telecom $n = 20,491$ $nz = 101,041$

Preconditioner	nzlu	Time	MFlops
ILU0	101,041	0.3	0.1
ILUTH (10^{-2})	82,521	1.3	0.3
ILUTH (10^{-3})	181,126	1.5	0.7
ILUK (10)	204,807	5.3	3.2

Method	Preconditioner	Time	# it	$\|\|r\|\|$	$\|\|Ax\|\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	197.1	500*	$0.25e-05$	$0.75e-05$	$0.89e-04$
	ILUTH (10^{-2})	190.5	500*	$0.61e-06$	$0.70e-06$	$0.69e-04$
	ILUTH (10^{-3})	224.2	500*	$0.13e-05$	$0.12e-05$	$0.26e-05$
	ILUK (10)	231.5	500*	$0.43e-06$	$0.88e-06$	$0.25e-05$
BCG	ILU0	240.1	500*	$0.79e+04$	$0.17e-03$	$0.13e-01$
	ILUTH (10^{-2})	223.1	500*	$0.11e-02$	$0.37e-04$	$0.23e-03$
	ILUTH (10^{-3})	307.3	500*	$0.36e+01$	$0.11e-03$	$0.15e-03$
	ILUK (10)	328.9	500*	$0.19e+00$	$0.12e-03$	$0.28e-02$
CGS	ILU0	236.0	500*	$0.40e+11$	$0.34e-03$	$0.10e-01$
	ILUTH (10^{-2})	218.8	500*	$0.34e-03$	$0.72e-04$	$0.39e-03$
	ILUTH (10^{-3})	26.9	44	$0.97e-10$	$0.16e-09$	$0.16e-09$
	ILUK (10)	321.3	500*	$0.13e+16$	$0.26e-03$	$0.18e-01$
BCGStab	ILU0	241.6	500*	$0.32e-02$	$0.20e-04$	$0.79e-03$
	ILUTH (10^{-2})	221.9	496	$0.79e-10$	$0.65e-10$	$0.61e-10$
	ILUTH (10^{-3})	44.4	72	$0.26e-10^{s}$	$0.10e-10$	$0.99e-11$
	ILUK (10)	328.8	500*	$0.14e-06$	$0.76e-07$	$0.93e-07$
QMR 2	ILU0	299.4	500*	$0.39e-05$	$0.55e-05$	$0.27e-03$
	ILUTH (10^{-2})	282.6	500*	$0.85e-05$	$0.57e-05$	$0.14e-04$
	ILUTH (10^{-3})	368.6	500*	$0.76e-07$	$0.30e-07$	$0.29e-07$
	ILUK (10)	389.5	500*	$0.15e-07$	$0.71e-09$	$0.67e-09$
QMR 3	ILU0	305.3	500*	$0.46e-05$	$0.68e-05$	$0.16e-03$
	ILUTH (10^{-2})	288.3	500*	$0.64e-05$	$0.71e-05$	$0.77e-02$
	ILUTH (10^{-3})	374.9	500*	$0.31e-05$	$0.38e-05$	$0.51e-05$
	ILUK (10)	393.9	500*	$0.13e-05$	$0.40e-05$	$0.45e-04$

SOR

ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.
1.0	$0.63e-04$	122.9	1,000*	$0.11e-05$	$0.12e-05$

Block SOR

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-2}$	1.6	$0.86e-10$	55.6	303	$0.38e-12$	$0.36e-12$	1,337
$\gamma = 10^{-1}$	1.2	$0.49e-08$	253.2	956	$0.18e-08$	$0.17e-08$	14,139
equal	1.4	$0.17e-08$	99.0	185	$0.15e-10$	$0.14e-10$	0
other	1.6	$0.64e-10$	221.4	465	$0.10e-12$	$0.92e-13$	2

IAD

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-2}$	1.0	$0.23e-13$	3.1	1	$0.18e-15$	$0.17e-15$	1,337
$\gamma = 10^{-1}$	1.0	$0.37e-14$	60.3	1	$0.65e-17$	$0.61e-17$	14,139
equal	1.0	$0.93e-10$	10.4	7	$0.22e-12$	$0.21e-12$	0
other	1.3	$0.91e-10$	16.7	21	$0.11e-12$	$0.99e-13$	2

Table A.18: Numerical Results for *telecom*.

telecom_gm2 $n = 20,491$ $nz = 101,041$

Preconditioner	nzlu	Time	MFlops
ILU0	101,041	0.3	0.1
ILUTH (10^{-2})	183,997	3.0	0.6
ILUTH (10^{-3})	375,622	3.6	1.5
ILUK (10)	199,053	7.5	3.2

Method	Preconditioner	Time	# it	$\|r\|$	$\|Ax\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	109.3	500*	$0.11e-04$	$0.38e-06$	$0.38e-06$
	ILUTH (10^{-2})	214.5	500*	$0.60e-06$	$0.78e-06$	$0.73e-04$
	ILUTH (10^{-3})	278.7	500*	$0.25e-05$	$0.50e-06$	$0.69e-06$
	ILUK (10)	222.3	500*	$0.41e-05$	$0.38e-06$	$0.41e-06$
BCG	ILU0	234.7	500*	$0.45e+07$	$0.13e-03$	$0.12e-01$
	ILUTH (10^{-2})	298.4	500*	$0.24e-02$	$0.51e-03$	$0.12e-02$
	ILUTH (10^{-3})	450.3	500*	$0.56e-03$	$0.15e-03$	$0.30e-03$
	ILUK (10)	314.4	500*	$0.14e+03$	$0.33e-03$	$0.11e-01$
CGS	ILU0	231.9	500*	$0.55e+15$	$0.19e-03$	$0.85e-02$
	ILUTH (10^{-2})	291.2	500*	$0.12e+00$	$0.45e-04$	$0.54e-03$
	ILUTH (10^{-3})	36.1	41	$0.43e-10$	$0.95e-11$	$0.90e-11$
	ILUK (10)	308.7	500*	$0.35e-04$	$0.52e-06$	$0.50e-06$
BCGStab	ILU0	212.5	448	$0.13e-10^{8}$	$0.11e-11$	$0.10e-10$
	ILUTH (10^{-2})	387.6	500*	$0.17e-05$	$0.14e-05$	$0.10e-04$
	ILUTH (10^{-3})	55.0	62	$0.76e-10$	$0.88e-11$	$0.83e-11$
	ILUK (10)	206.6	330	$0.26e-10^{8}$	$0.23e-12$	$0.22e-12$
QMR 2	ILU0	293.5	500*	$0.26e-03$	$0.20e-04$	$0.25e-04$
	ILUTH (10^{-2})	356.1	500*	$0.62e-05$	$0.63e-05$	$0.10e-04$
	ILUTH (10^{-3})	509.2	500*	$0.64e-07$	$0.63e-08$	$0.60e-08$
	ILUK (10)	373.6	500*	$0.12e-03$	$0.80e-05$	$0.11e-04$
QMR 3	ILU0	302.8	500*	$0.20e-04$	$0.99e-05$	$0.23e-04$
	ILUTH (10^{-2})	363.7	500*	$0.33e-05$	$0.43e-05$	$0.14e-03$
	ILUTH (10^{-3})	515.0	500*	$0.11e-08$	$0.23e-09$	$0.22e-09$
	ILUK (10)	382.2	500*	$0.39e-04$	$0.35e-06$	$0.33e-06$

