Filters (cont.)

CS 554 – Computer Vision
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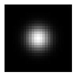
Bilkent University

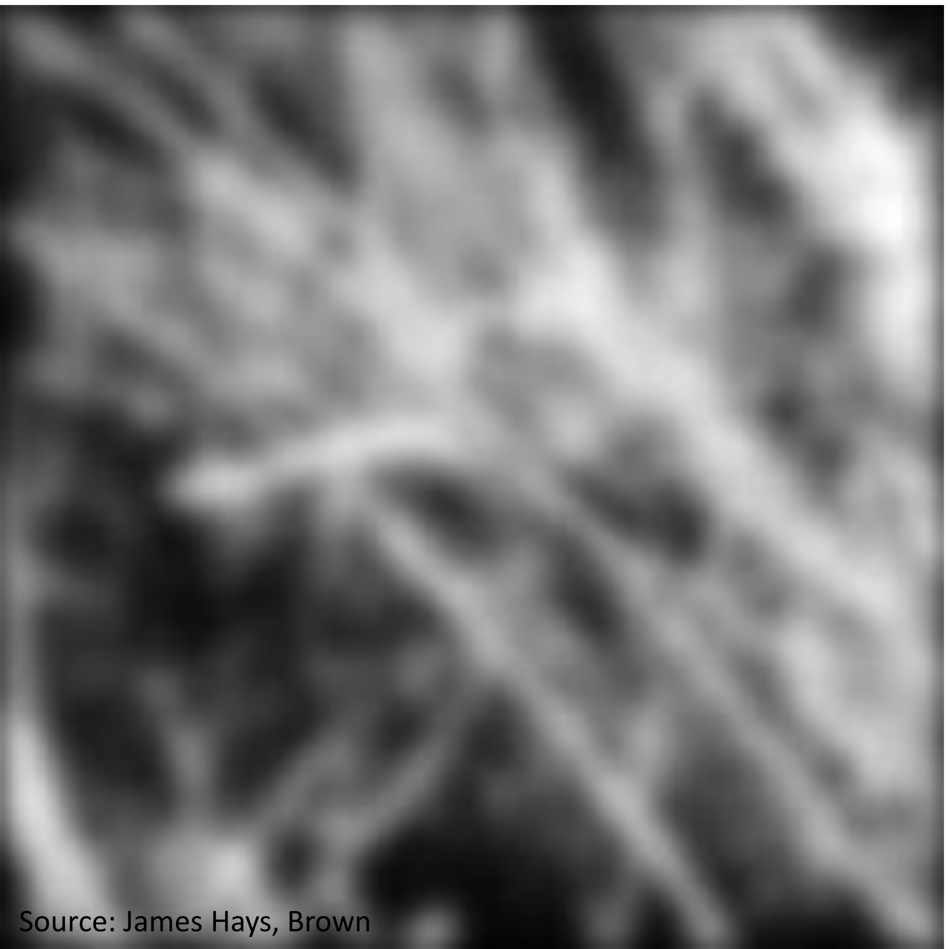
Today's topics

- Image Formation
- Image filters in spatial domain
  - Filter is a mathematical operation of a grid of numbers
  - Smoothing, sharpening, measuring texture
- Image filters in the frequency domain
  - Filtering is a way to modify the frequencies of images
  - Denoising, sampling, image compression
- Templates and Image Pyramids
  - Filtering is a way to match a template to the image
  - Detection, coarse-to-fine registration

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

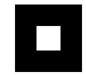
Gaussian

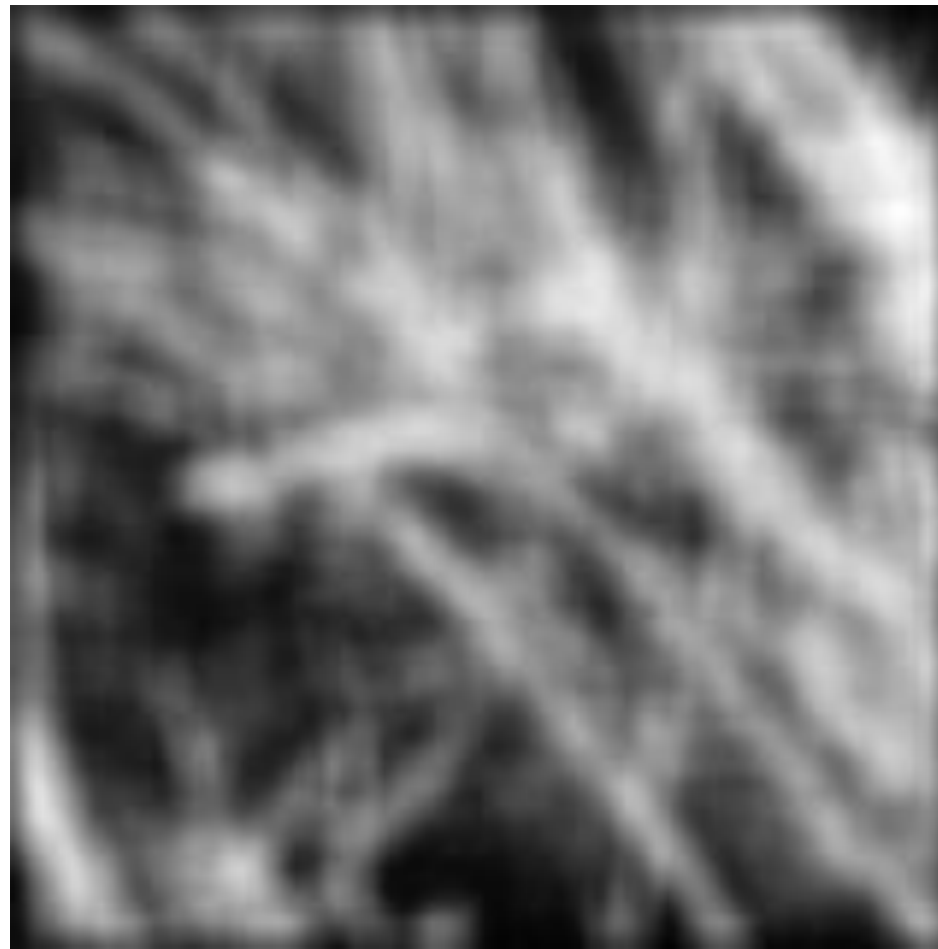




Source: James Hays, Brown

Box filter





Why does a lower resolution image still make sense to us? What do we lose?

