# CS481: Bioinformatics Algorithms

Can Alkan
EA509
calkan@cs.bilkent.edu.tr

**http://www.cs.bilkent.edu.tr/~calkan/teaching/cs481/**

# EXACT STRING MATCHING

Eileen Kraemer

# The problem of String Matching

Given a string ‘t’, the problem of string matching deals with finding whether a pattern ‘p’ occurs in ‘t’ and if ‘p’ does occur then returning position in ‘t’ where ‘p’ occurs.

# Brute force (O(mn))

```
n <- |t|
m <- |p|
i <= 1
while i < n
  if p == t[i, i+m-1]
     return i;
  else
     i = i + 1;
```

# SimpleStringSearch

| *t[0]* | *t[1]* | *t[2]* | *t[3]* | *t[4]* | *t[5]* | *t[6]* | *t[7]* | *t[8]* | *t[9]* | *t10]* |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| ***p[0]*** | ***p[1]*** | ***p[2]*** | ***p[3]*** |
|---|---|---|---|
| A | B | C | D |

**Y** **Y** **Y** **N**

# SimpleStringSearch

| t[0] | t[1] | t[2] | t[3] | t[4] | t[5] | t[6] | t[7] | t[8] | t[9] | t10] |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| p[0] | p[1] | p[2] | p[3] |
|---|---|---|---|
| A | B | C | D |

**N**

# SimpleStringSearch

| *t[0]* | *t[1]* | *t[2]* | *t[3]* | *t[4]* | *t[5]* | *t[6]* | *t[7]* | *t[8]* | *t[9]* | *t10]* |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| ***p[0]*** | ***p[1]*** | ***p[2]*** | ***p[3]*** |
|---|---|---|---|
| A | B | C | D |

**N**

# SimpleStringSearch

| t[0] | t[1] | t[2] | t[3] | t[4] | t[5] | t[6] | t[7] | t[8] | t[9] | t10] |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| p[0] | p[1] | p[2] | p[3] |
|---|---|---|---|
| A | B | C | D |

**N**

# SimpleStringSearch

| t[0] | t[1] | t[2] | t[3] | t[4] | t[5] | t[6] | t[7] | t[8] | t[9] | t10] |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| p[0] | p[1] | p[2] | p[3] |
|---|---|---|---|
| A | B | C | D |

**N**

# SimpleStringSearch

| t[0] | t[1] | t[2] | t[3] | t[4] | t[5] | t[6] | t[7] | t[8] | t[9] | t10] |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| p[0] | p[1] | p[2] | p[3] |
|---|---|---|---|
| A | B | C | D |

**N**

# SimpleStringSearch

| t[0] | t[1] | t[2] | t[3] | t[4] | t[5] | t[6] | t[7] | t[8] | t[9] | t10] |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| | p[0] | p[1] | p[2] | p[3] |
|---|---|---|---|---|
| | A | B | C | D |
| | Y | Y | Y | Y |

# Straightforward string searching

- Worst case:
  - Pattern string always matches completely except for last character
  - Example: search for XXXXXXY in target string of XXXXXXXXXXXXXXXXXXXXX
  - Outer loop executed once for every character in target string
  - Inner loop executed once for every character in pattern
  - O(mn), where m = |p| and n = |t|
- OK if patterns are short, but better algorithms exist

# SimpleStringSearch

| *t[0]* | *t[1]* | *t[2]* | *t[3]* | *t[4]* | *t[5]* | *t[6]* | *t[7]* | *t[8]* | *t[9]* | *t10]* |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| ***p[0]*** | ***p[1]*** | ***p[2]*** | ***p[3]*** |
|---|---|---|---|
| A | B | C | D |

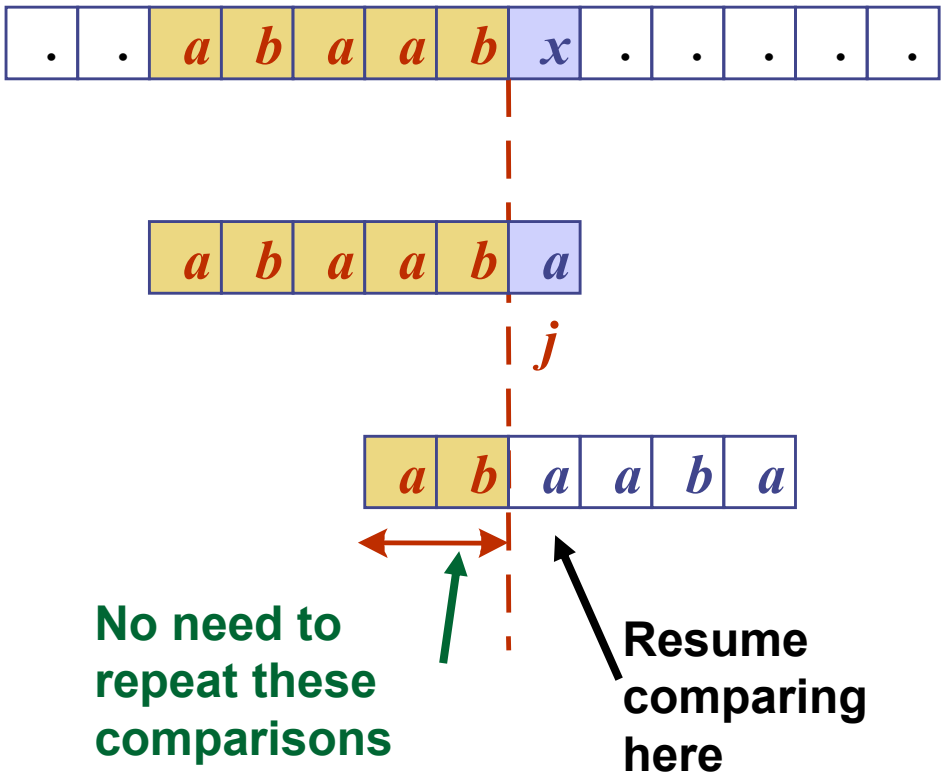
**Y** **Y** **Y** **N**

# Knuth-Morris-Pratt

- O(m+n)
- Key idea:
  - if pattern fails to match, slide pattern to right by as many boxes as possible without permitting a match to go unnoticed

# The KMP Algorithm - Motivation

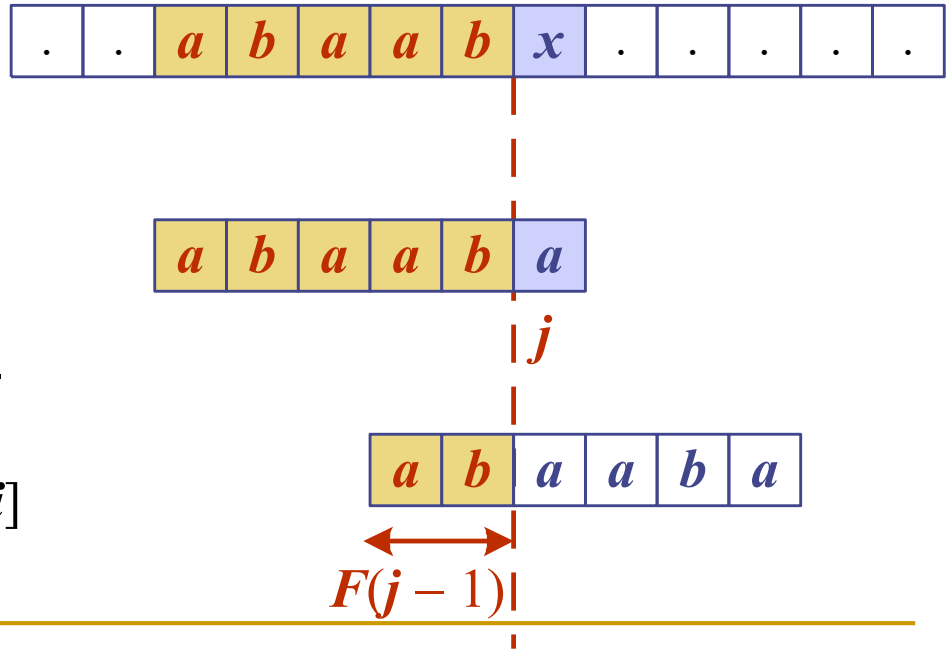
- Knuth-Morris-Pratt's algorithm compares the pattern to the text in **left-to-right**, but shifts the pattern more intelligently than the brute-force algorithm.
- When a mismatch occurs, what is the **most** we can shift the pattern so as to avoid redundant comparisons?
- Answer: the largest prefix of $P[0..j]$ that is a suffix of $P[1..j]$