SOR

ω	$\|\Delta x\|$	Time	# it	$\|Ax\|$	Bk. Err.
1.0	$0.77e-04$	114.5	1,000*	$0.12e-05$	$0.14e-05$

Block SOR

Partition.	ω	$\|\Delta x\|$	Time	# it	$\|Ax\|$	Bk. Err.	# Bl.
$\gamma = 10^{-2}$	1.6	$0.86e-10$	51.7	303	$0.38e-12$	$0.36e-12$	1,337
$\gamma = 10^{-1}$	1.2	$0.45e-06$	246.0	1,000*	$0.14e-07$	$0.13e-07$	14,139
equal	1.3	$0.75e-10$	41.1	47	$0.70e-12$	$0.66e-12$	0
other	1.4	$0.84e-10$	103.9	154	$0.32e-12$	$0.31e-12$	2

IAD

Partition.	ω	$\|\Delta x\|$	Time	# it	$\|Ax\|$	Bk. Err.	# Bl.
$\gamma = 10^{-2}$	1.0	$0.23e-13$	2.9	1	$0.18e-15$	$0.17e-15$	1,337
$\gamma = 10^{-1}$	1.0	$0.39e-14$	41.9	1	$0.67e-17$	$0.63e-17$	14,139
equal	1.3	$0.78e-10$	20.9	16	$0.82e-13$	$0.77e-13$	0
other	1.1	$0.18e-10$	15.7	11	$0.79e-13$	$0.75e-13$	2

Table A.19: Numerical Results for *telecom_gm2*.

telecom_gm1 $n = 20,491$ $nz = 101,041$

Preconditioner	nzlu	Time	MFlops
ILU0	101,041	0.3	0.1
ILUTH (10^{-2})	212,153	4.1	0.6
ILUTH (10^{-3})	362,354	5.1	1.3
ILUK (10)	204,386	16.0	8.7

Method	Preconditioner	Time	# it	$\|\|r\|\|$	$\|\|Ax\|\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	189.4	500*	$0.46e-05$	$0.43e-05$	$0.18e-03$
	ILUTH (10^{-2})	224.3	500*	$0.80e-06$	$0.71e-06$	$0.49e-04$
	ILUTH (10^{-3})	152.7	280	$0.17e-10$	$0.28e-11$	$0.26e-11$
	ILUK (10)	223.8	500*	$0.80e-05$	$0.12e-06$	$0.12e-06$
BCG	ILU0	230.3	500*	$0.14e+04$	$0.63e-03$	$0.15e-01$
	ILUTH (10^{-2})	319.7	500*	$0.12e+02$	$0.89e-03$	$0.32e-02$
	ILUTH (10^{-3})	435.1	500*	$0.86e-02$	$0.11e-03$	$0.15e-03$
	ILUK (10)	317.9	500*	$0.10e+01$	$0.39e-04$	$0.40e-04$
CGS	ILU0	234.5	500*	$0.73e+12$	$0.20e-03$	$0.79e-02$
	ILUTH (10^{-2})	161.5	252	$0.77e-10$	$0.35e-10$	$0.33e-10$
	ILUTH (10^{-3})	31.4	36	$0.82e-10$	$0.21e-10$	$0.20e-10$
	ILUK (10)	317.9	500*	$0.59e+10$	$0.20e-04$	$0.21e-04$
BCGStab	ILU0	233.6	500*	$0.86e-05$	$0.46e-05$	$0.57e-05$
	ILUTH (10^{-2})	320.0	500*	$0.47e-05$	$0.48e-05$	$0.13e-04$
	ILUTH (10^{-3})	40.5	47	$0.61e-10^{s}$	$0.69e-11$	$0.65e-11$
	ILUK (10)	166.9	261	$0.98e-10$	$0.66e-13$	$0.62e-13$
QMR 2	ILU0	290.7	500*	$0.28e-04$	$0.24e-04$	$0.41e-02$
	ILUTH (10^{-2})	379.5	500*	$0.83e-05$	$0.57e-05$	$0.50e-04$
	ILUTH (10^{-3})	497.8	500*	$0.43e-08$	$0.10e-08$	$0.98e-09$
	ILUK (10)	376.1	500*	$0.16e-07$	$0.18e-10$	$0.17e-10$
QMR 3	ILU0	300.6	500*	$0.18e-04$	$0.12e-04$	$0.38e-03$
	ILUTH (10^{-2})	386.9	500*	$0.71e-06$	$0.42e-06$	$0.39e-06$
	ILUTH (10^{-3})	502.8	500*	$0.31e-08$	$0.83e-09$	$0.78e-09$
	ILUK (10)	387.1	500*	$0.31e-05$	$0.76e-08$	$0.71e-08$

SOR

ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.
1.0	$0.13e-03$	112.0	1,000*	$0.12e-05$	$0.19e-05$

Block SOR

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-2}$	1.6	$0.86e-10$	51.6	303	$0.38e-12$	$0.36e-12$	1,337
$\gamma = 10^{-1}$	1.2	$0.49e-08$	232.2	956	$0.18e-08$	$0.17e-08$	14,139
equal	1.5	$0.18e-10$	152.2	238	$0.19e-12$	$0.18e-12$	0
other	1.7	$0.87e-10$	213.4	370	$0.31e-12$	$0.29e-12$	2

IAD

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-2}$	1.0	$0.23e-13$	2.7	1	$0.18e-15$	$0.17e-15$	1,337
$\gamma = 10^{-1}$	1.0	$0.37e-14$	56.8	1	$0.65e-17$	$0.61e-17$	14,139
equal	1.5	$0.73e-10$	21.8	19	$0.10e-11$	$0.98e-12$	0
other	1.3	$0.99e-10$	19.3	18	$0.24e-12$	$0.22e-12$	2

