Introduction To Modeling & Simulation (Part 1)

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Introduction To Modeling and Simulation (Outline)

• What is Modeling and Simulation (M&S) ?
• Complexity Types
• Model Types
• Simulation Types
• M&S Terms and Definitions
What is M&S?

- Discipline of **understanding** and **evaluating** the **interaction** of parts of a **real** or **theoretical system** by;
  - Designing its representation (**model**) and
  - Executing (running) the model including the time and space dimension (**simulation**).

What is a System?

- An unit or process, which **exists** and **operates in time and space** through the **interaction** of its parts.
What is a Model?

- A simplified representation of a real or theoretical system at some particular point in time or space intended to provide understanding of the system.

What Level of Model Detail?

- Whether a model is good or not depends on the extent to which it provides understanding.
- All the models are simplification of reality.
- Exact copy of a reality can only be the reality itself.
- There is always a trade off as to what level of detail is included in the model:
  - Too little detail: risk of missing relevant interactions.
  - Too much detail: Overly complicated to understand.
What is a Simulation?

- The manipulation of a model in such a way that it operates in time or space to summarize it.

Why Simulation?

- Enable one to perceive the interactions that would otherwise be apparent because of their separation in time or space.
Advantages of M&S

• Choose correctly:
  – M&S lets you test every aspect of a proposed change or addition without committing resources to their acquisition.

• Compress and expand time:
  – M&S allows you to speed up or slow down phenomena so that you can investigate them better.

Advantages of M&S

• Understand why:
  – Managers often want to know why certain phenomena occur in a real system.
  – M&S lets you determine answers to “why” questions by reconstructing (replaying) the scene and taking a closer look at what has happened during the run.
Advantages of M&S

• Explore possibilities:
  – You can explore new policies, operating procedures or methods without the need of experimenting with the real world systems.

• Diagnose problems:
  – Some systems are so complex that it is impossible to consider all the interactions taking place in a given moment.
  – With M&S, you can better understand the interactions among the variables that make up the complex system.

• Identify constraints:
  – Bottlenecks in a system is an effect rather than a cause.
  – Doing bottleneck analysis with M&S, you can discover the cause of the delays in work process, information, material or other processes.
Advantages of M&S

• Develop understanding:
  – M&S provides understanding about how a system really operates rather than indicating someone’s predictions about how a system will operate.

• Visualize the plan:
  – M&S lets you see your system actually running.
  – That allows you to detect design flaws that appear credible.

• Build consensus:
  – Instead of saying one person’s opinion about a system, you actually show how the system works, so provide an objective opinion.

• Prepare for change:
  – Using M&S, you can ask what-if questions for determining future improvements and new designs on a system.
Advantages of M&S

• **Invest wisely:**
  – M&S is a wise investment since
  • Typical cost of a simulation study is substantially less than generally 1% of the total amount being expended for the implementation of a design or redesign.

Advantages of M&S

• **Train the team:**
  – M&S can provide excellent training when design for that purpose.
  – In training, team provides decision inputs to the simulation as it progress, and observes the outputs.
  – After simulation ends, further evaluation can be provided by after action review (AAR).
Advantages of M&S

• **Specify requirements:**
  – M&S can be used to determine requirements for a system design by simulating different possible configurations of a system.

Disadvantages of M&S

• **Model building requires special training:**
  – M&S is an art that is learned over time and through experience.
  – Two models of the same system developed by two different individuals may have similarities, but it is unlikely be the same.
  – Building a realistic model may require domain knowledge that can only be acquired from a subject matter expert.
Disadvantages of M&S

• Simulation results may be difficult to interpret:
  – Since most simulation results are essentially random variables,
  • It may be hard to determine whether an observation is a result of system interrelationships or just randomness.

Disadvantages of M&S

• Simulation modeling and analysis can be time consuming and expensive:
  – Economizing on resources for modeling and analysis may result in a simulation not sufficient enough for the problem, and may consume time, effort and money for nothing.
Disadvantages of M&S

- **Simulation may be used inappropriately:**
  - Simulation is used in some cases when analytical solution is possible, or even preferable.

How to Use a Simulation?