# KMP Failure Function

- Knuth-Morris-Pratt's algorithm preprocesses the pattern to find matches of prefixes of the pattern with the pattern itself
- The **failure function** $F(j)$ is defined as the size of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$
- Knuth-Morris-Pratt's algorithm modifies the brute-force algorithm so that if a mismatch occurs at $P[j] \neq T[i]$ we set $j \leftarrow F(j\text{-}1)$

| $j$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P[j]$ | $a$ | $b$ | $a$ | $a$ | $b$ | $a$ |
| $F(j)$ | 0 | 0 | 1 | 1 | 2 | 3 |



# The KMP Algorithm

- The failure function can be represented by an array and can be computed in $O(m)$ time
- At each iteration of the while-loop, either
  - $i$ increases by one, or
  - the shift amount $i - j$ increases by at least one (observe that $F(j-1) < j$)
- Hence, there are no more than $2n$ iterations of the while-loop
- Thus, KMP's algorithm runs in optimal time $O(m + n)$

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

# Computing the Failure Function

- The failure function can be represented by an array and can be computed in $O(m)$ time
- The construction is similar to the KMP algorithm itself
- At each iteration of the while-loop, either
  - $i$ increases by one, or
  - the shift amount $i - j$ increases by at least one (observe that $F(j-1) < j$)
- Hence, there are no more than $2m$ iterations of the while-loop

```
Algorithm failureFunction(P)
    F[0] ← 0
    i ← 1
    j ← 0
    m ← length(P)
    while i < m
        if P[i] = P[j]
        {we have matched j + 1 chars}
            F[i] ← j + 1
            i ← i + 1
            j ← j + 1
        else if j > 0 then
        {use failure function to shift P}
            j ← F[j − 1]
        else
            F[i] ← 0 { no match }
            i ← i + 1
```

# Example

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *a* | *b* | *a* | *c* | *a* | *a* | *b* | *a* | *c* | *c* | *a* | *b* | *a* | *c* | *a* | *b* | *a* | *a* | *b* | *b* |

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| *a* | *b* | *a* | *c* | *a* | *b* |

| 7 | | | | |
|---|---|---|---|---|
| *a* | *b* | *a* | *c* | *a* | *b* |

| 8 | 9 | 10 | 11 | 12 | |
|---|---|---|---|---|---|
| *a* | *b* | *a* | *c* | *a* | *b* |

| 13 | | | | | |
|---|---|---|---|---|---|
| *a* | *b* | *a* | *c* | *a* | *b* |

| 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|
| *a* | *b* | *a* | *c* | *a* | *b* |

| $j$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P[j]$ | *a* | *b* | *a* | *c* | *a* | *b* |
| $F(j)$ | 0 | 0 | 1 | 0 | 1 | 2 |

# The Boyer-Moore Algorithm

- Similar to KMP in that:
  - Pattern compared against target
  - On mismatch, move as far to right as possible
- Different from KMP in that:
  - Compare the patterns from right to left instead of left to right
- Does that make a difference?
  - Yes – much faster on long targets; many characters in target string are never examined at all

# Boyer-Moore example

| *t[0]* | *t[1]* | *t[2]* | *t[3]* | *t[4]* | *t[5]* | *t[6]* | *t[7]* | *t[8]* | *t[9]* | *t10]* |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| ***p[0]*** | ***p[1]*** | ***p[2]*** | ***p[3]*** |
|---|---|---|---|
| A | B | C | D |

**N**

**There is no E in the pattern : thus the pattern can't match if *any* characters lie under t[3]. So, move four boxes to the right.**

# Boyer-Moore example

| t[0] | t[1] | t[2] | t[3] | t[4] | t[5] | t[6] | t[7] | t[8] | t[9] | t10] |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| p[0] | p[1] | p[2] | p[3] |
|---|---|---|---|
| A | B | C | D |

N

**Again, no match. But there is a B in the pattern. So move two boxes to the right.**

# Boyer-Moore example

| t[0] | t[1] | t[2] | t[3] | t[4] | t[5] | t[6] | t[7] | t[8] | t[9] | t10] |
|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | E | F | G | A | B | C | D | E |

| p[0] | p[1] | p[2] | p[3] |
|---|---|---|---|
| A | B | C | D |
| Y | Y | Y | Y |

# Boyer-Moore : another example

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *t[k]* | *t[k+1]* | … | | *t[k+i]* | | | | *t[k+m-1]* | |
| | | | … | | c | E | … | R | G | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ***p[0]*** | ***p[1]*** | ***…*** | ***p[i-1]*** | ***p[i]*** | ***p[i+1]*** | ***…*** | | ***p[m-1]*** |
| L | E | … | S | D | E | … | R | G |
| | | | | **N** | **Y** | **Y** | **Y** | **Y** |

**Problem: determine d, the number of boxes that the pattern can be moved to the right.**

**d should be smallest integer such that t[k+m-1]= p[m-1-d], t[k+m-2] = p[m-2-d], … t[k+i] = p[i-d]**

# The Boyer-Moore Algorithm

- We said:
  - d should be smallest integer such that:
    - T[k+m-1] = p[m-1-d]
    - T[k+m-2] = p[m-2-d]
    - T[k+i] = p[i-d]
  - Reminder:
    - k = starting index in target string
    - m = length of pattern
    - i = index of mismatch in pattern string
  - Problem: statement is valid only for d<= i
    - Need to ensure that we don't "fall off" the left edge of the pattern

# Boyer-Moore : another example

| t[k] | | | | | | t[k+5] | | | t[k+8] | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | c | X | Y | Z | |

| p[0] | p[1] | p[2] | p[3] | p[4] | p[5] | p[6] | p[7] | p[8] |
|---|---|---|---|---|---|---|---|---|
| Y | Z | W | X | Y | Z | X | Y | Z |
| | | | | | N | Y | Y | Y |