Table A.20: Numerical Results for *telecom_gm1*.

qnatm $n = 104,525$ $nz = 593,115$

Preconditioner	nzlu	Time	MFlops
ILU0	593,115	2.0	0.5
ILUTH (10^{-2})	1,020,335	798.4	6.6
ILUTH (10^{-3})	1,073,171	1,064.9	11.0
ILUK (10)	1,046,092	952.4	12.8

Method	Preconditioner	Time	# it	$\|\|r\|\|$	$\|\|Ax\|\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	1,117.0	500*	$0.27e-04$	$0.18e-03$	$0.14e-02$
	ILUTH (10^{-2})	204.3	80	$0.30e-10$	$0.20e-10$	$0.62e-10$
	ILUTH (10^{-3})	207.0	80	$0.47e-10$	$0.32e-10$	$0.95e-10$
	ILUK (10)	130.0	40	$0.39e-10$	$0.19e-10$	$0.56e-10$
BCG	ILU0	1,395.0	500*	$0.27e-01$	$0.43e-02$	$0.11e+00$
	ILUTH (10^{-2})	1,759.0	500*	$0.20e+01$	$0.22e-02$	$0.27e+00$
	ILUTH (10^{-3})	1,787.0	500*	$0.73e-06$	$0.64e-06$	$0.19e-05$
	ILUK (10)	1,768.0	500*	$0.20e-01$	$0.10e-02$	$0.16e+00$
CGS	ILU0	105.2	38	$0.33e-10$	$0.22e-10$	$0.66e-10$
	ILUTH (10^{-2})	87.7	25	$0.21e-10$	$0.14e-10$	$0.43e-10$
	ILUTH (10^{-3})	86.1	24	$0.64e-10$	$0.45e-10$	$0.14e-09$
	ILUK (10)	81.6	23	$0.54e-11$	$0.26e-11$	$0.79e-11$
BCGStab	ILU0	125.3	45	$0.78e-10^{s}$	$0.52e-10$	$0.16e-09$
	ILUTH (10^{-2})	87.1	25	$0.17e-10^{s}$	$0.11e-10$	$0.34e-10$
	ILUTH (10^{-3})	87.3	24	$0.80e-10$	$0.54e-10$	$0.16e-09$
	ILUK (10)	86.4	24	$0.93e-11$	$0.45e-11$	$0.14e-10$
QMR 2	ILU0	1,706.0	500*	$0.32e-08$	$0.60e-08$	$0.18e-07$
	ILUTH (10^{-2})	2,068.0	500*	$0.38e-08$	$0.36e-08$	$0.11e-07$
	ILUTH (10^{-3})	2,109.0	500*	$0.87e-09$	$0.58e-09$	$0.18e-08$
	ILUK (10)	2,095.0	500*	$0.18e-08$	$0.17e-08$	$0.52e-08$
QMR 3	ILU0	1,731.0	500*	$0.21e-07$	$0.14e-07$	$0.43e-07$
	ILUTH (10^{-2})	2,092.1	500*	$0.24e-07$	$0.16e-07$	$0.48e-07$
	ILUTH (10^{-3})	2,130.0	500*	$0.10e-07$	$0.70e-08$	$0.21e-07$
	ILUK (10)	2,121.0	500*	$0.49e-07$	$0.28e-07$	$0.86e-07$

SOR

ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.
1.2	$0.22e-09$	93.4	134	$0.24e-10$	$0.73e-10$

Block SOR

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-1}$	1.1	$0.66e-10$	76.6	44	$0.32e-11$	$0.98e-11$	91,350
equal	1.2	$0.90e-10$	111.6	113	$0.22e-10$	$0.66e-10$	0
other	1.2	$0.86e-10$	130.1	136	$0.24e-10$	$0.73e-10$	2

IAD

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-1}$	1.0	requires unreasonably long time (coupling matrix very large)					
equal	1.2	$0.89e-10$	112.7	58	$0.78e-11$	$0.23e-10$	0
other	1.2	$0.93e-10$	110.5	57	$0.13e-10$	$0.38e-10$	2

Table A.21: Numerical Results for *qnatm*.

qnatm_gm1 $n = 104,625$ $nz = 593,115$

Preconditioner	nzlu	Time	MFlops
ILU0	593,115	2.0	0.5
ILUTH (10^{-2})	950,290	1,679.4	4.9
ILUTH (10^{-3})	1,005,767	2,308.0	8.3
ILUK (10)	1,045,181	2,262.6	61.3

Method	Preconditioner	Time	# it	$\|\|r\|\|$	$\|\|Ax\|\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	88.9	40	$0.27e-10$	$0.56e-10$	$0.17e-09$
	ILUTH (10^{-2})	73.9	31	$0.12e-11$	$0.89e-12$	$0.26e-11$
	ILUTH (10^{-3})	74.9	31	$0.20e-11$	$0.11e-11$	$0.34e-11$
	ILUK (10)	51.3	20	$0.39e-10$	$0.30e-10$	$0.92e-10$
BCG	ILU0	1,389.0	500*	$0.62e-01$	$0.11e-02$	$0.78e-01$
	ILUTH (10^{-2})	1,697.0	500*	$0.46e-03$	$0.24e-03$	$0.11e-02$
	ILUTH (10^{-3})	1,741.0	500*	$0.23e-01$	$0.63e-03$	$0.37e-01$
	ILUK (10)	1,787.0	500*	$0.66e-02$	$0.15e-02$	$0.72e-01$
CGS	ILU0	61.6	22	$0.43e-11$	$0.80e-11$	$0.24e-10$
	ILUTH (10^{-2})	51.6	15	$0.94e-10$	$0.69e-10$	$0.21e-09$
	ILUTH (10^{-3})	49.3	14	$0.89e-10$	$0.46e-10$	$0.14e-09$
	ILUK (10)	50.5	14	$0.45e-11$	$0.78e-11$	$0.24e-10$
BCGStab	ILU0	69.6	25	$0.43e-10^{s}$	$0.80e-10$	$0.24e-09$
	ILUTH (10^{-2})	52.3	15	$0.37e-10$	$0.27e-10$	$0.82e-10$
	ILUTH (10^{-3})	53.6	15	$0.96e-10$	$0.54e-10$	$0.16e-09$
	ILUK (10)	47.7	13	$0.70e-10$	$0.66e-10$	$0.20e-09$
QMR 2	ILU0	1,713.0	500*	$0.21e-08$	$0.38e-08$	$0.12e-07$
	ILUTH (10^{-2})	2,025.0	500*	$0.42e-09$	$0.31e-09$	$0.93e-09$
	ILUTH (10^{-3})	2,065.0	500*	$0.40e-09$	$0.22e-09$	$0.68e-09$
	ILUK (10)	2,106.0	500*	$0.42e-09$	$0.93e-09$	$0.12e-08$
QMR 3	ILU0	1,732.0	500*	$0.28e-08$	$0.51e-08$	$0.16e-07$
	ILUTH (10^{-2})	2,041.0	500*	$0.32e-08$	$0.23e-08$	$0.70e-08$
	ILUTH (10^{-3})	2,082.0	500*	$0.25e-09$	$0.14e-09$	$0.41e-09$
	ILUK (10)	2,125.0	500*	$0.52e-08$	$0.38e-08$	$0.12e-07$