Source: James Hays, Brown

Image: http://www.flickr.com/photos/igorms/136916757/

Slide: Hoiem

Thinking in terms of frequency

Source: James Hays, Brown

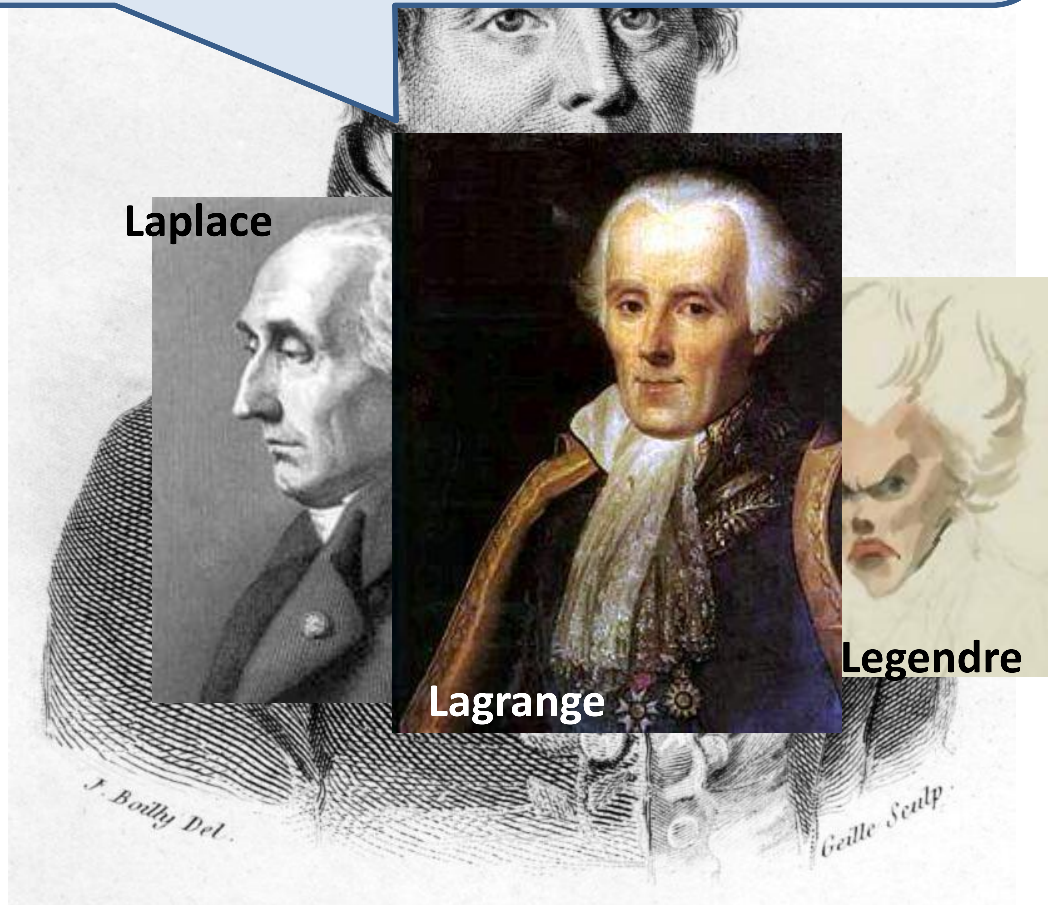
Jean Baptiste Joseph Fourier (1768-1830)

had crazy idea (1807):

> *...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.*

Any *univariate function can* *rewritten as a weighted sum* *sines and cosines of different* *frequencies.*

- Don’t believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it’s (mostly) true!
  - called Fourier Series
  - there are some subtle restrictions



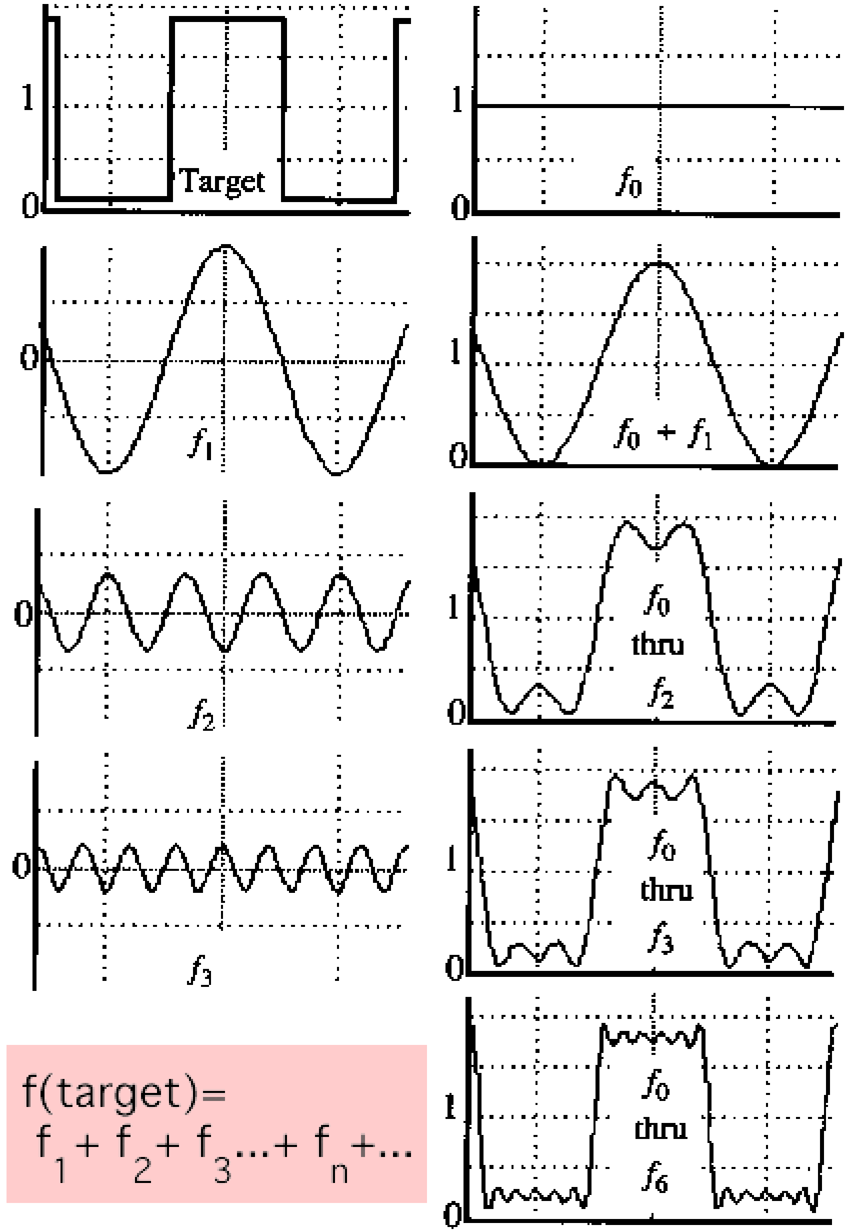
Source: James Hays, Brown

A sum of sines

Our building block:

$$A\sin(\omega x + \phi)$$

Add enough of them to get any signal *f(x)* you want!

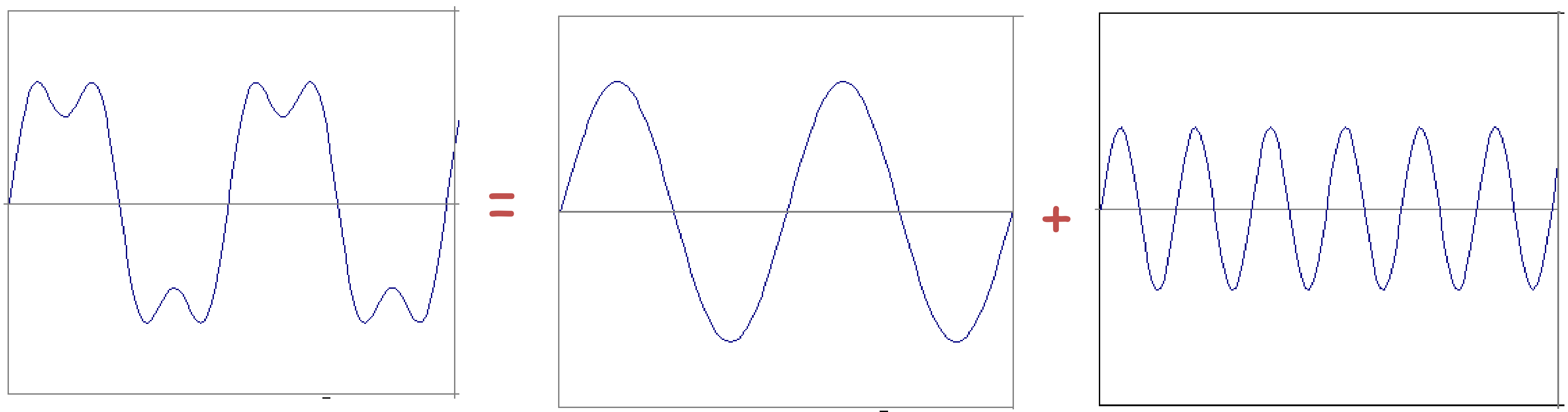


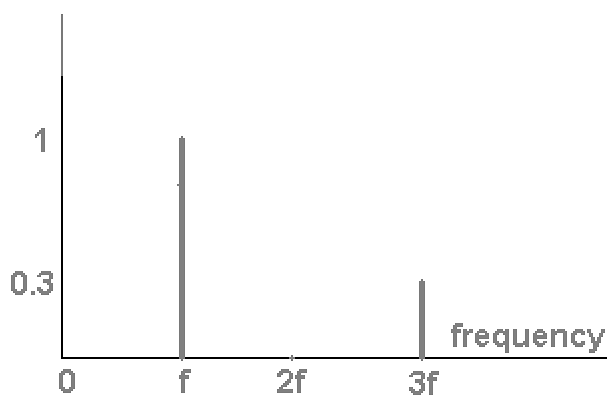
f(target)= $f_1 + f_2 + f_3 ... + f_n + ...$

