Varol Akman

Unobstructed Shortest Paths in Polyhedral Environments

Springer-Verlag
Berlin Heidelberg New York London Paris Tokyo
Author
Varol Akman
Interactive Systems, Centrum voor Wiskunde en Informatica
Kruislaan 413, 1098 SJ Amsterdam, The Netherlands

CR Subject Classification (1987): F.2.2, G.1.5-6, I.1.2, I.2.9, I.3.4-5


This work is subject to copyright. All rights are reserved, whether the whole or part of the material
is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation,
broadcasting, reproduction on microfilms or in other ways, and storage in data banks. Duplication
of this publication or parts thereof is only permitted under the provisions of the German Copyright
Law of September 9, 1965, in its version of June 24, 1985, and a copyright fee must always be
paid. Violations fall under the prosecution act of the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1987
Printed in Germany
Printing and binding: Druckhaus Beitz, Hemsbach/Bergstr.
2145/3140-543210
Dedicated to my parents and my brother
For me there is only the traveling on paths that have heart, on any path that may have heart. There I travel, and the only worthwhile challenge is to traverse its full length. And there I travel, looking, looking, breathlessly.
- DON JUAN

Namely, because the shape of the whole universe is most perfect and, in fact, designed by the wisest creator, nothing in all of the world will occur in which no maximum or minimum rule is somehow shining forth.
- LEONHARD EULER

Whenever the local endpoint of the unknown quantity describes a straight line or a circle, a plane locus results, and when it describes a parabola, hyperbola, or ellipse, a solid locus results.
- PIERRE DE FERMAT

Is the three-dimensional shortest path problem NP-complete? For one, it does not seem to be in NP at all, because of the difficulty with the skew lines mentioned before. The difficulty is reminiscent of a similar one with the Euclidean traveling salesman problem and the precision required in evaluating tours, although the present situation is far more complex, and furthermore, there is no obvious discretization to help avoid the issue.
- CHRISTOS PAPADIMITRIOU
Table of Contents

1. Introduction 1
   1.1. Overview 1
   1.2. Prerequisites and Notation 2
   1.3. FINDPATH - Problem Statement 3
   1.4. Motivation 9
   1.5. Related Research on Motion Planning 12
      1.5.1. More Practical Works 12
      1.5.2. Complexity-Theoretic Works 15
      1.5.3. Voronoi-Based Works 17
   1.6. Organization 18
2. Solution of the General Instance of FINDPATH 19
   2.1. A Brute-Force Algorithm 19
   2.2. Efficiency Considerations 28
3. Solutions of Two Specific Instances of FINDPATH 34
   3.1. Shortest Paths on a Convex Polyhedron 34
   3.2. Shortest Paths around a Convex Polyhedron 46
4. Two Voronoi-Based Techniques for FINDPATH 52
   4.1. Partitioning the Boundary of a Convex Polyhedron 52
   4.2. Partitioning the Free Space around Polyhedra 56
   4.3. Critique of Recent Algorithms for 3-Space 66
5. Desirable Functionalities of a Geometer’s Workbench 70
   5.1. General Considerations 70
   5.2. SP - A Program to Compute Shortest Paths 73
6. Conclusion and Future Work 85
   6.1. Results 85
   6.2. Future Research and Open Problems 86
References 93
Foreword

The study of minimum paths on or around polyhedra in Euclidean 3-space is of growing importance in robotics. This work presents new algorithms based on extensions of the Voronoi diagram. Since experience with new algorithms is also important, this work also describes a workbench to allow experimentation.

This book is based on the Ph.D. research of my former student, Varol Akman, who graduated from the Electrical, Computer, and Systems Engineering Dept. of Rensselaer Polytechnic Institute in August 1985.

We wish to thank the other members of his committee: Frank DiCesare, Mukkai S. Krishnamoorthy, and James M. Tien. We also wish to thank Herbert Freeman for employing him in the Image Processing Lab at RPI, Michael J. Wozny for employing him in the Center for Interactive Computer Graphics, and Edwin Rogers for granting free access to the Computer Science Dept. facilities.

This research has been supported by the National Science Foundation under grants ECS 80-21504 and ECS 83-51942, by the Schlumberger-Doll Research Center, Ridgefield, Connecticut, USA, by a Fulbright Award, and by the Middle East Technical University, Ankara, Turkey. The views expressed in this work are those of the author alone, and no endorsement is expressed or implied by the support.

Wm. Randolph Franklin
Rensselaer Polytechnic Institute
June 1986