SOR

ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.
1.2	$0.26e-09$	58.9	84	$0.32e-11$	$0.98e-11$

Block SOR

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-1}$	1.1	$0.66e-10$	71.7	44	$0.32e-11$	$0.98e-11$	91,350
equal	1.3	$0.60e-10$	44.4	35	$0.87e-11$	$0.26e-10$	0
other	1.3	$0.74e-10$	43.5	34	$0.20e-11$	$0.59e-11$	2

IAD

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-1}$	1.0	requires unreasonably long time (coupling matrix very large)					
equal	1.2	$0.87e-10$	90.4	35	$0.37e-11$	$0.11e-10$	0
other	1.3	$0.45e-10$	93.8	36	$0.69e-11$	$0.21e-10$	2

Table A.22: Numerical Results for *qnatm_gm1*.

mutex $n = 39,203$ $nz = 563,491$

Preconditioner	nzlu	Time	MFlops
ILU0	563,491	2.5	0.6
ILUTH (10^{-2})	301,347	814.8	0.9
ILUTH (10^{-3})	496,693	876.9	3.0
ILUK (10)	392,037	1,215.4	15.8

Method	Preconditioner	Time	# it	$\|\|r\|\|$	$\|\|Ax\|\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	11.8	10	$0.27e-11$	$0.24e-11$	$0.30e-11$
	ILUTH (10^{-2})	9.6	10	$0.35e-11$	$0.77e-11$	$0.93e-11$
	ILUTH (10^{-3})	13.7	12	$0.27e-11$	$0.27e-11$	$0.33e-11$
	ILUK (10)	15.9	15	$0.54e-11$	$0.80e-11$	$0.58e-11$
BCG	ILU0	1,030.0	500*	$0.95e-02$	$0.49e-02$	$0.11e-01$
	ILUTH (10^{-2})	775.1	500*	$0.32e-01$	$0.61e-02$	$0.84e-01$
	ILUTH (10^{-3})	972.9	500*	$0.82e-05$	$0.82e-05$	$0.99e-05$
	ILUK (10)	859.0	500*	$0.28e+00$	$0.13e-01$	$0.43e-01$
CGS	ILU0	9.8	5	$0.58e-10$	$0.52e-10$	$0.63e-10$
	ILUTH (10^{-2})	7.9	5	$0.15e-10$	$0.32e-10$	$0.39e-10$
	ILUTH (10^{-3})	11.4	6	$0.93e-10$	$0.93e-10$	$0.11e-09$
	ILUK (10)	13.4	8	$0.50e-10$	$0.44e-10$	$0.54e-10$
BCGStab	ILU0	9.9	5	$0.37e-10$	$0.33e-10$	$0.40e-10$
	ILUTH (10^{-2})	7.2	5	$0.92e-10^{s}$	$0.20e-09$	$0.25e-09$
	ILUTH (10^{-3})	11.4	6	$0.34e-10$	$0.33e-10$	$0.41e-10$
	ILUK (10)	12.7	8	$0.90e-10^{s}$	$0.79e-10$	$0.96e-10$
QMR 2	ILU0	1,099.0	500*	$0.28e-08$	$0.25e-08$	$0.30e-08$
	ILUTH (10^{-2})	891.1	500*	$0.28e-09$	$0.62e-09$	$0.75e-09$
	ILUTH (10^{-3})	1,083.0	500*	$0.12e-08$	$0.12e-08$	$0.14e-08$
	ILUK (10)	970.5	500*	$0.44e-09$	$0.39e-09$	$0.48e-09$
QMR 3	ILU0	1,108.0	500*	$0.79e-03$	$0.26e-03$	$0.26e-02$
	ILUTH (10^{-2})	895.2	500*	$0.95e-08$	$0.21e-07$	$0.26e-07$
	ILUTH (10^{-3})	1,063.0	500*	$0.39e-03$	$0.21e-03$	$0.27e-02$
	ILUK (10)	965.9	500*	$0.14e-06$	$0.13e-06$	$0.16e-06$

SOR

ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.
1.1	$0.93e-02$	9.7	19	$0.34e-12$	$0.41e-12$

Block SOR

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-3}$	1.1	$0.68e-10$	78.3	11	$0.57e-12$	$0.69e-12$	50
equal	1.1	$0.54e-10$	11.6	15	$0.25e-13$	$0.30e-13$	0
other	1.1	$0.36e-10$	11.6	18	$0.40e-12$	$0.48e-12$	2

IAD

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-3}$	1.1	$0.88e-11$	156.4	11	$0.15e-12$	$0.18e-12$	60
equal	1.1	$0.56e-10$	102.3	10	$0.47e-13$	$0.57e-13$	0
other	1.1	$0.21e-10$	95.6	12	$0.20e-12$	$0.24e-12$	2

Table A.23: Numerical Results for *mutex*.

mutex_alt1 $n = 39,203$ $nz = 563,491$

Preconditioner	nzlu	Time	MFlops
ILU0	563,491	2.6	0.6
ILUTH (10^{-2})	301,347	814.4	0.9
ILUK (10)	392,037	1,163.9	15.8