Source: James Hays, Brown

Frequency Spectra

- example : $g(t) = \sin(2\pi f\, t) + (1/3)\sin(2\pi(3f)\, t)$

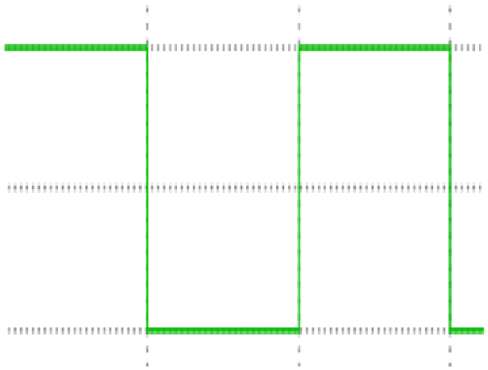




Source: James Hays, Brown

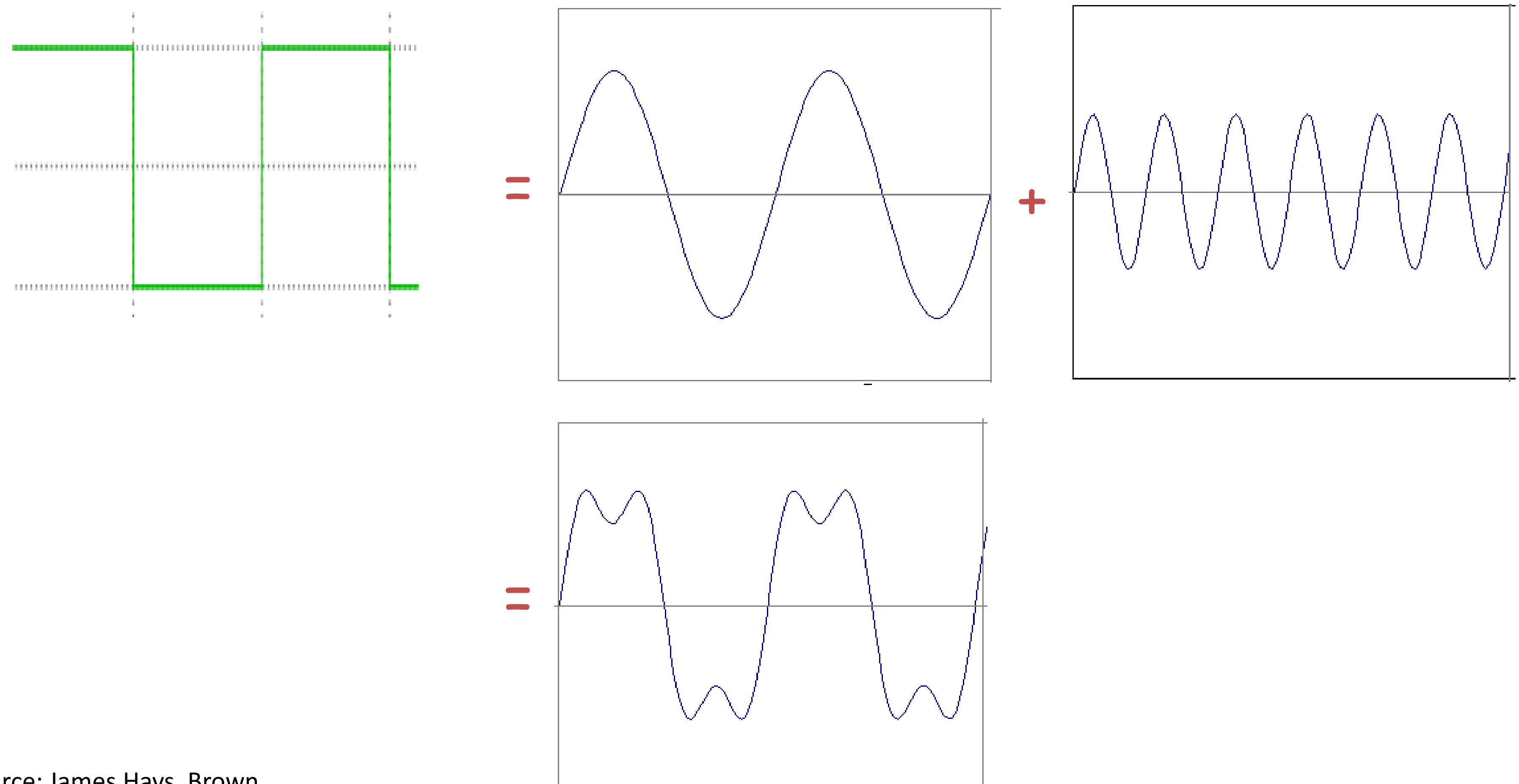