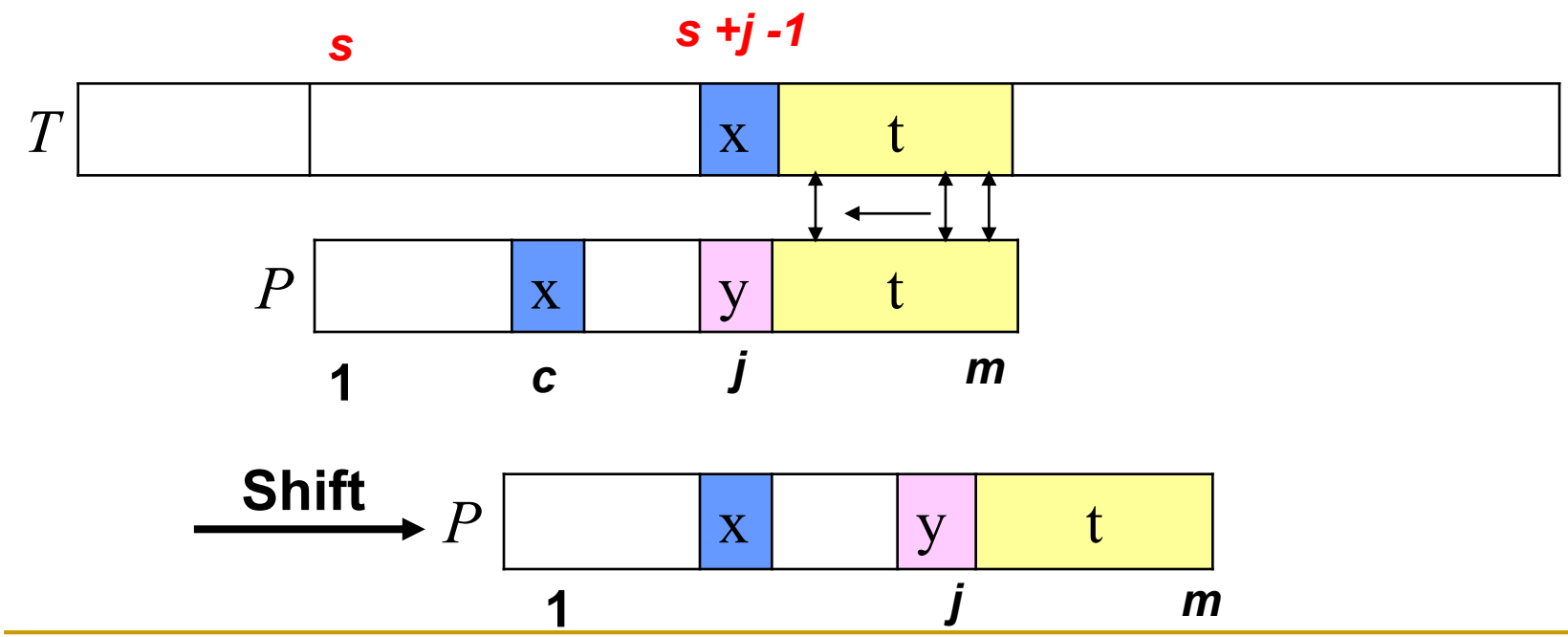
**If c == W, then d should be 3**

**If c == R, then d should be 7**

# Bad Character Rule

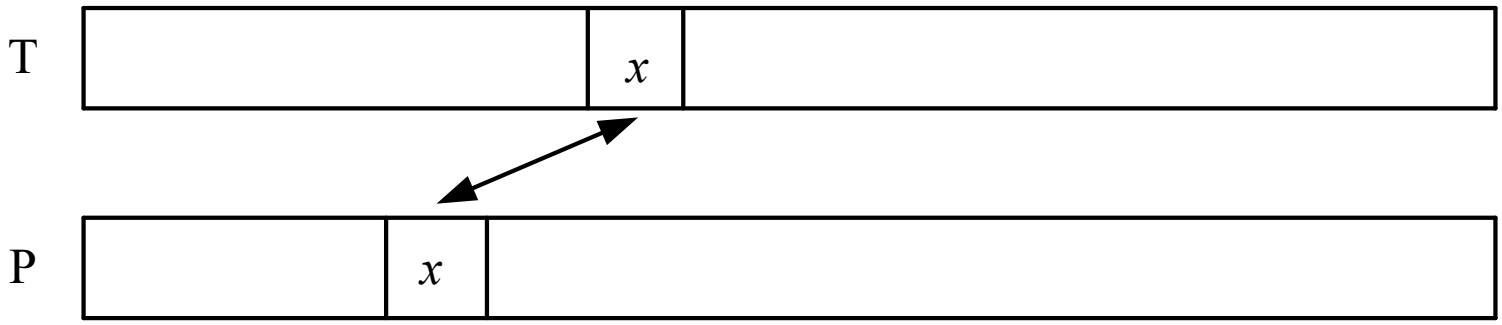
Suppose that $P_1$ is aligned to $T_s$ now, and we perform a pair-wise comparing between text $T$ and pattern $P$ from right to left. Assume that the first mismatch occurs when comparing $T_{s+j-1}$ with $P_j$.

Since $T_{s+j-1} \neq P_j$, we move the pattern $P$ to the right such that the largest position $c$ in the left of $P_j$ is equal to $T_{s+j-1}$. We can shift the pattern at least ($j$-$c$) positions right.



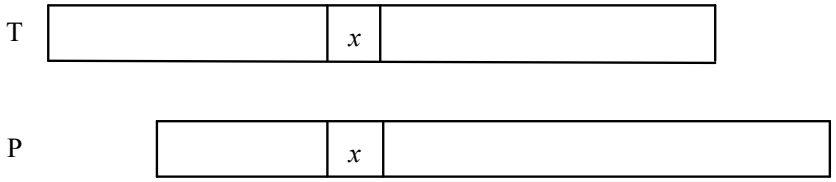
# Character Matching Rule

- Bad character rule uses Character Matching Rule.
- For any character $x$ in $T$, find the nearest $x$ in $P$ which is to the left of $x$ in $T$.

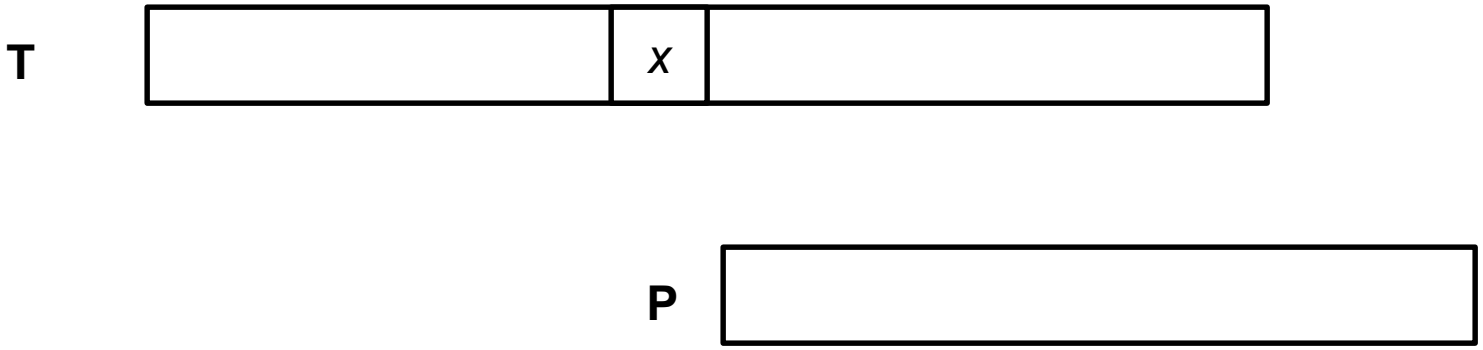


# Implication of Character M. Rule

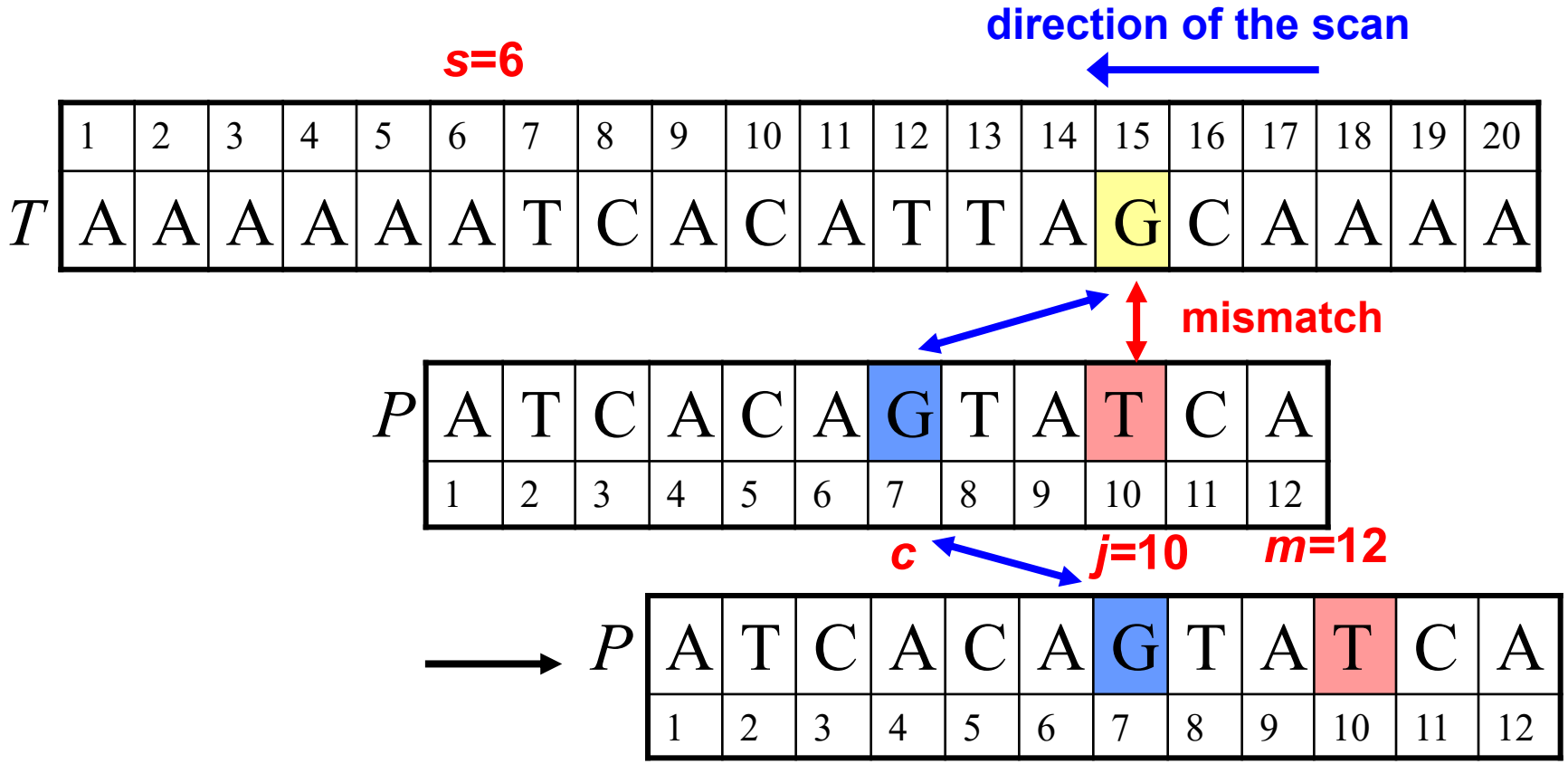
- Case 1. If there is a x in P to the left of T, move P so that the two x's match.