Method	Preconditioner	Time	# it	$\|\|r\|\|$	$\|\|Ax\|\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	7.2	6	$0.26e-12$	$0.10e-12$	$0.74e-13$
	ILUTH (10^{-2})	3.3	3	$0.27e-12$	$0.64e-12$	$0.45e-12$
	ILUK (10)	16.9	16	$0.88e-11$	$0.84e-14$	$0.60e-14$
BCG	ILU0	1,028.0	500*	$0.28e-02$	$0.14e-02$	$0.11e-02$
	ILUTH (10^{-2})	779.1	500*	$0.30e-07$	$0.87e-07$	$0.62e-07$
	ILUK (10)	858.6	500*	$0.19e+00$	$0.21e-04$	$0.15e-04$
CGS	ILU0	6.3	3	$0.40e-11$	$0.16e-11$	$0.11e-11$
	ILUTH (10^{-2})	3.4	2	$0.79e-16$	$0.19e-15$	$0.13e-15$
	ILUK (10)	10.5	6	$0.11e-11$	$0.13e-14$	$0.90e-15$
BCGStab	ILU0	5.4	3	$0.78e-10^s$	$0.31e-10$	$0.22e-10$
	ILUTH (10^{-2})	2.7	2	$0.71e-13^s$	$0.17e-12$	$0.12e-12$
	ILUK (10)	9.5	6	$0.51e-11^s$	$0.49e-14$	$0.35e-14$
QMR 1	ILU0	1,143.0	500*	$0.36e-08$	$0.14e-08$	$0.10e-08$
	ILUTH (10^{-2})	6.1	3	$0.92e-10$	$0.22e-09$	$0.16e-09$
	ILUK (10)	964.9	500*	$0.30e-04$	$0.14e-06$	$0.10e-06$
QMR 3	ILU0	37.8	16	$0.96e-10$	$0.38e-10$	$0.27e-10$
	ILUTH (10^{-2})	6.1	3	$0.92e-10$	$0.22e-09$	$0.16e-09$
	ILUK (10)	970.9	500*	$0.27e-04$	$0.14e-06$	$0.10e-06$

SOR

ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.
1.0	$0.82e-10$	5.2	10	$0.12e-14$	$0.82e-15$

Block SOR

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-6}$	1.0	$0.54e-12$	76.2	8	$0.41e-16$	$0.29e-16$	50
equal	1.0	$0.27e-10$	4.5	3	$0.76e-13$	$0.54e-13$	0
other	1.0	$0.82e-10$	6.8	10	$0.12e-14$	$0.82e-15$	2

IAD

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-6}$	1.0	$0.47e-10$	146.3	8	$0.14e-13$	$0.97e-14$	60
equal	1.0	$0.36e-11$	94.1	2	$0.41e-14$	$0.29e-14$	0
other	1.0	$0.36e-11$	84.8	4	$0.11e-15$	$0.76e-16$	2

Table A.24: Numerical Results for *mutex_alt1*.

mutex_alt1_gm6 $n = 39,203$ $nz = 563,491$

Preconditioner	nzlu	Time	MFlops
ILU0	563,491	2.3	0.6
ILUTH (10^{-2})	322,876	1,126.8	1.3
ILUTH (10^{-3})	464,251	1,127.3	1.9
ILUK (10)	382,280	1,547.8	83.7

Method	Preconditioner	Time	# it	$\|\|r\|\|$	$\|\|Ax\|\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	10.8	9	$0.81e-13$	$0.31e-18$	$0.22e-18$
	ILUTH (10^{-2})	5.2	5	$0.45e-12$	$0.71e-15$	$0.50e-15$
	ILUTH (10^{-3})	4.7	4	$0.17e-13$	$0.64e-15$	$0.45e-15$
	ILUK (10)	10.7	10	$0.20e-12$	$0.23e-15$	$0.16e-15$
BCG	ILU0	failed (division by 0 at step 12 of BCG)				
	ILUTH (10^{-2})	828.6	500*	$0.20e-05$	$0.53e-08$	$0.37e-08$
	ILUTH (10^{-3})	945.9	500*	$0.34e-02$	$0.21e-03$	$0.15e-03$
	ILUK (10)	875.8	500*	$0.35e-03$	$0.40e-06$	$0.28e-06$
CGS	ILU0	8.4	4	$0.10e-11$	$0.26e-18$	$0.18e-18$
	ILUTH (10^{-2})	5.1	3	$0.34e-15$	$0.54e-18$	$0.38e-18$
	ILUTH (10^{-3})	3.9	2	$0.11e-13$	$0.42e-15$	$0.30e-15$
	ILUK (10)	12.1	7	$0.38e-10$	$0.43e-13$	$0.31e-13$
BCGStab	ILU0	7.5	4	$0.38e-10^{s}$	$0.96e-17$	$0.69e-17$
	ILUTH (10^{-2})	4.4	3	$0.12e-10^{s}$	$0.19e-13$	$0.14e-13$
	ILUTH (10^{-3})	3.1	2	$0.38e-10^{s}$	$0.15e-11$	$0.10e-11$
	ILUK (10)	13.1	8	$0.22e-13^{s}$	$0.26e-16$	$0.18e-16$
QMR 2	ILU0	failed due to $\epsilon = 0$ at step 20 in QMR				
	ILUTH (10^{-2})	955.0	500*	$0.67e-06$	$0.11e-08$	$0.75e-09$
	ILUTH (10^{-3})	1,047.0	500*	$0.16e-07$	$0.12e-08$	$0.86e-09$
	ILUK (10)	failed due to $\epsilon = 0$ at step 20 in QMR				
QMR 3	ILU0	1,170.0	500*	$0.47e-06$	$0.48e-11$	$0.20e-11$
	ILUTH (10^{-2})	946.1	500*	$0.21e-07$	$0.33e-10$	$0.23e-10$
	ILUTH (10^{-3})	1,057.0	500*	$0.19e-08$	$0.74e-10$	$0.53e-10$
	ILUK (10)	failed due to $\epsilon = 0$ at step 20 in QMR				

SOR

ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.
1.0	not available	4.5	8	$0.43e-15$	$0.30e-15$

Block SOR

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-6}$	1.0	$0.13e-11$	77.3	8	$0.17e-14$	$0.12e-14$	52
equal	1.0	$0.50e-12$	64.5	8	$0.27e-16$	$0.19e-16$	22
other	1.0	$0.39e-12$	66.6	8	$0.16e-16$	$0.12e-16$	14

IAD

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-6}$	1.0	$0.11e-10$	142.2	8	$0.89e-14$	$0.64e-14$	62
equal	1.0	$0.23e-10$	134.9	8	$0.86e-14$	$0.61e-14$	41
other	1.0	$0.13e-11$	136.2	8	$0.44e-15$	$0.31e-15$	24

Table A.25: Numerical Results for *mutex_alt1_gm6*.

mutex_alt2 $n = 39,203$ $nz = 563,491$

Preconditioner	nzlu	Time	MFlops
ILU0	563,491	2.6	0.6
ILUTH (10^{-2})	301,347	803.7	0.9
ILUK (10)	392,037	1,158.4	15.8