Slides: Efros

Frequency Spectra



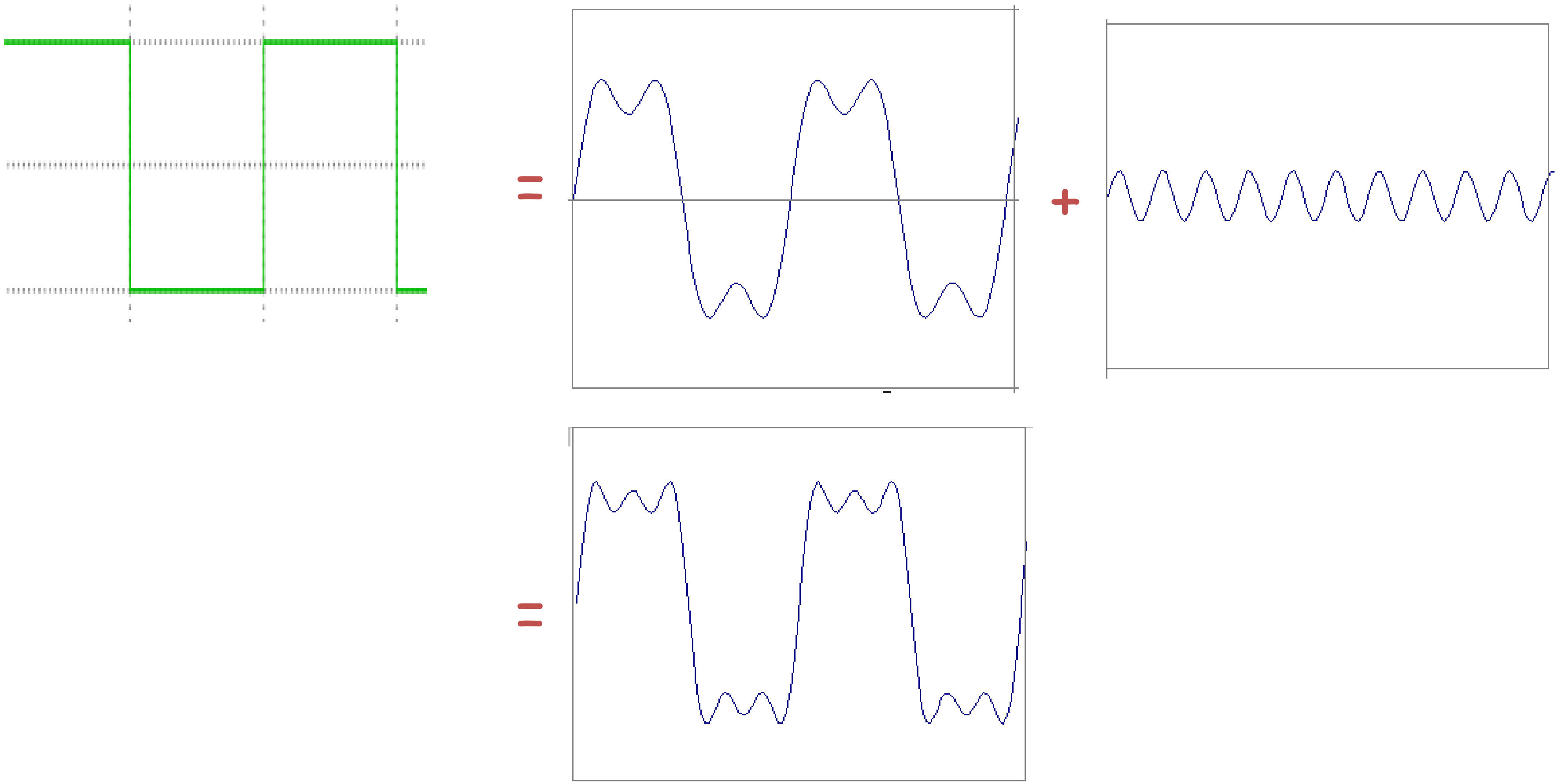
Source: James Hays, Brown

Frequency Spectra



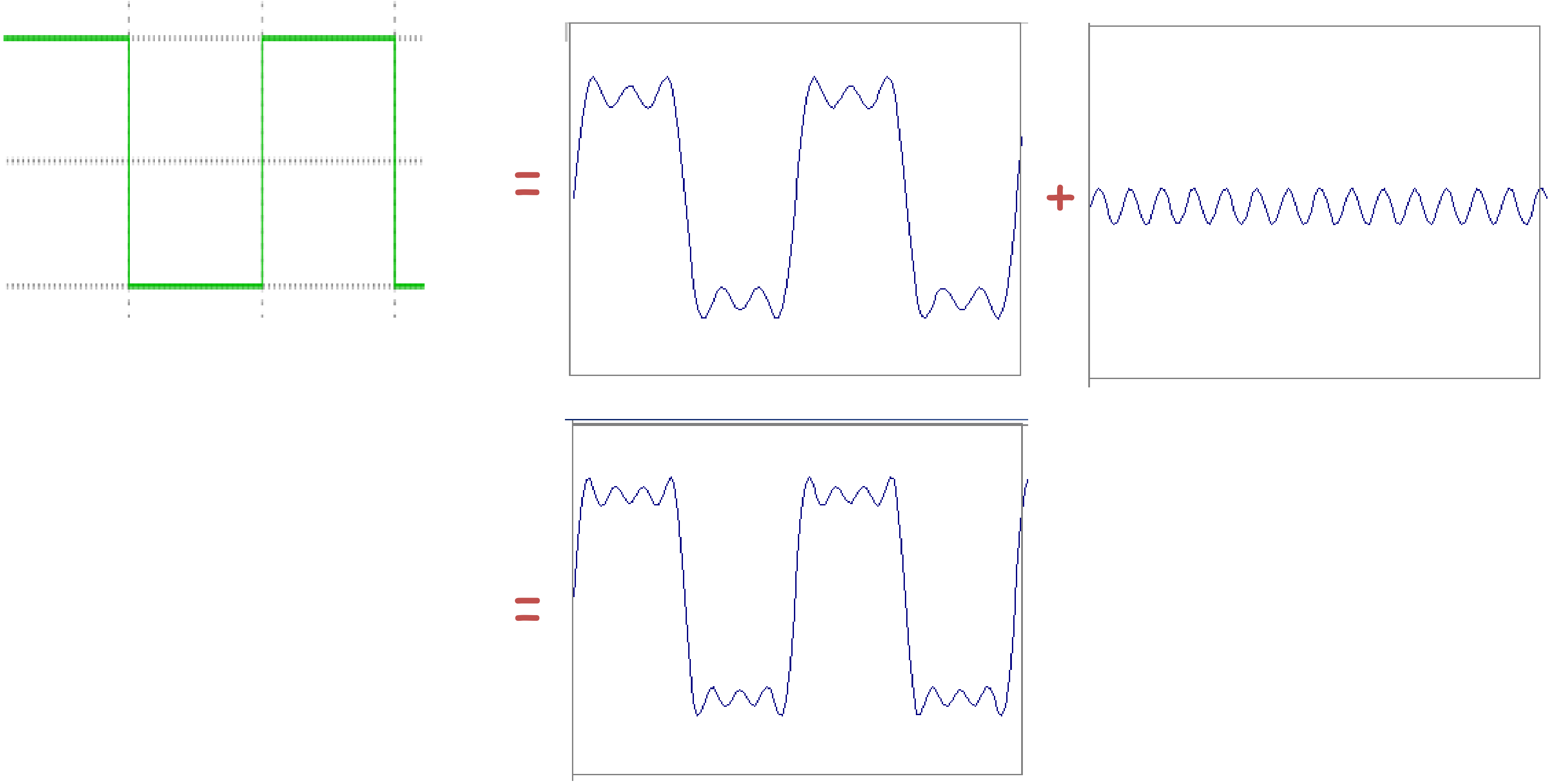
Source: James Hays, Brown

Frequency Spectra