- Develop a model,
- Simulate it,
- Analyze the results,
- Learn from the simulation,
- Revise the model & simulation,
- Continue the interactions until adequate level of understanding is developed.

- M&S is a discipline, but it is also very much an art form.
Steps in a M&S Study

- Problem formulation
- Setting of objectives and overall project plan
  - Model conceptualization
  - Data collection
  - Model transformation
  - Experimental design
  - Simulation runs & analysis
    - Validated? yes
    - No runs? yes
  - Documentation and reporting

Verified? no
Validated? yes

Some Application Areas
- Medical research, training & support
- Industrial engineering designs & presentations (Factory process design, manufacturing, ...)
- Civil engineering designs & presentations (Building design, city & infrastructure planning, ...)
- Mechanical engineering designs & presentations (Engine designs, aerodynamic design, ...)
- Nature sciences (Physic, chemistry, biology, meteorology, astronomy, ...)
- Geographic Information Systems (Earth modeling, ...)
- Military Decision Support (War modeling, ...)
- Training (Simulators, games, ...)
- Entertainment (Games, ...)
Practical Lecture Focus

- **Modeling & Simulation in:**
  - Defense Industry, and
  - Game Programming.
- **Includes:**
  - Earth modeling,
  - Entity modeling,
  - Behavior modeling,
  - Sensor & weapon systems modeling,
  - Distributed simulations,
  - Simulation based optimization and analysis.

Complexity Types

- Detail Complexity
- Dynamic Complexity
Detail Complexity

- Associated with systems which have many component parts.

Dynamic Complexity

- Associated with systems which have cause and effect separated by time and space.
- Great difficulty dealing with.
- Unable to readily see the connections between parts of the systems and their interactions.
Dynamic Complexity (Sample)

Model Types

- Mathematical Models
- Physical Models
- Process Models
Mathematical Models

- Models, properties of which are described by mathematical symbols and relations.
- Constructed using:
  - Procedures (algorithms)
  - Mathematical equations.

Mathematical Models (Sample)

<table>
<thead>
<tr>
<th>Chaparral Missile Properties (Parameters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Radius</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Guidance</td>
</tr>
<tr>
<td>Range</td>
</tr>
<tr>
<td>Velocity</td>
</tr>
<tr>
<td>War Head</td>
</tr>
<tr>
<td>Engine</td>
</tr>
</tbody>
</table>

\[
A = \text{Acceleration} \\
S = \text{Speed} \\
W = \text{Effective Radius} \\
E = \text{Effective Range} \\
S2 = \text{Target Velocity} \\
D = \text{Target Distance} \\
R = A + \frac{D}{(D/S + D/S2)} \\
R = \text{Probability of Hit}
\]
Physical Models

- Models, properties of which are described by physical structures and relations.
- Usually applied to high fidelity (detailed) system simulations such as simulators.

Physical Models (Sample)
Process Models

- Models the process a system performs.
- Represents dynamic relations by mathematical and logical functions.

Process Models (Sample)

<table>
<thead>
<tr>
<th>Time</th>
<th>Step Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>Search &amp; Classify Target</td>
</tr>
<tr>
<td>$T_2$</td>
<td>Decide To Fire</td>
</tr>
<tr>
<td>$T_3$</td>
<td>Perform Fire Computations</td>
</tr>
<tr>
<td>$T_4$</td>
<td>Command Fire</td>
</tr>
<tr>
<td>$T_5$</td>
<td>Fire Missile</td>
</tr>
<tr>
<td>$T_6$</td>
<td>Mission Complete</td>
</tr>
</tbody>
</table>

$T_1 + T_2 + T_3 + T_4 + T_5 + T_6 = \text{Mission Complete}$
Simulation Types
(WRT Entities Involved)

- Live
- Virtual
- Constructive

Live Simulations

- Real systems & actors
- Real environment

Possible results:
- Resource Waste
- Time Waste
- Possible Damages
Virtual Simulations

- Real/Virtual systems & actors
- Real/Virtual environment

*Usually used for training within simulators*

Constructive Simulations

- Virtual systems & actors
- Virtual environment

Objectives:
- Doing measurement, comparison, forecasting & concept analysis,
- Producing statistics
Simulation Types (WRT Time Advance)

- Discrete
- Continuous

Discrete (Event) Simulations

- Time is advanced from event time to event time rather than using a continuously advancing time clock.
Continuous Simulations

• Something that can only really be accomplished with an analog computer.