Method	Preconditioner	Time	# it	$\|\|r\|\|$	$\|\|Ax\|\|$	Bk. Err.
GMRES ($m = 20$)	ILU0	13.1	11	$0.32e-12$	$0.28e-15$	$0.20e-15$
	ILUTH (10^{-2})	2.5	2	$0.48e-13$	$0.11e-12$	$0.81e-13$
	ILUK (10)	17.0	16	$0.10e-11$	$0.22e-18$	$0.16e-18$
BCG	ILU0	23.1	11	$0.37e-10$	$0.14e-09$	$0.97e-10$
	ILUTH (10^{-2})	774.3	500*	$0.61e-04$	$0.26e-04$	$0.18e-04$
	ILUK	26.0	15	$0.95e-10$	$0.51e-10$	$0.36e-10$
CGS	ILU0	6.3	3	$0.93e-12$	$0.55e-15$	$0.39e-15$
	ILUTH (10^{-2})	1.9	1	$0.76e-15$	$0.18e-14$	$0.13e-14$
	ILUK (10)	21.6	13	$0.56e-15$	$0.66e-14$	$0.47e-14$
BCGStab	ILU0	5.4	3	$0.40e-11^s$	$0.24e-14$	$0.17e-14$
	ILUTH (10^{-2})	1.9	1	$0.17e-15$	$0.40e-15$	$0.28e-15$
	ILUK (10)	15.3	9	$0.63e-11$	$0.37e-15$	$0.26e-15$
QMR 2	ILU0	732.0	319	$0.73e-10$	$0.11e-11$	$0.79e-12$
	ILUTH (10^{-2})	889.0	500*	$0.37e-09$	$0.13e-08$	$0.96e-09$
	ILUK (10)	49.5	25	$0.92e-11$	$0.12e-11$	$0.85e-12$
QMR 3	ILU0	94.8	41	$0.93e-10$	$0.70e-10$	$0.50e-10$
	ILUTH (10^{-2})	894.0	500*	$0.17e-09$	$0.69e-09$	$0.49e-09$
	ILUK (10)	980.1	500*	$0.21e-03$	$0.20e-09$	$0.14e-09$

SOR

ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.
1.0	not available	4.7	9	$0.50e-16$	$0.35e-16$

Block SOR

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-9}$	1.0	$0.88e-10$	68.2	5	$0.31e-13$	$0.22e-13$	47
equal	1.0	$0.22e-15$	3.9	2	$0.17e-17$	$0.12e-17$	0
other	1.0	$0.15e-12$	6.2	9	$0.50e-16$	$0.35e-16$	2

IAD

Partition.	ω	$\|\|\Delta x\|\|$	Time	# it	$\|\|Ax\|\|$	Bk. Err.	# Bl.
$\gamma = 10^{-9}$	1.0	$0.24e-10$	134.9	6	$0.96e-14$	$0.68e-14$	57
equal	1.0	$0.79e-15$	92.1	2	$0.37e-17$	$0.26e-17$	0
other	1.0	$0.50e-14$	82.9	3	$0.50e-16$	$0.35e-16$	2

Table A.26: Numerical Results for *mutex_alt2*.

leaky $n = 8,258$ $nz = 197,474$

SOR

ω	$\|\Delta x\|$	Time	# it	$\|Ax\|$	Bk. Err.
1.0	$0.17e-09$	57.8	409	$0.75e-10$	$0.38e-09$

Block SOR

Partition.	ω	$\|\Delta x\|$	Time	# it	$\|Ax\|$	Bk. Err.	# Bl.
$\gamma = 10^{-9}$	1.0	$0.98e-10$	115.9	14	$0.41e-10$	$0.21e-09$	192
$\gamma = 10^{-1}$	1.0	$0.94e-10$	33.3	142	$0.11e-10$	$0.53e-10$	7,324
equal	1.1	$0.96e-10$	59.1	255	$0.83e-10$	$0.42e-09$	89
other	1.0	$0.99e-10$	73.8	313	$0.77e-10$	$0.39e-09$	109

IAD

Partition.	ω	$\|\Delta x\|$	Time	# it	$\|Ax\|$	Bk. Err.	# Bl.
$\gamma = 10^{-9}$	1.0	$0.41e+00$	267.0	1,000*	$0.21e-16$	$0.11e-15$	192
$\gamma = 10^{-1}$	1.0	requires unreasonably long time					
equal	1.0	$0.82e-10$	20.3	70	$0.34e-11$	$0.18e-10$	89
other	1.0	$0.78e-10$	24.9	80	$0.28e-11$	$0.14e-10$	109

Table A.27: Numerical Results for *leaky*.

Appendix B

The Nonzero Structures of the Matrices

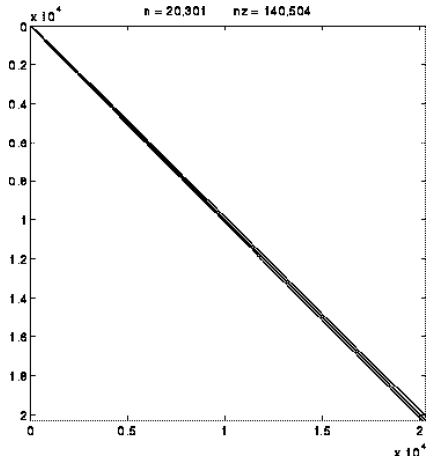

Figure B.1: Pushout Threshold.

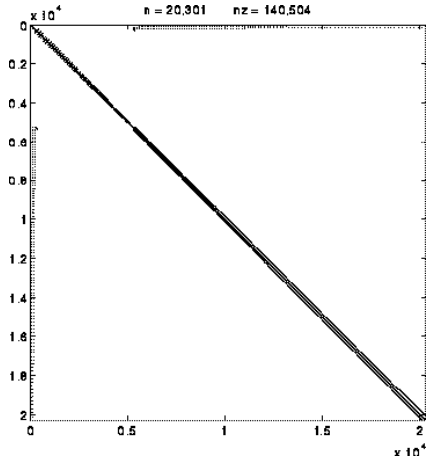

Figure B.2: easy_gm4.

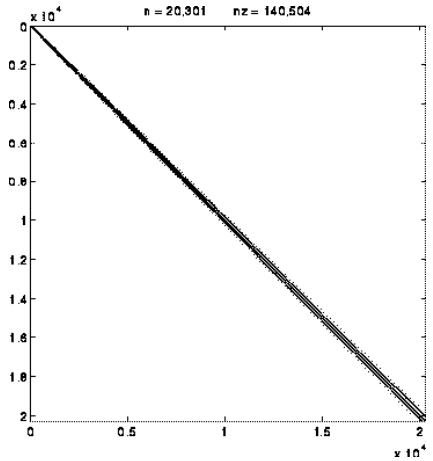

Figure B.3: easy_gm3.

