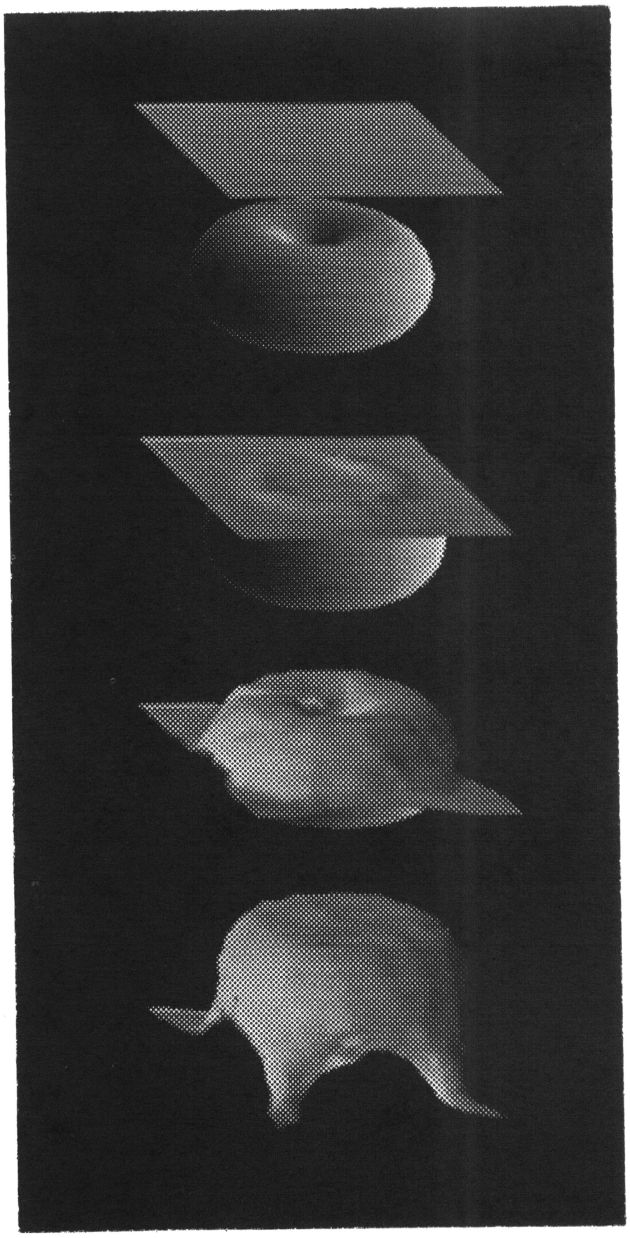


Figure 6: An elastic surface drops over a toroid with a small hole.

This is due to the nature of the weighting functions.

The advantages of the external force formulations are

- The elastic properties of the materials are represented as external forces, instead of using the stiffness matrix approach. In this way, the problem of automatically constructing the stiffness matrix is avoided.
- Since the stiffness matrix is not formed, models could be animated faster than the other approaches. The linear system of equations that should be solved to compute animation frames contains only mass and damping values which are the diagonal entries. This allows us to use simple linear system solving methods.
- The elastic properties of the materials could be given by setting the spring constants to proper values.
- Since such formulation model a deformable object using a finite number of grid points, it is possible to give different elastic properties to different parts of a model.

External force formulations will thus promise to be more suitable to fast animation of deformable models. The accuracy and realism needs more refinement and development.

## References


[1] D. Terzopoulos, J. Platt, A. H. Barr, and K. Fleischer "Elastically Deformable Models," *ACM Computer Graphics (Proc. SIGGRAPH'87)*, Vol 21, No 4, pp. 205–214, July 1987.

[2] D. Terzopoulos and K. Fleischer "Modeling Inelastic Deformation: Viscoelasticity, Plasticity, Fracture," *ACM Computer Graphics (Proc. SIGGRAPH'88)*, Vol 22, No 4, pp. 269–278, August 1988.

[3] U. Güdükbay. *Physically-Based Animation of Elastically Deformable Models.* PhD thesis, Department of Computer Engineering and Information Science, Bilkent University, Ankara, Turkey, 1994.

[4] D. Metaxas and D. Terzopoulos "Dynamic Deformation of Solid Primitives with Constraints," *ACM Computer Graphics (Proc. SIGGRAPH'92)*, Vol 26, No 2, pp. 309–312, July 1992.

[5] M. Carignan, Y. Yang, N. M. Thalmann, and D. Thalmann "Dressing Animated Synthetic Actors with Complex Deformable Clothes," *ACM Computer Graphics (Proc. SIGGRAPH'92)*, Vol 26, No 2, pp. 99–104, July 1992.

[6] J. Platt and A. H. Barr "Constraint Methods for Flexible Models," *ACM Computer Graphics (Proc. SIGGRAPH'88)*, Vol 22, No 4, pp. 279–288, August 1988.

[7] X. Tu and T. Terzopoulos "Artificial Fishes: Physics, Locomotion, Perception, Behavior," *ACM Computer Graphics–Annual Conference Series*, pp. 43–50, July 1994.

[8] M. P. Do Carmo. *Differential Geometry of Curves and Surfaces.* Prentice-Hall, Englewood Cliffs, NJ, 1974.

[9] N. M. Thalmann and D. Thalmann "Six-hundred Indexed References on Computer Animation," *The Journal of Visualization and Computer Animation*, Vol 3, No 3, pp. 147–174, July-September 1992.

[10] A. Witkin, M. Gleischer, and W. Welch "Interactive Dynamics," *ACM Computer Graphics (Proc. SIGGRAPH'90)*, Vol 24, No 4, pp. 11–22, August 1990.