• An approximation for continuous simulations (combined discrete continuous sim.) is;
  – Making the time step of the simulation sufficiently small so there are no transitions within the system between time steps.
  – So the simulation is stepped in time increments.

Simulation Types
(WRT Results)

• Deterministic
• Stochastic
Deterministic Simulations

- A model that does **not contain probability**.
- Every run will result the same.
- Single run is enough to evaluate the result.

Stochastic Simulations

- A model that **contains probability**.
- Units, process, events or their parameters are **initiated randomly** using random numbers.
- If different runs are initiated with **different random number seeds**,  
  - Every run will **result differently**.
- **Multiple runs are required** to evaluate the results.
- Statistics such as **averages, standard deviations** are used for evaluation.
Simulation Types (WRT Design)

- Traditional
- Agent-Based

Traditional Simulations

- Simulations where the characteristics of a population are averaged together, and
- The model attempts to simulate changes in these averaged characteristics for the whole population.
Traditional Simulations
(Screen Shoot of a GPSS Program)

• GPSS is a traditional computer simulation language that stands for general-purpose simulation systems.

An internet cafe simulation

Traditional Simulations
(A Sample GPSS Program)

• Statistical values are used to model & simulate the system.

Mean inter arrival times for different time of day

<table>
<thead>
<tr>
<th>Time of Day</th>
<th>Arrival Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00-10:00</td>
<td>60 minutes</td>
</tr>
<tr>
<td>10:00-11:00</td>
<td>45 minutes</td>
</tr>
<tr>
<td>11:00-14:00</td>
<td>30 minutes</td>
</tr>
<tr>
<td>14:00-18:00</td>
<td>25 minutes</td>
</tr>
<tr>
<td>18:00-21:00</td>
<td>20 minutes</td>
</tr>
<tr>
<td>21:00-24:00</td>
<td>15 minutes</td>
</tr>
<tr>
<td>23:00-01:00</td>
<td>10 minutes</td>
</tr>
</tbody>
</table>

INITIAL X$INTMEAN 60
INITIAL X$WAITMEAN 30
CALCULATE VARIABLE X$WAITMEAN+5#FN$WAITTIME
GENERATE X$INTMEAN,FN$EXPO
ASSIGN 1,V$CALCWAIT ; max waiting time is in parameter 1 of command
ENDO
FUNCTION RN1,C24 ;Exponential Distribution
FUNCTION RN3,C25 ;Standard normal Dist. Function

CS-503
Traditional Simulations
(A Sample GPSS Program)

Simulating 1 day in GPSS

```
ONEDAY  GENERATE 961,1 ; Internet Cafe Open at 09:00, 16*60 Min = 1 Day
SAVEVALUE INTMEAN,60 ; After 09:00 Inter Arrival Mean = 60 min
SAVEVALUE COMMUN,60 ; Computer Usage Mean = 60 min
ADVANCE 60 ; 1 Hours
SAVEVALUE INTMEAN,45 ; After 10:00 Inter Arrival Mean = 45 min
ADVANCE 60 ; 1 Hours
SAVEVALUE INTMEAN,30 ; After 11:00 Inter Arrival Mean = 30 min
SAVEVALUE COMMUN,30 ; Computer Usage Mean = 30 min
ADVANCE 180 ; 3 Hours
SAVEVALUE INTMEAN,25 ; After 14:00 Inter Arrival Mean = 25 min
ADVANCE 240 ; 4 Hours
SAVEVALUE INTMEAN,20 ; After 18:00 Inter Arrival Mean = 20 min
ADVANCE 180 ; 3 Hours
SAVEVALUE INTMEAN,15 ; After 21:00 Inter Arrival Mean = 15 min
ADVANCE 60 ; 1 Hours
SAVEVALUE INTMEAN,10 ; After 24:00 Inter Arrival Mean = 10 min
ADVANCE 60 ; 1 Hours
TERMINE 1 ; Internet Cafe Closed At 01:00
```
Traditional Simulations (Sample GPSS Results)

