Recursion

CS 201

Introduction

- Recursion is an extremely powerful problem-solving technique
  - It breaks a problem into smaller identical problems and uses the same function to solve these smaller problems
  - It is an alternative to iterative solutions, which use loops

- Facts about recursive solutions
  - A recursive function calls itself
  - Each recursive call solves an identical but a smaller problem
  - Base case must be defined (it enables to stop the recursive calls)
  - Eventually, one of the smaller problems must be the base case

Simple example: Write a global function that displays a given C-style string backward

Recursive solution:

- Each recursive call diminishes the string length by 1
- Base case: displaying the empty string backward

```
void displayBackward( char* str ) {

   if ( str[0] == '\0' )
      return;

   displayBackward( str + 1 );
   cout << str[0];
}
```

Recursion and efficiency: Fibonacci function

Recurrence relation:

$$F(n) = F(n-1) + F(n-2)$$

Base cases:

$$F(1) = 1$$
$$F(2) = 1$$

```
int recursiveFib( int n ) {
   if ( n <= 2 )
      return 1;

   return recursiveFib( n - 1) + recursiveFib( n - 2 );
}
```

```
int iterativeFib( int n ) {
   int previous = 1;
   int current = 1;
   int next = 1;          // result when n is 1 or 2

   // compute next Fibonacci values when n >= 3
   for ( int i = 3; i <= n; i++ ) {
      next = current + previous;
      previous = current;
      current = next;
   }
   return next;
}
```

Recursion and efficiency

- Some recursive solutions are so inefficient that they should not be used
- Factors contributing to this inefficiency
  - Inherent inefficiency of some recursive algorithms (such as the `recursiveFib` function)
  - Overhead associated with function calls
- Do not use a recursive solution if it is inefficient and there is a clear and efficient iterative solution

More examples: Write a recursive function for the binary search algorithm

A high-level pseudocode for binary search

```
if ( anArray is of size 1 )
    determine if anArray's item is equal to the searched value
else {
    find the midpoint of anArray
    determine which half of anArray contains the searched value
    if ( the value is in the first half of anArray )
        binarySearch( first half of anArray, value )
    else
        binarySearch( second half of anArray, value )
}
```

Implementation issues

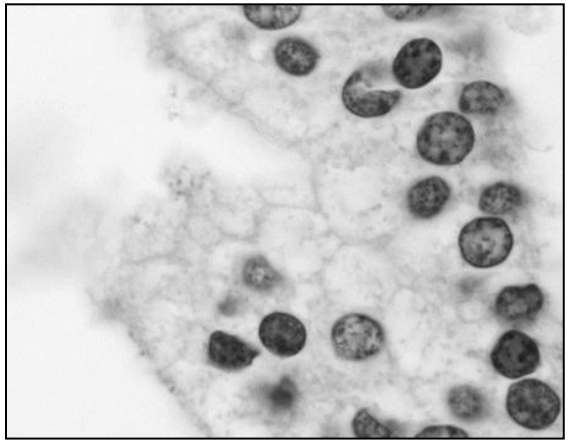
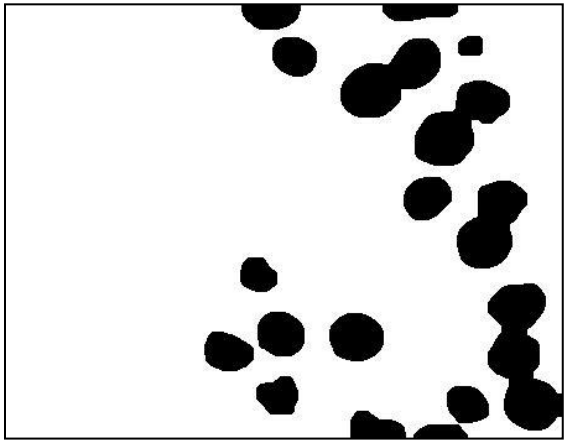
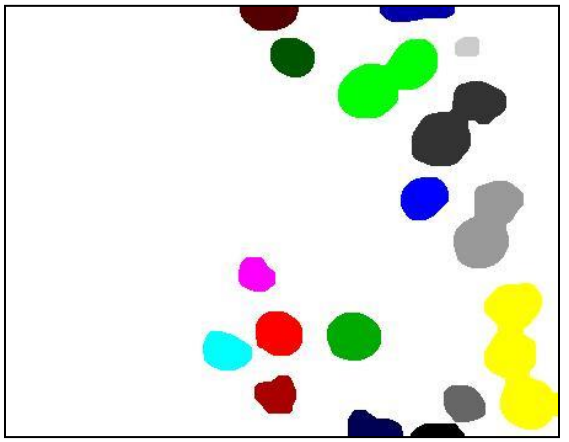
- How to pass "half of `anArray`" to the function?
- How to determine the base case(s)?
- How to return the result?