- Case 2: If no such a x exists in P, move P to the right of x



Ex: Suppose that P1 is aligned to T6 now. We compare pairwise between T and P from right to left. Since T16,17 = P11,12 = “CA” and T15 =“G” ≠P10 = “T”. Therefore, we find the rightmost position c=7 in the left of P10 in P such that Pc is equal to “G” and we can move the window at least (10-7=3) positions.



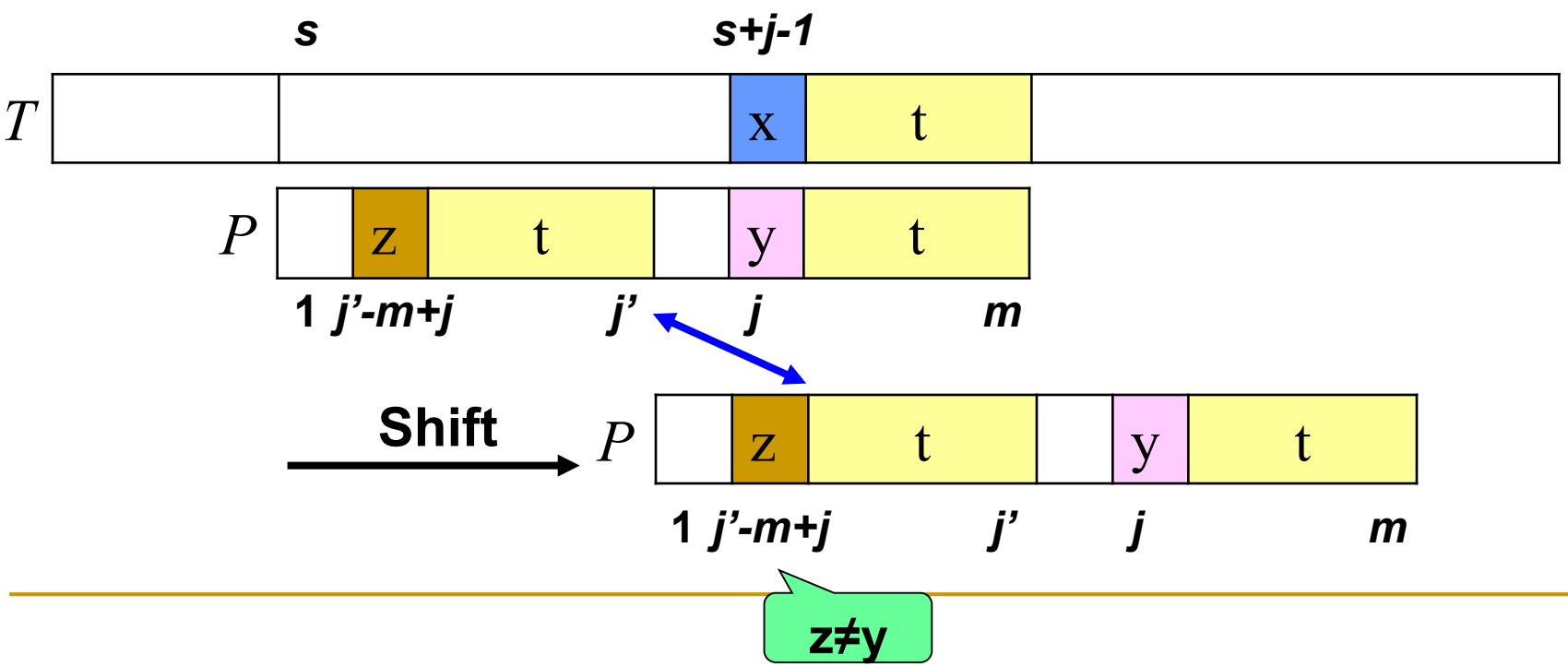
# Good Suffix Rule 1

- If a mismatch occurs in $T_{s+j-1}$, we match $T_{s+j-1}$ with $P_{j'-m+j}$ , where $j'$ ($m-j+1 \leqq j' < m$) is the **largest position** such that

  (1) $P_{j+1,m}$ **is a suffix of** $P_{1,j'}$

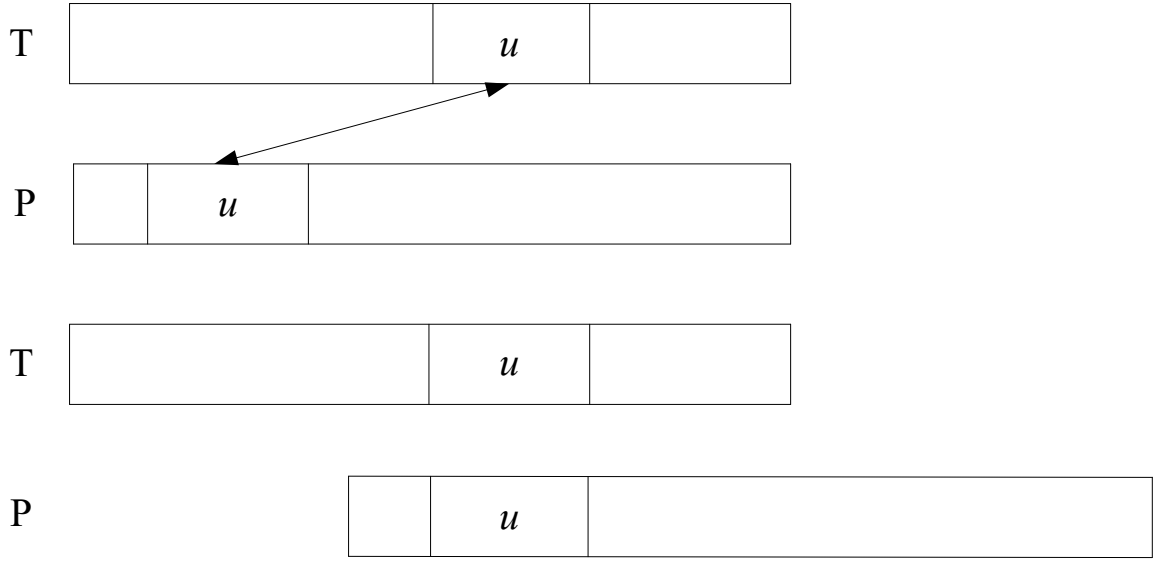
  (2) $P_{j'-(m-j)} \neq P_j$.

- We can move the window at least ($m-j'$) position(s).