Queue waiting frequencies

<table>
<thead>
<tr>
<th>Time Range</th>
<th>Frequency</th>
<th>Cumulative %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 minute</td>
<td>20</td>
<td>62.50%</td>
</tr>
<tr>
<td>1...5 minutes</td>
<td>1</td>
<td>65.63%</td>
</tr>
<tr>
<td>5...10 minutes</td>
<td>3</td>
<td>75.00%</td>
</tr>
<tr>
<td>10...15 minutes</td>
<td>1</td>
<td>78.13%</td>
</tr>
<tr>
<td>15...20 minutes</td>
<td>3</td>
<td>85.00%</td>
</tr>
<tr>
<td>20...25 minutes</td>
<td>1</td>
<td>90.63%</td>
</tr>
<tr>
<td>25...30 minutes</td>
<td>1</td>
<td>93.75%</td>
</tr>
<tr>
<td>30...35 minutes</td>
<td>2</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Frequency of leaving time for too much waited customers

<table>
<thead>
<tr>
<th>Time Range</th>
<th>Frequency</th>
<th>Cumulative %</th>
</tr>
</thead>
<tbody>
<tr>
<td>25...30 minutes</td>
<td>1</td>
<td>33.33%</td>
</tr>
<tr>
<td>30...35 minutes</td>
<td>2</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Agent-Based Simulations

- Differs from traditional kinds of simulations in that some or all of the simulated entities are modeled in terms of agents.
- Explicitly attempts to model specific behaviors of specific individuals.
- Contrasted to methods where the characteristics of a population are averaged together.
- Supports structure preserving modeling of the simulated reality.
Agent-Based Simulations
(Domain Examples)

• Vehicles and pedestrians in traffic situations.
• Actors in financial markets.
• Consumer behavior.
• Humans and machines in battlefields.
• People in crowds.
• Animals and/or plants in eco-systems.
• Artificial creatures in computer games.

Agent-Based Simulations
(Advantages)

• Distributed control, supporting parallel computations on separate machines.
• Supports simulation of pro-active behaviour.
• Ability to add or delete entities during a simulation.
• Easy to swap (exchange) an agent with the corresponding simulated entity,
  – i.e., a real person or a physical machine, (even during a simulation) making the simulation scenarios very dynamic.
Agent-Based Simulations (Advantages)

• Facilitates simulation of group behavior in highly dynamic situations,
  – Thereby allowing the study of "emergent behavior" that is hard to grasp with traditional methods.
• Well-suited for the simulation of situations where there are a large number of heterogeneous individuals who may behave somewhat differently.

Agent-Based Simulations (Agent Characterization)

• What is referred to as an agent covers a spectrum ranging from ordinary objects to full agents.
• May characterize them with the following dimensions:
  – Interaction
  – Communication language
  – Control/autonomy
  – Pro-activeness
  – Spatial explicitness
  – Mobility
  – Adaptivity
  – Modelling concepts
Agent-Based Simulations (Agent Characterization)

- **Interaction:**
  - From collaborative to no interaction at all.

- **Communication language:**
  - From KQML via simple signals (e.g. procedure calls) to none at all.

- **Control/autonomy:**
  - From each agent being a separate process (or thread) to one single process (monolithic system).

Agent-Based Simulations (Agent Characterization)

- **Pro-activeness:**
  - From pro-active to full reactive.

- **Spatial explicitness:**
  - From each agent being assigned a location in physical geometrical space to no notion of space.

- **Mobility:**
  - From each agent being able to move around in the simulated physical space to all agents being stationary.
Agent-Based Simulations
(Agent Characterization)

• **Adaptivity:**
  – From learning to static behaviour.

• **Modelling concepts:**
  – From mentalistic (e.g., Belief Desire Intention [BDI]) to non-mentalistic.