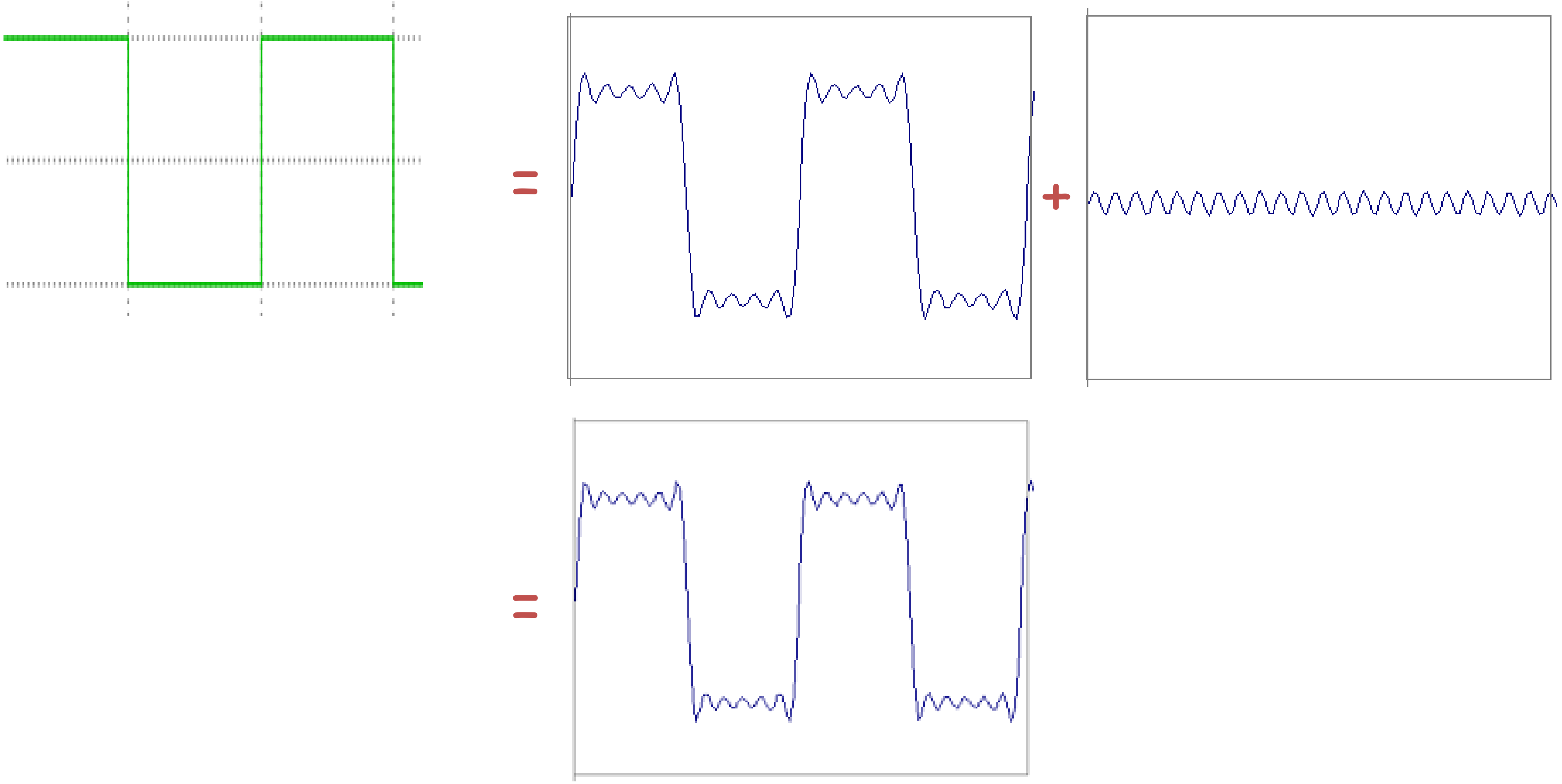
Source: James Hays, Brown

Frequency Spectra



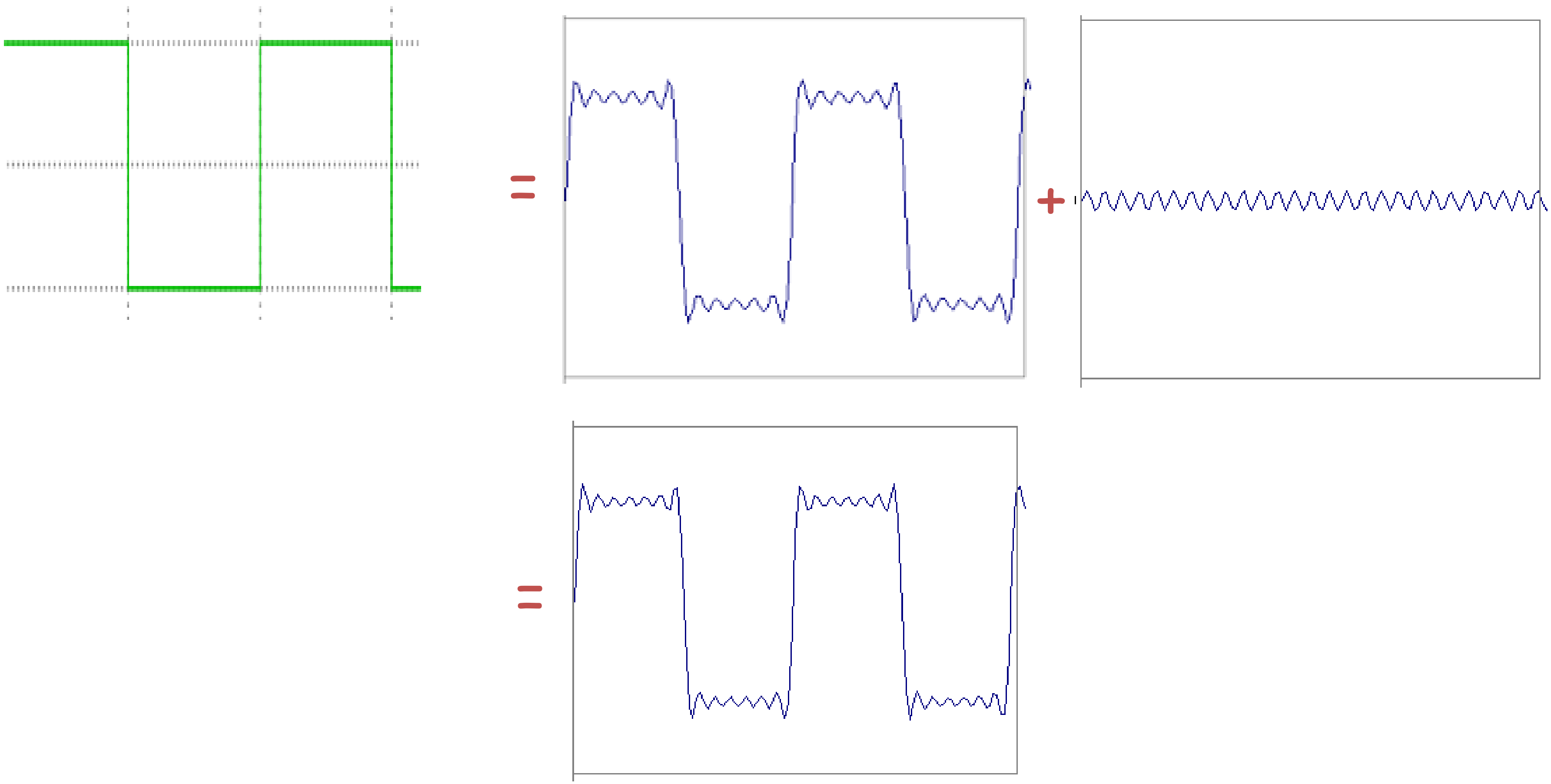
Source: James Hays, Brown

Frequency Spectra



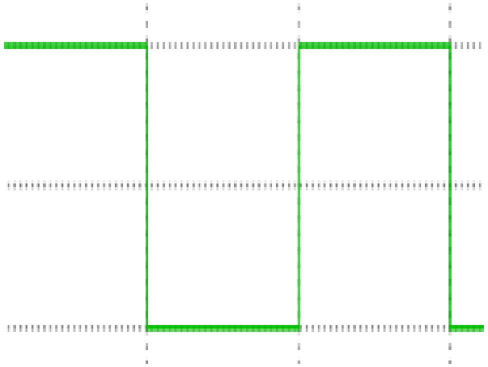
Source: James Hays, Brown