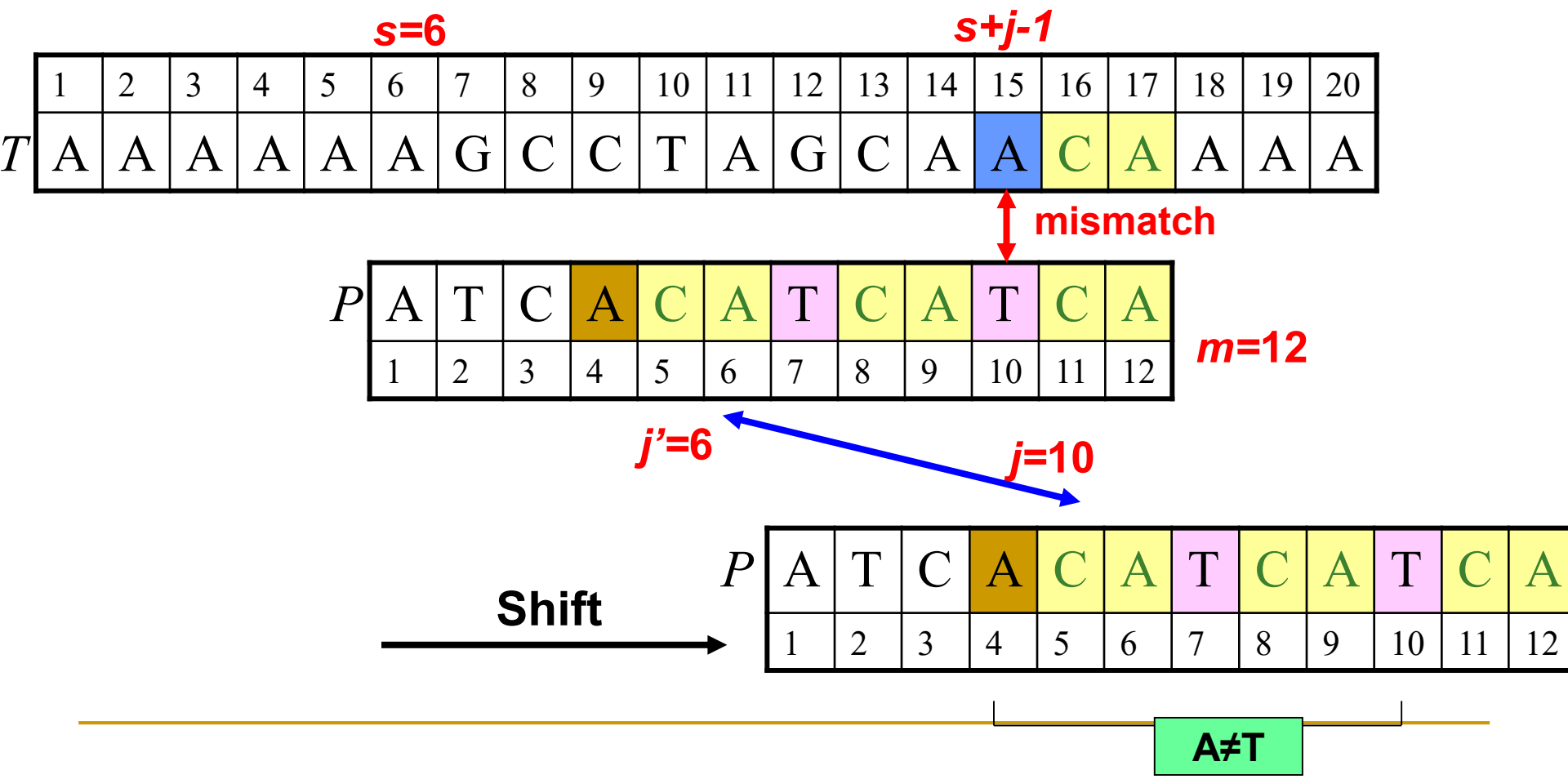


# The Substring Matching Rule

- For any substring u in T, find a nearest u in P which is to the left of it. If such a u in P exists, move P;



Ex: Suppose that P1 is aligned to T6 now. We compare pair-wise between P and T from right to left. Since T16,17 = “CA” = P11,12 and T15 =“A” ≠P10 = “T”. We find the substring “CA” in the left of P10 in P such that “CA” is the suffix of P1,6 and the left character to this substring “CA” in P is not equal to P10 = “T”. Therefore, we can move the window at least m-j’ (12-6=6) positions right.

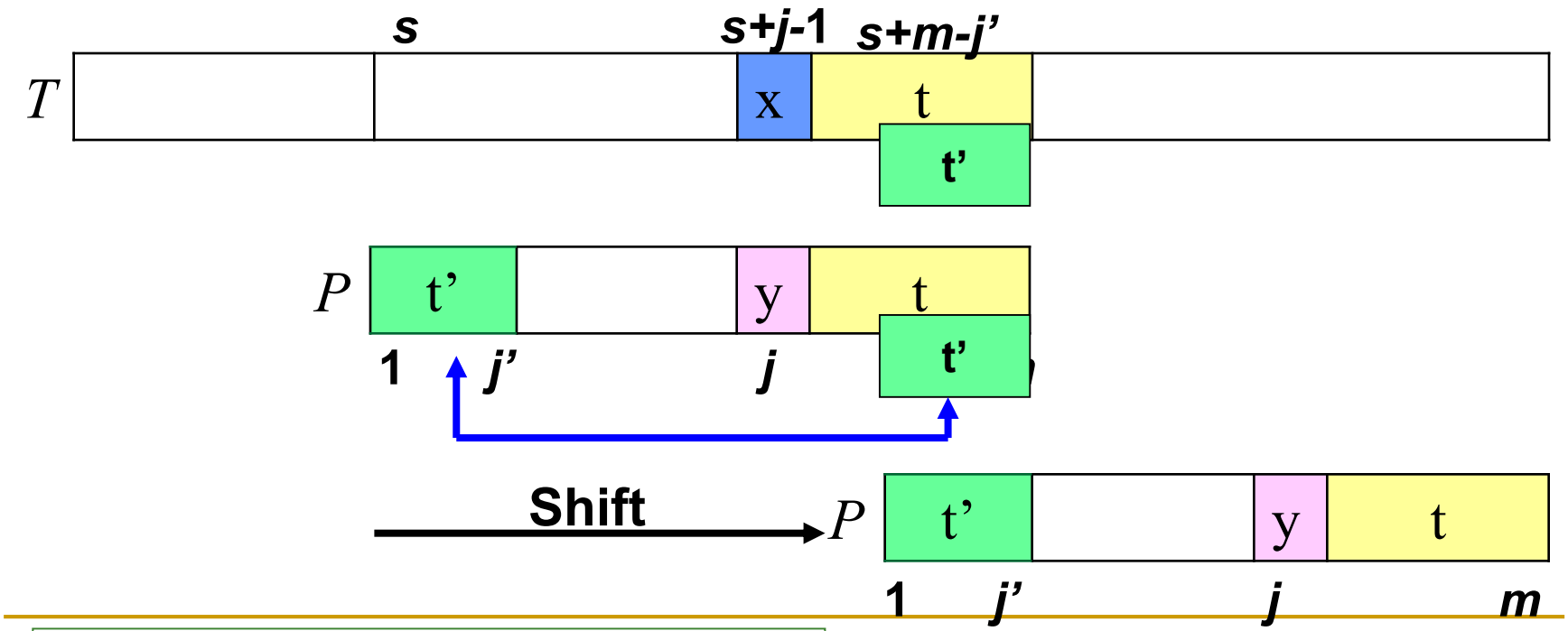


# Good Suffix Rule 2

**Good Suffix Rule 2 is used only when Good Suffix Rule 1 can not be used. That is, t does not appear in P(1, j). Thus, t is unique in P.**

- If a mismatch occurs in $T_{s+j-1}$, we match $T_{s+m-j'}$ with $P_1$, where $j'$ ($1 \leqq j' \leqq m-j$) is **the largest position** such that