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Frequently Used M&S Terms and Definitions

• Entity, Attributes, State Variables & Event
• Replication
• Pixel, Polygon & Voxel
• Fidelity & Resolution
• Aggregation & Disaggregation
• Interoperability & Reusability
• Frame
• Simulator
• Computer Generated Forces
• Distributed Simulation
• High Level Architecture (HLA)
• Conceptual Model of The Mission Space (CMMS)
• Verification & Validation
Entity

- A representation of an object that requires explicit definition.
- An entity can be:
  - Dynamic: Moves through the system
    - E.g. A customer
  - Static (resource): Serves other entities
    - E.g. A bank teller

Attributes

- Local values that defines the characteristics of an entity.
- A soldier attributes could be:
  - Max running speed: 12 km/h
  - Head direction left limit: -80 degree
  - Head direction right limit: +80 degree
  - Max ammunition level: 20 bullets
  - Max target detection range: 2 km
State Variables

• The collection of all information needed to define what is happening within an entity or system to sufficient level at a given point in time.

• A soldier state variables could be:
  – Body posture: standing, running, ...
  – Head direction: -80 ... +80 degrees
  – Ammunition level: 0 .. 20 bullets
  – Health: alive, injured, dead

Event

• An occurrence that changes the state of a system.

• Event types:
  – Internal (endogenous) events
    • E.g. Beginning of a service at a bank.
  – External (exogenous) events
    • E.g. arrival of customers for service.
**Replication**

- A single simulation run is a random sequence of events that illustrated only one of the branches of all possible event flow combinations.
- Therefore, reaching a conclusion based on just a single run is not an appropriate way of analysis.
- To minimize effect of randomness, simulations are run multiple times with the same scenario and the results are averaged.
- Each of these runs are called a replication.

**Pixel**

- The smallest piece of information in an image data.
- Normally arranged in a regular 2D grid, and are often represented using dots or squares.
Value of Pixels

- Intensity/value of each pixel is variable:
  - In color systems, each pixel has typically 3 or 4 components such as red, green, blue and alpha.
  - In digital elevation models, each pixel is typically a height value such as elevation from sea level.

Polygon

- A plane figure that is bounded by a closed path or circuit, composed of a finite sequence of straight line segments.
- Segments are called edges or sides.
- Points where two edges meet are the polygon's vertices or corners.
- Interior of the polygon is called its body.
Characterization of Polygons

- **Convex**: Any line drawn through the polygon (and not tangent to an edge or corner) meets its boundary exactly twice.
- **Concave**: Non-convex.

- **Simple**: The boundary of the polygon does not cross itself. All convex polygons are simple.

Characterization of Polygons

- **Star-shaped**: There exists a point that whole interior is visible from without crossing any edge. The polygon must be simple.
- **Self-intersecting**: Boundary of the polygon crosses itself.

- **Star-Polygon**: A polygon which self-intersects in a regular way.
- **Polygon with Holes**: A polygon having interior boundaries that generates holes.
Polygonal Models

- A three dimensional model of an object that is created by building polygons (usually triangles) from the points in a point cloud.

- A faceted three dimensional model of an object.

Voxel

- A volume element, representing a value on a regular grid in three dimensional space.
- This is analogous to a pixel, which represents 2D image data.
- Voxels are frequently used in the visualization and analysis of medical and scientific data.
Voxel Models

• A three-dimensional model of an object that is represented by voxels (created with voxelization).

Resolution

• The level of detail a model is represented.
Fidelity

- The degree to which a model or simulation reproduces the state and behaviour of a real world system.
- Fidelity is therefore a measure of the realism of a model or simulation.
- A high resolution model does not always mean a high fidelity model.

Aggregation & Disaggregation

- The grouping/ungrouping of a number of entities for low/high fidelity modeling and/or visualization
Interoperability

• The ability of diverse systems to work together (inter-operate).

If two systems are interoperable, they can work together.

Reusability

• The likelihood a segment of source code can be used again to add new functionalities with slight or no modification.
• Reusable modules and classes;
  – Reduce implementation time,
  – Increase the likelihood that prior testing and use has eliminated bugs,
  – Localizes code modifications when a change in implementation is required.
Frame

- One of the many single;
  - Photographic images in a motion picture, or
  - Time instant in a simulation run.

Simulator

- A software & hardware integrated system that creates an environment that is as close as possible to reality for the purpose of training or research.
Computer Generated Forces

- Simulated military entities (e.g. soldiers, tanks) capable of acting autonomously in a simulation environment using artificial intelligence techniques.