More examples: Write a recursive function for the binary search algorithm

```
int binarySearch( int* arr, int low, int high, int key ) {
   if ( low > high )
      return -1;

   int mid = (low + high) / 2;

   if ( arr[mid] == key )
      return mid;

   if ( arr[mid] > key )
      return binarySearch( arr, low, mid - 1, key );

   return binarySearch( arr, mid + 1, high, key );
}
```

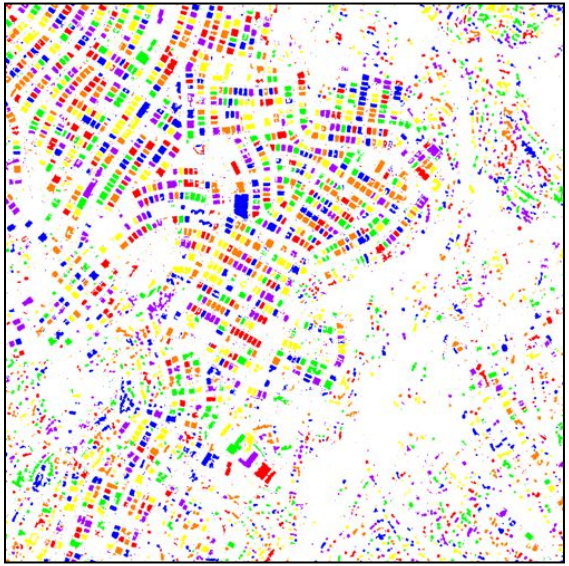
More examples: Write a recursive function that finds the connected components of a given black-and-white image

Application 1: Suppose that we want to locate cell nuclei in a gray-level image whose pixel intensities are in between 0 and 255. To find the nucleus locations, one may first obtain a black-and-white image, whose intensities are either 0 or 1, using some image processing techniques (e.g., thresholding). Then, s/he may identify each connected component of the 1-pixels as a cell nucleus.

More examples: Write a recursive function that finds the connected components of a given black-and-white image

Application 2: Similarly, in the image below, we want to identify individual buildings. Connected component analysis can be used after obtaining a black-and-white image of buildings.



More examples: Write a recursive function that finds the connected components of a given black-and-white image

```
int** findConnectedComponents( int** arr, int row, int column ) {

   int** labels, i, j, currLabel;

   labels = new int* [ row ];
   for ( i = 0; i < row; i++ ) {
      labels[ i ] = new int [ column ];
      for ( j = 0; j < column; j++ )
         labels[ i ][ j ] = 0;
   }

   currLabel = 1;
   for ( i = 0; i < row; i++ )
      for ( j = 0; j < column; j++ )
         if ( arr[ i ][ j ]  &&  !labels[ i ][ j ] )
            fourConnectivity( arr, labels, row, column, i, j, currLabel++ );

   return labels;
}
```

More examples: Write a recursive function that finds the connected components of a given black-and-white image

```
void fourConnectivity( int** arr, int** labels, int row, int column,
                       int i, int j, int currLabel ) {

   if ( arr[i][j] == 0 )
      return;
   if ( labels[i][j] > 0 )
      return;

   labels[i][j] = currLabel;

   if ( i - 1 >= 0 )
      fourConnectivity( arr, labels, row, column, i - 1, j, currLabel );
   if ( i + 1 < row )
      fourConnectivity( arr, labels, row, column, i + 1, j, currLabel );
   if ( j - 1 >= 0 )
      fourConnectivity( arr, labels, row, column, i, j - 1, currLabel );
   if ( j + 1 < column )
      fourConnectivity( arr, labels, row, column, i, j + 1, currLabel );
}
```