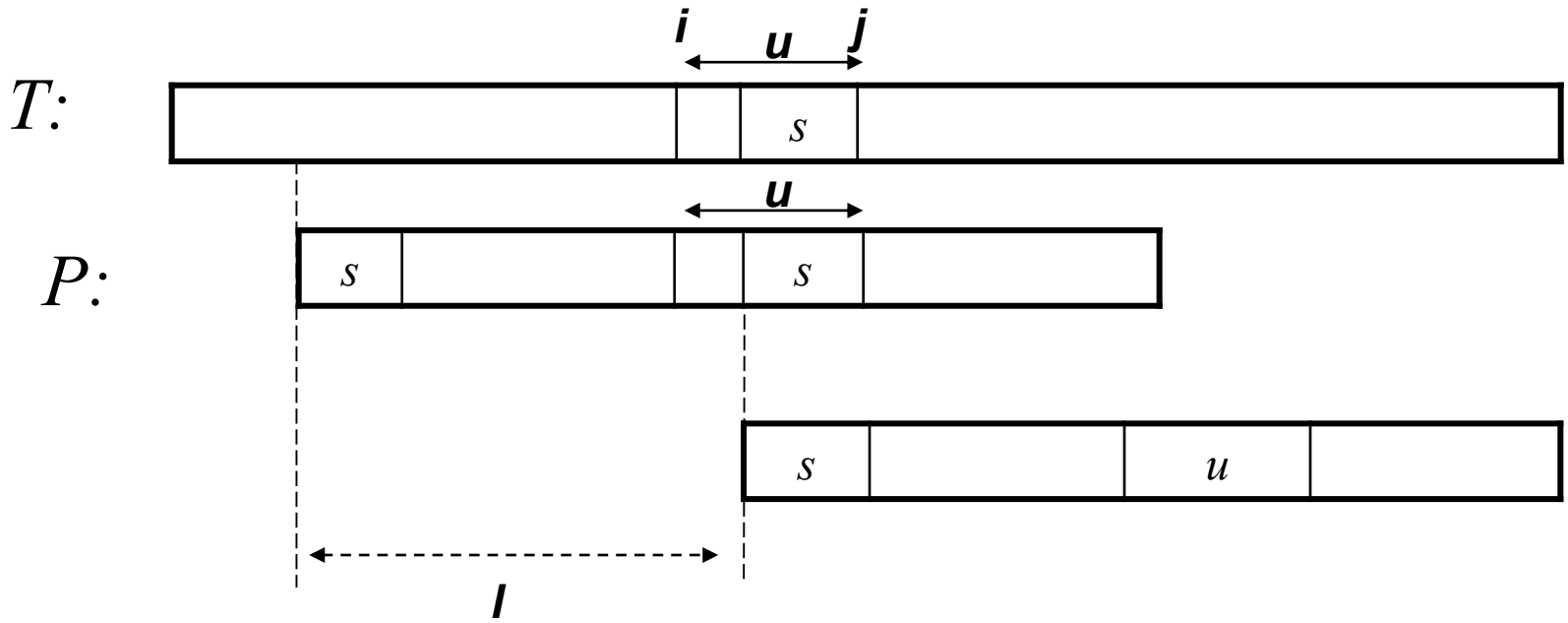
  ***$P_{1,j'}$* is a suffix of *$P_{j+1,m}$*.**




**P.S. : t' is suffix of substring t.**

# Unique Substring Rule

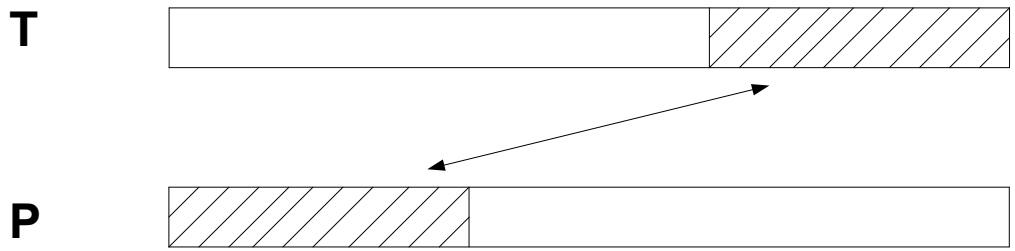
- The substring $u$ appears in $P$ exactly once.
- If the substring $s$ matches with $T_{i,j}$, no matter whether a mismatch occurs in some position of $P$ or not, we can slide the window by $l$.




The string $s$ is the longest prefix of $P$ which equals to a suffix of $u$.

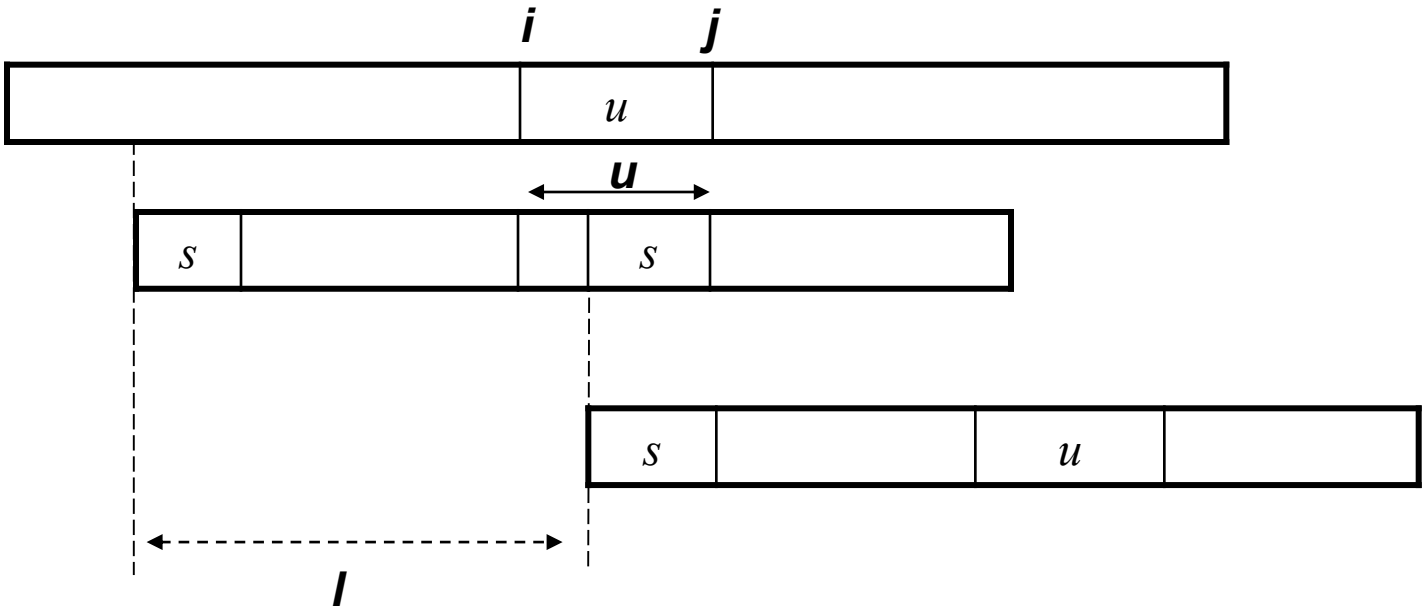
# The Suffix to Prefix Rule

- For a window to have any chance to match a pattern, in some way, there must be a suffix of the window which is equal to a prefix of the pattern.

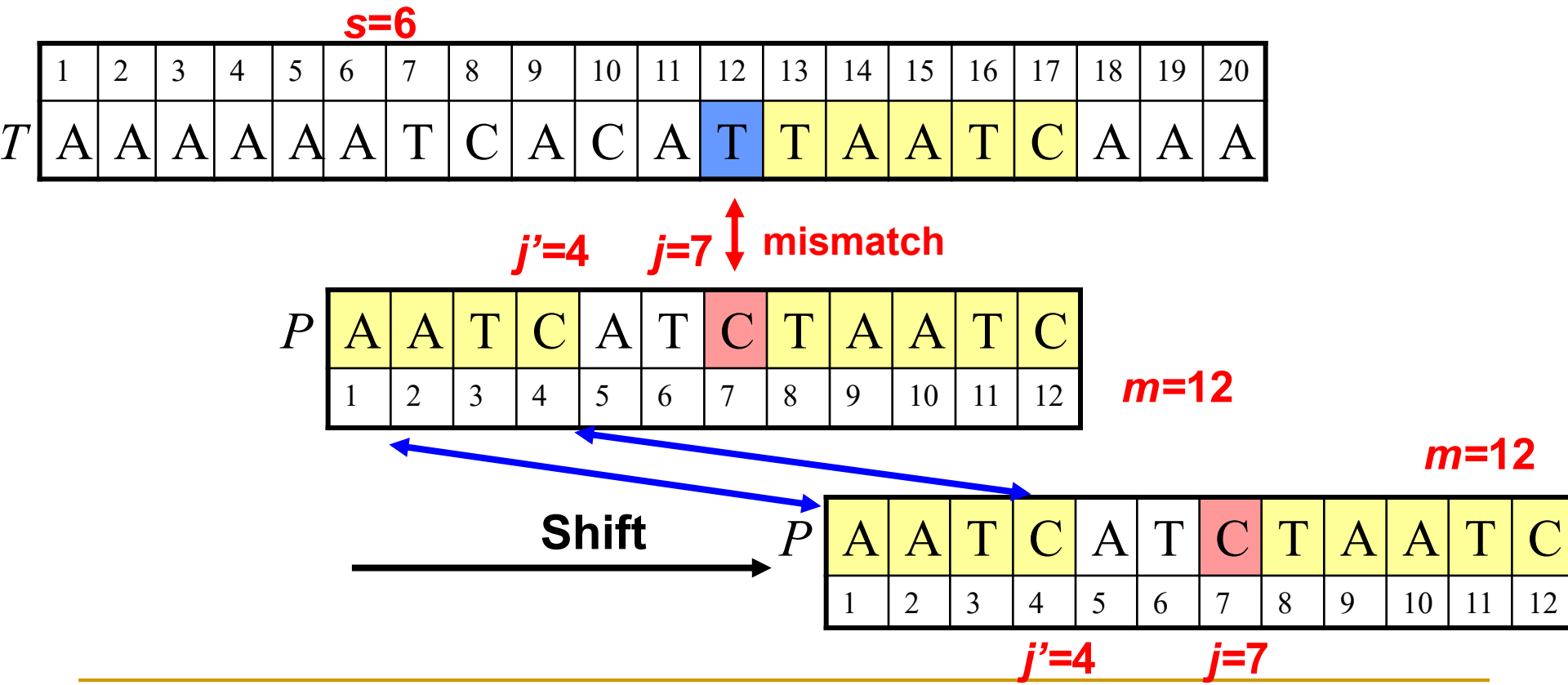


# The Suffix to Prefix Rule

- Note that the above rule also uses Rule 1.
- It should also be noted that the unique substring is the shorter and the more right-sided the better.
- A short u guarantees a short (or even empty) s which is desirable.