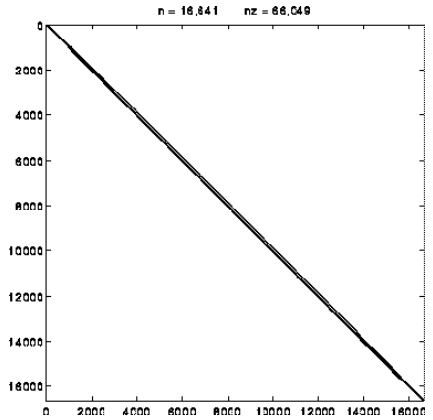

Figure B.4: 2D.

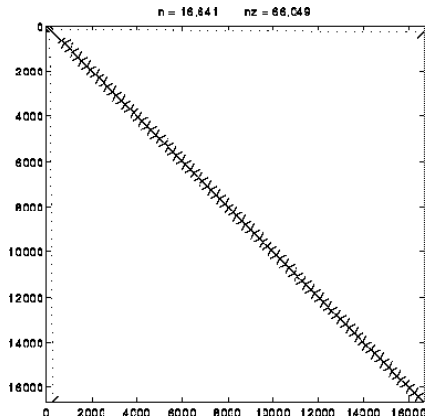

Figure B.5: 2D_gm3.

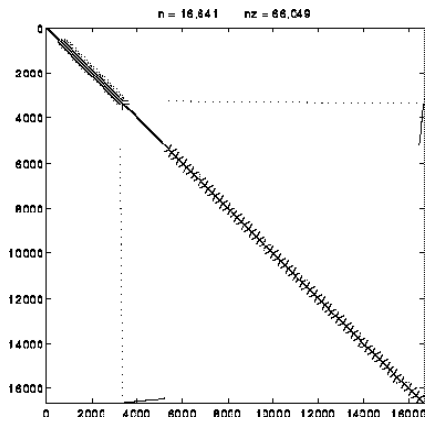

Figure B.6: 2D_gm2.

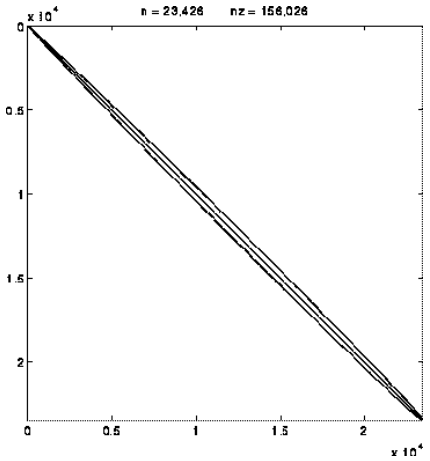

Figure B.7: ncd.

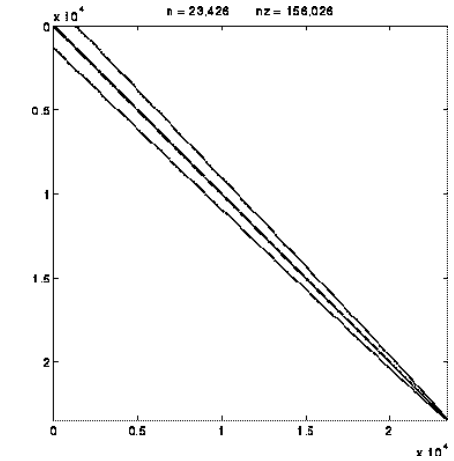

Figure B.10: ncd_alt1_gm5.

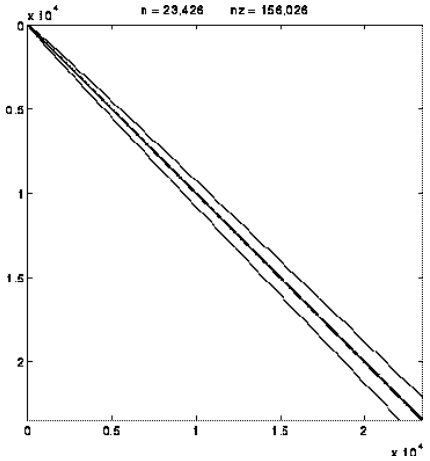

Figure B.8: ncd_gm4.

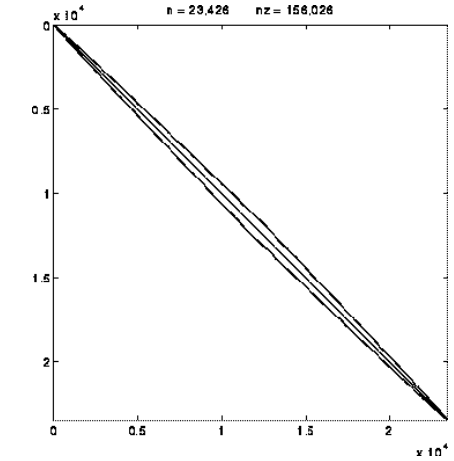

Figure B.11: ncd_alt1_gm4.

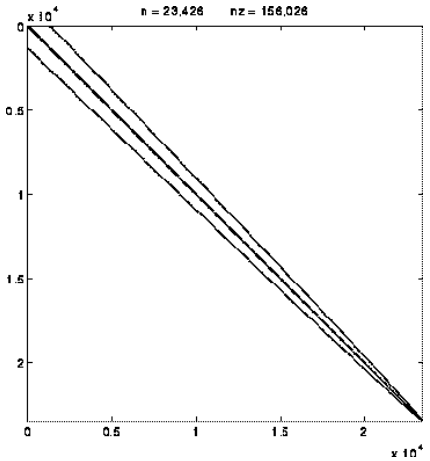

Figure B.9: ncd_alt1_gm7.

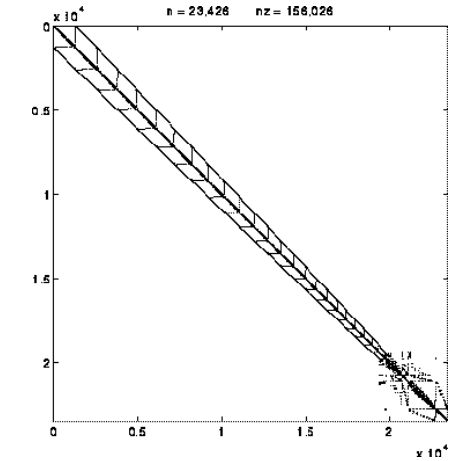

Figure B.12: ncd_alt2_gm7.