Frequency Spectra

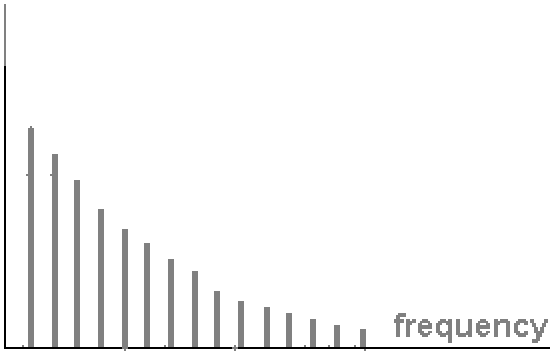


Source: James Hays, Brown

Frequency Spectra



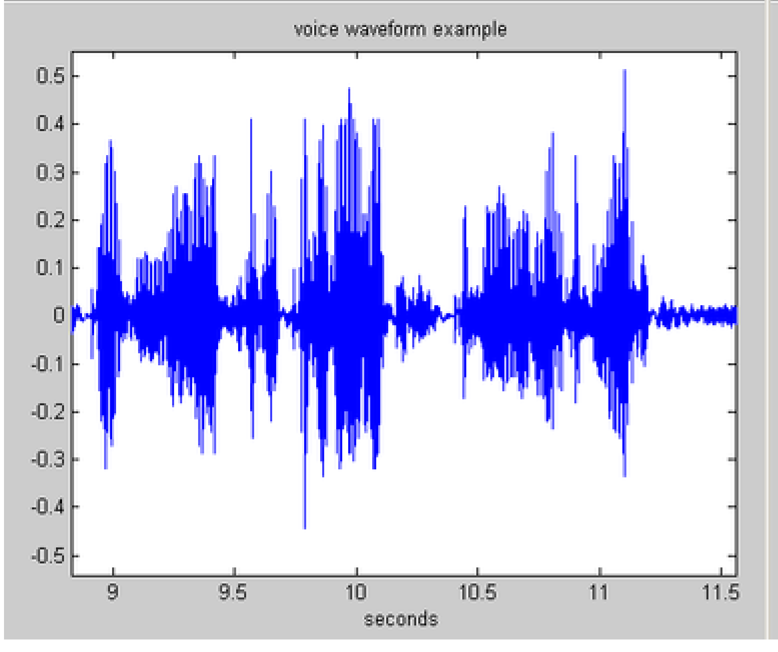
$$= A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kt)$$