Distributed Simulation

- An integrated simulation that is partitioned into a number of smaller simulations over different computational units (e.g. processors, computers).
- Provides higher scalability and multi user interaction.
- A system, whose performance improves proportional to the computational capacity added, is said to be a scalable system.
High Level Architecture (HLA)

- An IEEE (1516) standard for developing distributed simulations.
- The concept was developed by Defense Modeling & Simulation Office (DMSO).
  - Current technology was not providing tools necessary to achieve DoD M&S Master Plan.
  - A standard was needed for the interoperability of developed simulations.
High Level Architecture (HLA)

- Simulations (Federates) → Integrated Simulation (Federation)
- In practice: A Centralized Approach
- All Simulations Communicate via RTI

RUN TIME INFRASTRUCTURE (RTI)

- Federation Management
- Declaration Management
- Object Management
- Ownership Management
- Time Management
- Data Distribution Management

Conceptual Model of The Mission Space (CMMS)

- Developing simulations for some specific domains such as military operations requires knowledge of the domain (mission space).
- But that mission space knowledge is not usually readily available for the developers.
  - It is incomplete, ambiguous or defined in an informal way.
- In such a case,
  - It is not possible to develop a high fidelity model and simulation.
Conceptual Model of The Mission Space (CMMS)

- CMMS is a bridge between subject matter experts (SME*) and developers, which describes in a consistent way how the real world runs within a particular domain.

SME* : A person who is an expert in a particular area.

Steps of CMMS Development

- Collect authoritative simulation context information;
  To develop scope of simulation, which describes the domain that a simulation is to address.
- Identify entities and processes;
  That must be represented for the simulation to accomplish its objectives.
- Develop simulation elements;
  To represent entities and processes.
- Address relationships among simulation elements;
  To ensure that constraints and boundary conditions imposed by the simulation context are accommodated.
Representing CMMS Formally

Verification & Validation

- Real-world system under investigation is abstracted by a conceptual model.
- Conceptual model is then coded into operational model.
- Hopefully, operation model is a correct representation of real-world system.
- We need more than hope.
- To ensure correctness, we have to perform verification and validation.
Verification

• Determination of whether the computer implementation of the conceptual model is correct.

• Question:
  – Does the operational model accurately reflects the conceptual model?

• To get an answer:
  – Examine the simulation program in details and compare to the conceptual model.

Verification

• Commonsense ways to perform verification:
  – Follow the principles of structured programming (detailed plans, top-down design, flow charts, etc.).
  – Make operational model as self-documenting as possible (comments, graphical software).
  – Have computer code checked by more than one person.
Verification

• Commonsense ways to perform verification:
  – Check to see that values of input data are being used appropriately (e.g. units).
  – For a variety of input-data values, ensure that outputs are reasonable.
  – Use an interactive run controller or debugger to check that program operates as intended (e.g. execute model step by step).
  – Visualisation is a very useful verification tool (e.g. detect actions that are illogical).

Validation

• Determination of whether the conceptual model can be substituted for the real system for the purpose of experimentation.
• A variety of subjective and objective techniques can be used to validate the conceptual model.
Validation

• Subjective techniques to perform validation:
  – Face validation: Model must appear reasonable to the subject matter experts.
  – Sensitivity analysis: When model input is changed, output should change in a predictable direction.
  – Extreme condition test: Check whether model behaves properly when input data are at the extremes.

Validation

• Subjective techniques to perform validation:
  – Validation of conceptual model assumptions:
    • Check structural and data assumptions with appropriate personnel (experts, consultants).
    • No one person knows everything about the entire system.
    • So, many people are required.
Validation

• Subjective techniques to perform validation:
  – Consistency checks:
    • Continue to examine operation model over time.
    • Detect significant changes in real-system that would effect correctness of simulation model.

• Subjective techniques to perform validation:
  – Turing tests:
    • Experimentally compare model outputs to system outputs with experts.
    • Make experts distinguish the ones that are simulated.
    • If a substantial number of simulated ones are identified, simulation model is inadequate.
Validation

• Objective techniques to perform validation:
  – Validating input-output transformations:
    • Compare model output to the output of real-system if possible (e.g. using t-test).
  – Validation using historical input data:
    • Drive operational model with historical records.
    • Output should stay within acceptable statistical error of those observed from real-world system.