- Ex: Suppose that $P_1$ is aligned to $T_6$ now. We compare pair-wise between $P$ and $T$ from right to left. Since $T_{12} \neq P_7$ and there is no substring $P_{8,12}$ in left of $P_8$ to exactly match $T_{13,17}$. We find a longest suffix "AATC" of substring $T_{13,17}$, the longest suffix is also prefix of $P$. We shift the window such that the last character of prefix substring to match the last character of the suffix substring. Therefore, we can shift at least 12-4=8 positions.



- Let ***B*(*a*)** be the rightmost position of *a* in *P[1..j]*. The function will be used for applying ***bad character rule***.

| *j* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *P* | A | T | C | A | C | A | T | C | A | T | C | A |

| Σ | A | C | G | T |
|---|---|---|---|---|
| *B[12]* | 12 | 11 | 0 | 10 |
| … | | | | |
| *B[10]* | 9 | 8 | 0 | 10 |
| … | | | | |

- We can move our pattern right at least ***j-B[j]*($T_{s+j-1}$)** position by above ***B*** function.

| *j* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *T* | A | G | C | T | A | G | C | C | T | G | C | A | C | G | T | A | C | A |

| *j* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *P* | A | T | C | A | C | A | T | C | A | T | C | A |

**Move at least**
**10-*B[10]*(*G*) = 10 positions**

**Let *Gs*(*j*) be the largest number of shifts by *good suffix rule* when a mismatch occurs for comparing $P_j$ with some character in *T*.**

- $gs_1(j)$ be the largest $k$ such that $P_{j+1,m}$ **is a suffix of** $P_{1,k}$ **and** $P_{k-m+j} \neq P_j$ where $m-j+1 \leqq k < m$ ; 0 if there is no such $k$.

  ($gs_1$ is for Good Suffix Rule 1)

- $gs_2(j)$ be the largest $k$ such that $P_{1,k}$ **is a suffix of** $P_{j+1,m}$ where $1 \leqq k \leqq m-j$; 0 if there is no such $k$.

  ($gs_2$ is for Good Suffix Rule 2.)

- $Gs(j) = m - \max\{gs_1, gs_2\}$, if $j = m$ , $Gs(j)=1$.

| $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P$ | | | | | | | | | | | | |
| $gs_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 0 | 0 | 6 | 1 | 0 |
| $gs_2$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 0 |
| $Gs$ | 8 | 8 | 8 | 8 | 8 | 8 | 3 | 8 | 11 | 6 | 11 | 1 |

**$gs_1(7)=9$**

**$\because P_{8,12}$ is a suffix of $P_{1,9}$ and $P_4 \neq P_7$**

**$gs_2(7)=4$**

**$\because P_{1,4}$ is a suffix of $P_{8,12}$**

| $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P$ | **A** | **T** | **C** | **A** | **C** | **A** | **T** | **C** | **A** | **T** | **C** | **A** |
| $gs_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 0 | 0 | 6 | 1 | 0 |
| $gs_2$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 0 |
| $Gs$ | 8 | 8 | 8 | 8 | 8 | 8 | 3 | 8 | 11 | 6 | 11 | 1 |

**s=6** **s+j-1**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | A | A | A | A | A | A | G | C | C | T | A | G | C | A | A | C | A | A | A | A |

**mismatch**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P$ | A | T | C | A | C | A | T | C | A | T | C | A |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

**m=12**

**j'=6** **j=10**

**Shift**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P$ | A | T | C | A | C | A | T | C | A | T | C | A |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

# Time Complexity

- Use good character or bad suffix rule
  - The one that skips more
- The preprocessing phase in $O(m+\Sigma)$ complexity
- If you are searching for ALL matches, worst case:
  - $O(mn)$ when P is in T at all positions
    - T=AAAAAAAAAAA; P=AAAA
  - $O(m+n)$ when P is not in T

# BRUTE FORCE – EXAMPLE #2

# Brute force

```
T = ABABABCABABABCABABAC
P = ABABAC
            x
```

Comparisons: 6

```
T = ABABABCABABABCABABAC
P =   ABABAC
     x
```

Comparisons: 1

```
T = ABABABCABABABCABABAC
P =    ABABAC
              x
```

Comparisons: 5

```
T = ABABABCABABABCABABAC
P =      ABABAC
            x
```

Comparisons: 1

```
T = ABABABCABABABCABABAC
P =         ABABAC
              x
```

Comparisons: 3

# Brute force

```
T = ABABABCABABABCABABAC
P =          ABABAC
             x
```

Comparisons: 1

```
T = ABABABCABABABCABABAC
P =           ABABAC
              x
```

Comparisons: 1

```
T = ABABABCABABABCABABAC
P =             ABABAC
                      x
```

Comparisons: 6

```
T = ABABABCABABABCABABAC
P =               ABABAC
                  x
```

Comparisons: 1

```
T = ABABABCABABABCABABAC
P =                ABABAC
                         x
```

Comparisons: 5

# Brute force

```
T = ABABABCABABABCABABAC
P =                ABABAC
                   x
```

Comparisons: 1

```
T = ABABABCABABABCABABAC
P =                 ABABAC
                     x
```

Comparisons: 3

```
T = ABABABCABABABCABABAC
P =                  ABABAC
                      x
```

Comparisons: 1

```
T = ABABABCABABABCABABAC
P =                   ABABAC
                       x
```

Comparisons: 1

```
T = ABABABCABABABCABABAC
P =                    ABABAC
```

*match*

Comparisons: 6

Total comparisons: 41

# KMP – EXAMPLE #2

# Knuth-Morris-Pratt

T = ABABABCABABABCABABAC
P = ABABAC

Reminder: $F(j)$ is defined as the size of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$

| | A | B | A | B | A | C |
|---|---|---|---|---|---|---|
| F | 0 | 0 | 1 | 2 | 3 | 0 |

```
Algorithm failureFunction(P)
    F[0] ← 0
    i ← 1
    j ← 0
    m ← length(P)
    while i < m
        if P[i] = P[j]
        {we have matched j + 1 chars}
            F[i] ← j + 1
            i ← i + 1
            j ← j + 1
        else if j > 0 then
        {use failure function to shift P}
            j ← F[j − 1]
        else
            F[i] ← 0 { no match }
            i ← i + 1
```

# Knuth-Morris-Pratt

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | A | B | A | B | A | C |
| **F** | 0 | 0 | 1 | 2 | 3 | 0 |

**T = ABABABCABABABCABABAC**
**P = ABABAC**

**x**

**j = 5, i = 5**

**New j = F[4] = 3 (shift 5-3 = 2)**
**New i = 5 (no change)**

**i=5**
**T = ABABABCABABABCABABAC**
**P = ABABAC**
**j=3**

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

**Comparisons = 6**

# Knuth-Morris-Pratt

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | A | B | A | B | A | C |
| **F** | 0 | 0 | 1 | 2 | 3 | 0 |

**T = ABABABCABABABCABABAC**
**P = ABABAC**
**x**

**j = 4, i = 6**

**New j = F[3] = 2 (shift 4-2 = 2)**
**New i = 6 (no change)**

**i=6**
**T = ABABABCABABABCABABAC**
**P = ABABAC**
**j=2**

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

**Comparisons = 6+5 = 11**

# Knuth-Morris-Pratt

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | A | B | A | B | A | C |
| **F** | 0 | 0 | 1 | 2 | 3 | 0 |