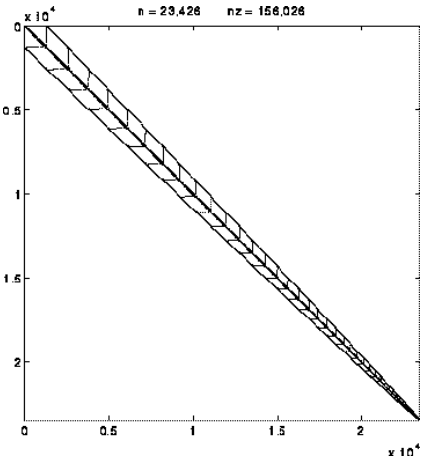

Figure B.13: ncd_alt2_gm6.

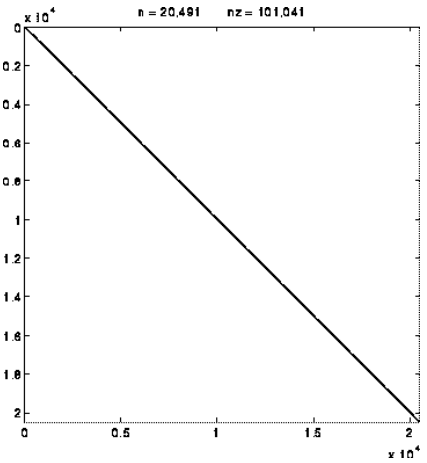

Figure B.14: telecom.

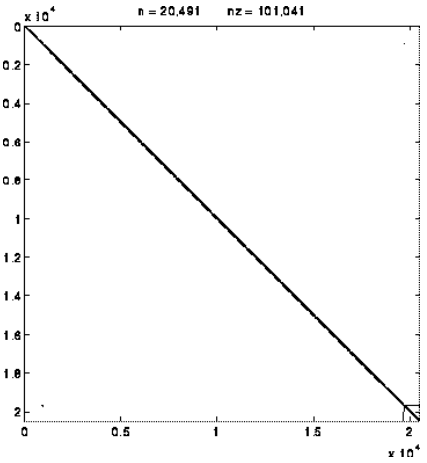

Figure B.15: telecom_gm2.

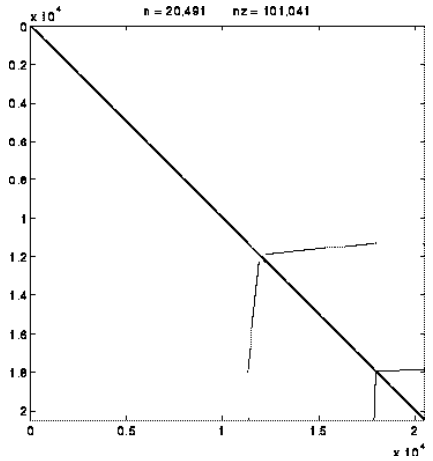

Figure B.16: telecom_gm1.

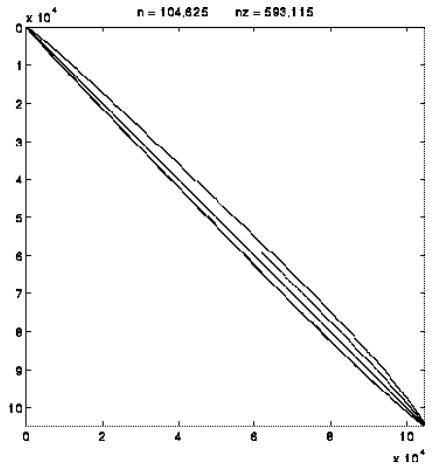

Figure B.17: qnatm.

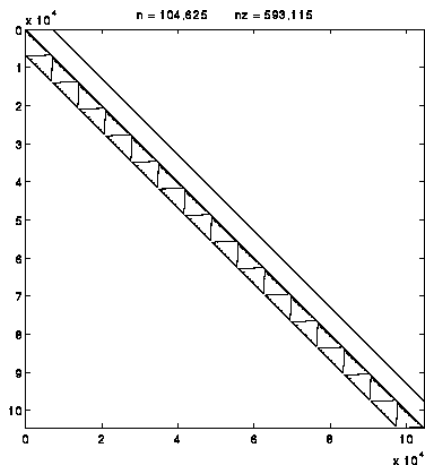

Figure B.18: qnatm_gm1.

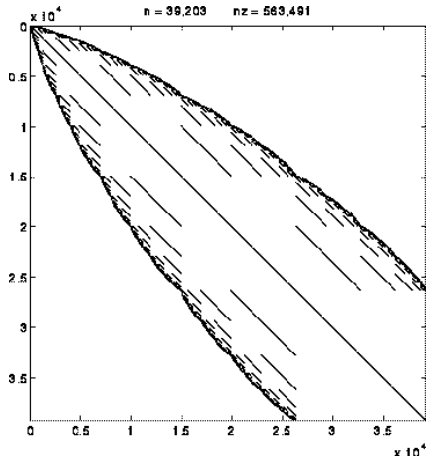

Figure B.19: mutex.

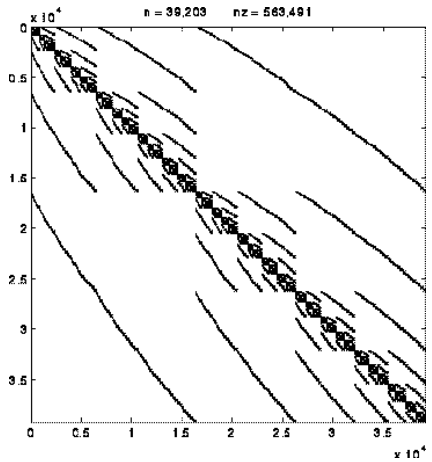

Figure B.20: mutex_alt1_gm6.

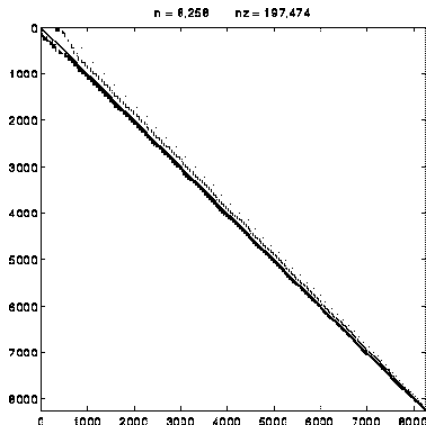

Figure B.21: leaky.

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