[11] A. Pentland and J. Williams "Good Vibrations: Modal Dynamics for Graphics and Animation," *ACM Computer Graphics (Proc. SIGGRAPH'89)*, Vol 23, No 3, pp. 215–222, July 1989.

[12] J. A. Thingvold and E. Cohen "Physical Modeling with B-spline Surfaces for Interactive Design and Animation," *ACM Computer Graphics (Proc. SIGGRAPH'90)*, Vol 24, No 4, pp. 129–137, August 1990.

[13] A. Witkin, K. Fleischer, and A. H. Barr "Energy Constraints on Parameterized Models," *ACM Computer Graphics (Proc. SIGGRAPH'87)*, Vol 21, No 4, pp. 225–232, July 1987.

[14] D. E. Breen, D. H. House, and P. H. Getto "Physically-Based Particle Model of Woven Cloth," *The Visual Computer*, Vol 8, No 5–6, pp. 264–277, June 1992.

[15] J. Weil "The Synthesis of Cloth Objects," *ACM Computer Graphics (Proc. SIGGRAPH'86)*, Vol 20, No 4, pp. 49–54, August 1986.

[16] C. R. Feynman. "Modeling the Appearance of Cloth,". Master's thesis, Department of Electrical Engineering and Computer Science, MIT, Cambridge, MA, 1986.

[17] Y. Tokad. *Analysis of Engineering Systems — Part III.* Bilkent University, Ankara, Turkey, 1990.

[18] U. Güdükbay, B. Özgüç, and Y. Tokad "An Animation System for Rigid and Deformable Models," *Computers and Graphics*, Vol 17, No 1, pp. 71–77, January-February 1993.

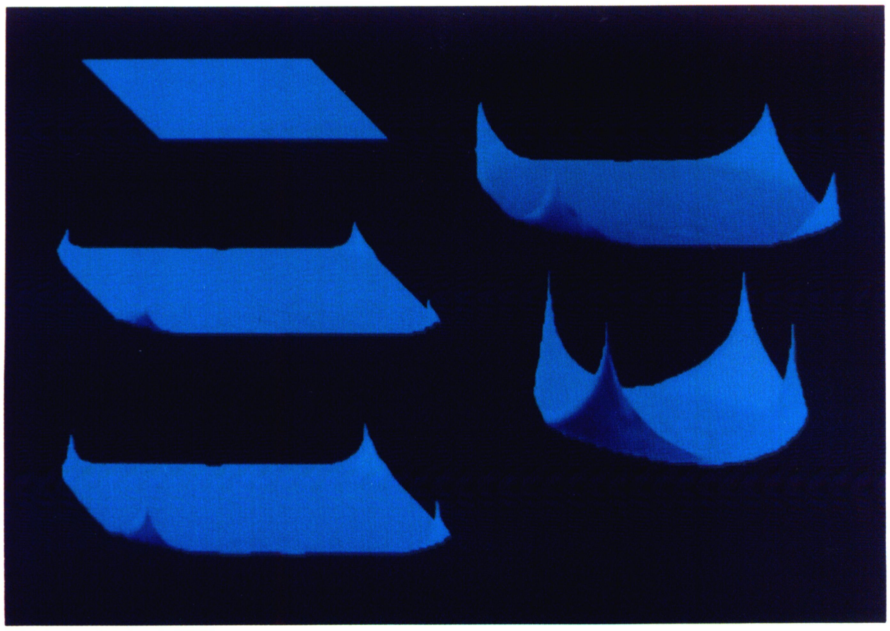


Figure 5. A stretchy sheet, constrained from its four corners, falls.

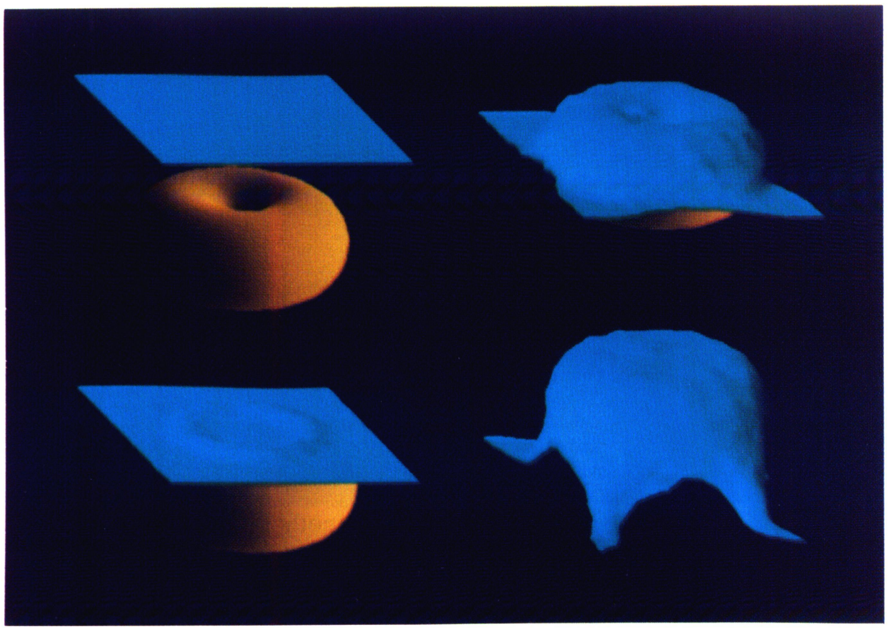


Figure 6. An elastic surface drops over a toroid with a small hole.