**T = ABABABCABABABCABABAC**
**P = ABABAC**
**x**

**j = 2, i = 6**

**New j = F[1] = 0 (shift 2-0 = 2)**
**New i = 6 (no change)**

**i=6**
**T = ABABABCABABABCABABAC**
**P = ABABAC**
**j=0**

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

**Comparisons = 11 + 3 = 14**

# Knuth-Morris-Pratt

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | A | B | A | B | A | C |
| **F** | 0 | 0 | 1 | 2 | 3 | 0 |

**T = ABABABCABABABCABABAC**
**P = ABABAC**
**x**

**j = 0, i = 6**

**New j = 0**
**New i = 7 (shift 1)**

**i=7**
**T = ABABABCABABABCABABAC**
**P = ABABAC**
**j=0**

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

**Comparisons = 14+1 = 15**

# Knuth-Morris-Pratt

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | A | B | A | B | A | C |
| **F** | 0 | 0 | 1 | 2 | 3 | 0 |

**T = ABABABCABABABCABABAC**
**P = ABABAC**
**x**

**j = 5, i = 12**

**New j = F[4] = 3 (shift 5-3=2)**
**New i = 12 (no change)**

**i=12**
**T = ABABABCABABABCABABAC**
**P = ABABAC**
**j=3**

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

**Comparisons = 15+6=21**

# Knuth-Morris-Pratt

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | A | B | A | B | A | C |
| **F** | 0 | 0 | 1 | 2 | 3 | 0 |

**T = ABABABCABABABCABABAC**
**P = ABABAC**
**x**

**j = 4, i = 13**

**New j = F[3] = 2 (shift 4-2=2)**
**New i = 13 (no change)**

**i=13**
**T = ABABABCABABABCABABAC**
**P = ABABAC**
**j=2**

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

**Comparisons = 21+5=26**

# Knuth-Morris-Pratt

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | A | B | A | B | A | C |
| **F** | 0 | 0 | 1 | 2 | 3 | 0 |

**T = ABABABCABABABCABABAC**
**P = ABABAC**
**x**

**j = 2, i = 13**

**New j = F[1] = 0 (shift 2-0=0)**
**New i = 13 (no change)**

**i=13**
**T = ABABABCABABABCABABAC**
**P = ABABAC**
**j=0**

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

**Comparisons = 26+3 = 29**

# Knuth-Morris-Pratt

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | A | B | A | B | A | C |
| **F** | 0 | 0 | 1 | 2 | 3 | 0 |

**T = ABABABCABABABCABABAC**
**P = ABABAC**
**x**

**j = 0, i = 13**

**New j = 0 (no shift)**
**New i = 14 (shift by 1)**

**i=14**
**T = ABABABCABABABCABABAC**
**P = ABABAC**
**j=0**

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j -1]
            else
                i ← i + 1
                j ← 0
    return −1 { no match }
```

**Comparisons = 29+6=35**

**Brute force: 41**

# BOYER-MOORE – EXAMPLE #2

# Boyer-Moore

T = ABABABCABABABCABABAC
P = ABABAC

Comparison: 1

x

T = ABABABCABABABCABABAC
P = ABABAC

Bad character rule

T = ABABABCABABABCABABAC
P = ABABAC

Good suffix rule
Note: no suffix matches in the previous step!!!

Pick bad character rule shift:

T = ABABABCABABABCABABAC
P = ABABAC

# Boyer-Moore

T = ABABABCABABABCABABAC
P = ABABAC

x

Comparison: 1

T = ABABABCABABABCABABAC
P = ABABAC

Bad character rule

T = ABABABCABABABCABABAC
P = ABABAC

Good suffix rule
Note: no suffix matches in the previous step!!!

Pick either shift:

T = ABABABCABABABCABABAC
P = ABABAC

# Boyer-Moore

T = ABABABCABABABCABABAC
P = ABABAC
x

Comparison: 1

T = ABABABCABABABCABABAC
P = ABABAC

Bad character rule

T = ABABABCABABABCABABAC
P = ABABAC

Good suffix rule
Note: no suffix matches in the previous step!!!

Pick bad character rule shift:

T = ABABABCABABABCABABAC
P = ABABAC

# Boyer-Moore

T = ABABABCABABABCABABAC
P = ABABAC
x

Comparison: 1

T = ABABABCABABABCABABAC
P = ABABAC

Bad character rule

T = ABABABCABABABCABABAC
P = ABABAC

Good suffix rule
Note: no suffix matches in the previous step!!!

Pick bad character rule shift:

T = ABABABCABABABCABABAC
P = ABABAC

# Boyer-Moore

T = ABABABCABABABCABABAC
P = ABABAC
x

Comparison: 1

T = ABABABCABABABCABABAC
P = ABABAC

Bad character rule

T = ABABABCABABABCABABAC
P = ABABAC

Good suffix rule
Note: no suffix matches in the previous step!!!

Pick bad character rule shift:

T = ABABABCABABABCABABAC
P = ABABAC

# Boyer-Moore

T = ABABABCABABABCABABAC
P = ABABAC
x

Comparison: 1

T = ABABABCABABABCABABAC
P = ABABAC

Bad character rule

T = ABABABCABABABCABABAC
P = ABABAC

Good suffix rule

Pick either:

T = ABABABCABABABCABABAC
P = ABABAC

# Boyer-Moore

T = ABABABCABABABCABABAC
P = ABABAC
x

Comparison: 1

T = ABABABCABABABCABABAC
P = ABABAC

Bad character rule

T = ABABABCABABABCABABAC
P = ABABAC

Good suffix rule

Pick bad character rule shift:

T = ABABABCABABABCABABAC
P = ABABAC

# Boyer-Moore

T = ABABABCABABABCABABAC
P = ABABAC
x

Comparison: 1

T = ABABABCABABABCABABAC
P = ABABAC

Bad character rule

T = ABABABCABABABCABABAC
P = ABABAC

Good suffix rule

Pick bad character rule shift and match:

T = ABABABCABABABCABABAC
P = ABABAC

Comparison: 6

Total comparisons: 14

# BOYER-MOORE – EXAMPLE #3

# Boyer-Moore

T = ABABABCABABABCABCBAB
P = ABCBAB
x

Comparison: 4

T = ABABABCABABABCABCBAB
P = ABCBAB

Bad character rule

T = ABABABCABABABCABCBAB
P = ABCBAB

Good suffix rule 2
Suffix (BAB) = prefix(P) = AB

Pick good suffix rule 2:

T = ABABABCABABABCABCBAB
P = ABCBAB

# Boyer-Moore

T = ABABABCABABABCABCBAB
P = ABCBAB
x

Comparison: 1

T = ABABABCABABABCABCBAB
P = ABCBAB

Bad character rule

T = ABABABCABABABCABCBAB
P = ABCBAB

Good suffix rule
No suffix match in previous step

Pick either:

T = ABABABCABABABCABCBAB
P = ABCBAB

# Boyer-Moore

T = ABABABCABABABCABCBAB
P = ABCBAB
x

Comparison: 4

T = ABABABCABABABCABCBAB
P = ABCBAB

Bad character rule

T = ABABABCABABABCABCBAB
P = ABCBAB

Good suffix rule 2
Suffix (BAB) = prefix(P) = AB

Pick good suffix rule 2:

T = ABABABCABABABCABCBAB
P = ABCBAB

# Boyer-Moore

T = ABABABCABABABCABCBAB
P = ABCBAB
x

Comparison: 1

T = ABABABCABABABCABCBAB
P = ABCBAB

Bad character rule

T = ABABABCABABABCABCBAB
P = ABCBAB

Good suffix rule
No suffix match in previous step

Pick either:

T = ABABABCABABABCABCBAB
P = ABCBAB

# Boyer-Moore

T = ABABABCABABABCABCBAB
P = ABCBAB
x

Comparison: 3

T = ABABABCABABABCABCBAB
P = ABCBAB

Bad character rule

T = ABABABCABABABCABCBAB
P = ABCBAB

Good suffix rule 1 or 2
Suffix (AB) = prefix(P) = AB

Pick good suffix rule and match:

T = ABABABCABABABCABCBAB
P = ABCBAB

Comparison: 6

Brute force & KMP for this example: exercise

**Total comparisons: 19**