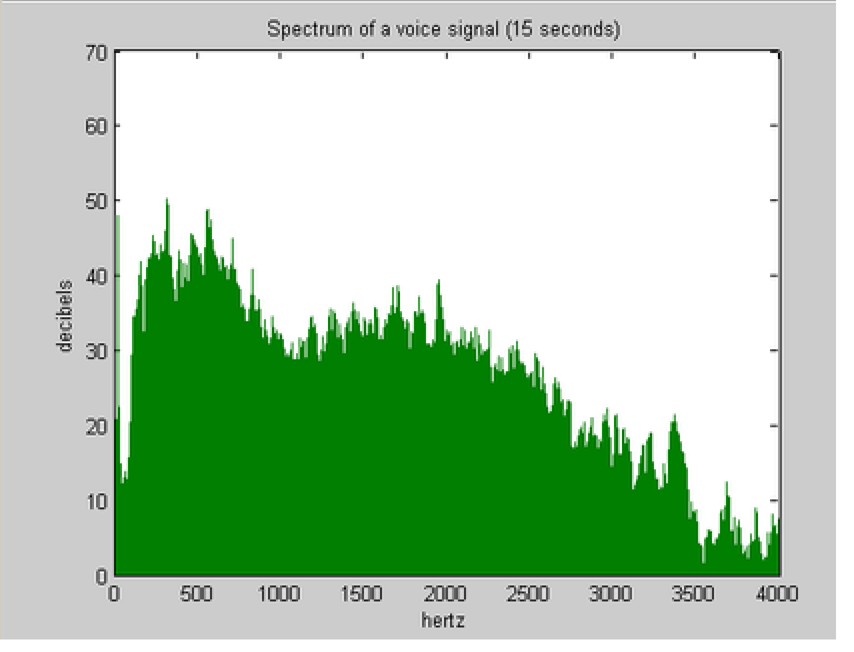


Source: James Hays, Brown

Example: Music

- We think of music in terms of frequencies at different magnitudes

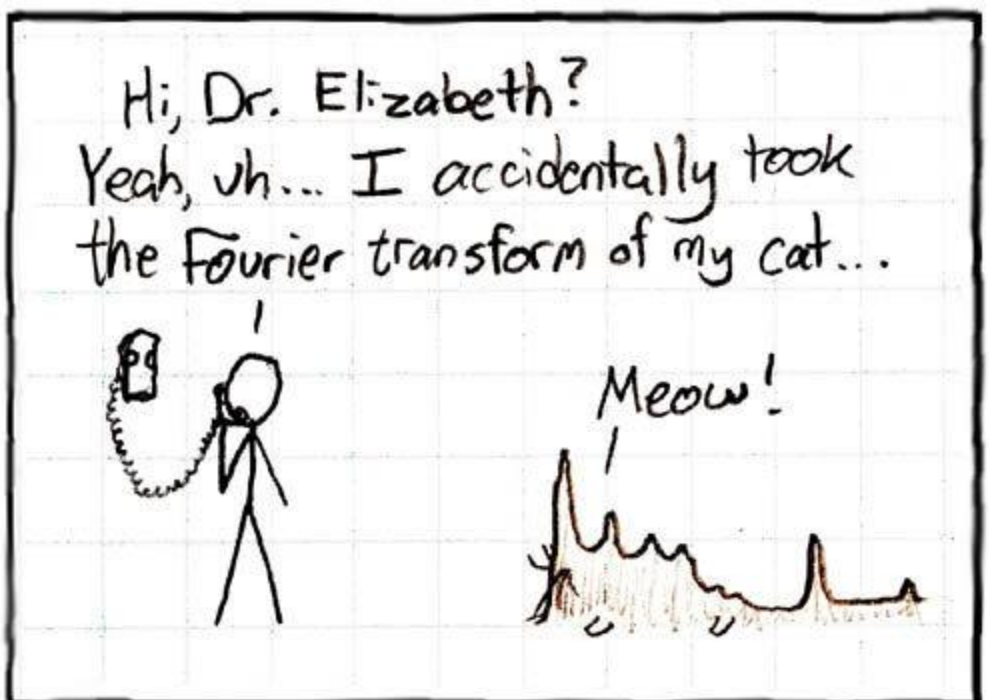




Source: James Hays, Brown

Other signals

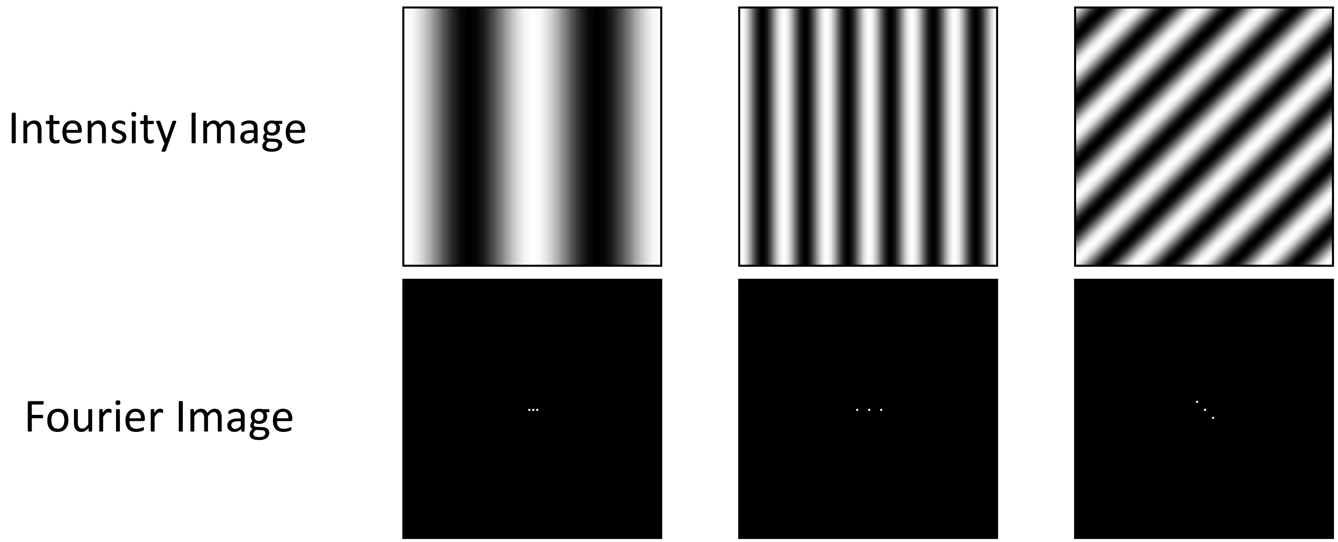
- We can also think of all kinds of other signals the same way



xkcd.com

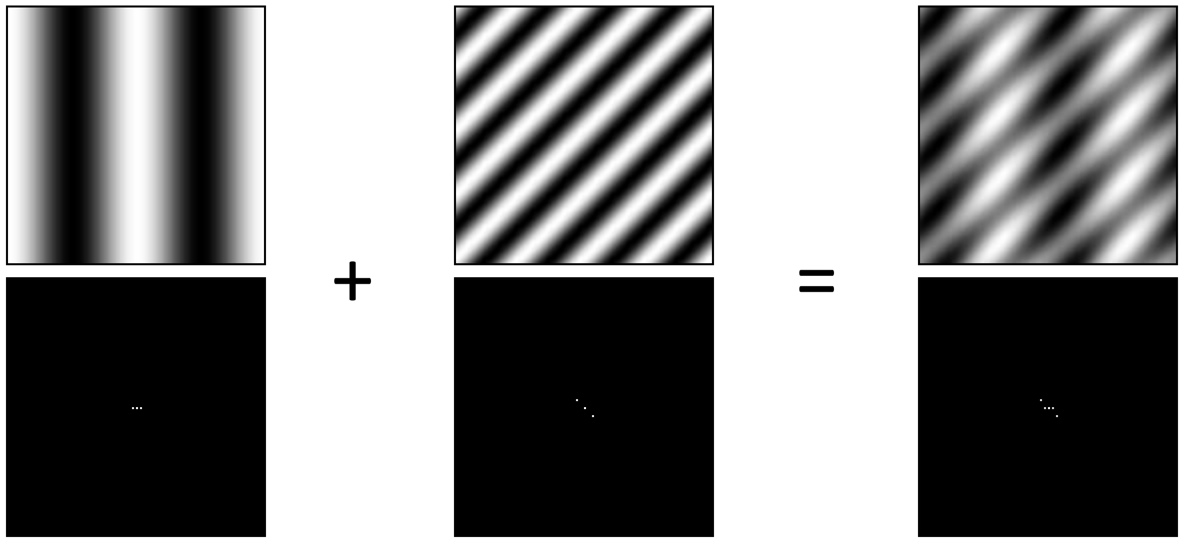
Source: James Hays, Brown

Fourier analysis in images



http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering

Signals can be composed



http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering
More: http://www.cs.unm.edu/~brayer/vision/fourier.html

Source: James Hays, Brown

Fourier Transform

- Fourier transform stores the magnitude and phase at each frequency
  - Magnitude encodes how much signal there is at a particular frequency
  - Phase encodes spatial information (indirectly)
  - For mathematical convenience, this is often notated in terms of real and complex numbers

Amplitude: $A = \pm\sqrt{R(\omega)^2 + I(\omega)^2}$ Phase: $\phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$

Source: James Hays, Brown

The Convolution Theorem

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$\mathrm{F}[g * h] = \mathrm{F}[g]\,\mathrm{F}[h]$$

- The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

$$\mathrm{F}^{-1}[gh] = \mathrm{F}^{-1}[g] * \mathrm{F}^{-1}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

Source: James Hays, Brown

Properties of Fourier Transforms

- Linearity $\mathcal{F}[ax(t) + by(t)] = a\mathcal{F}[x(t)] + b\mathcal{F}[y(t)]$

- Fourier transform of a real signal is symmetric about the origin

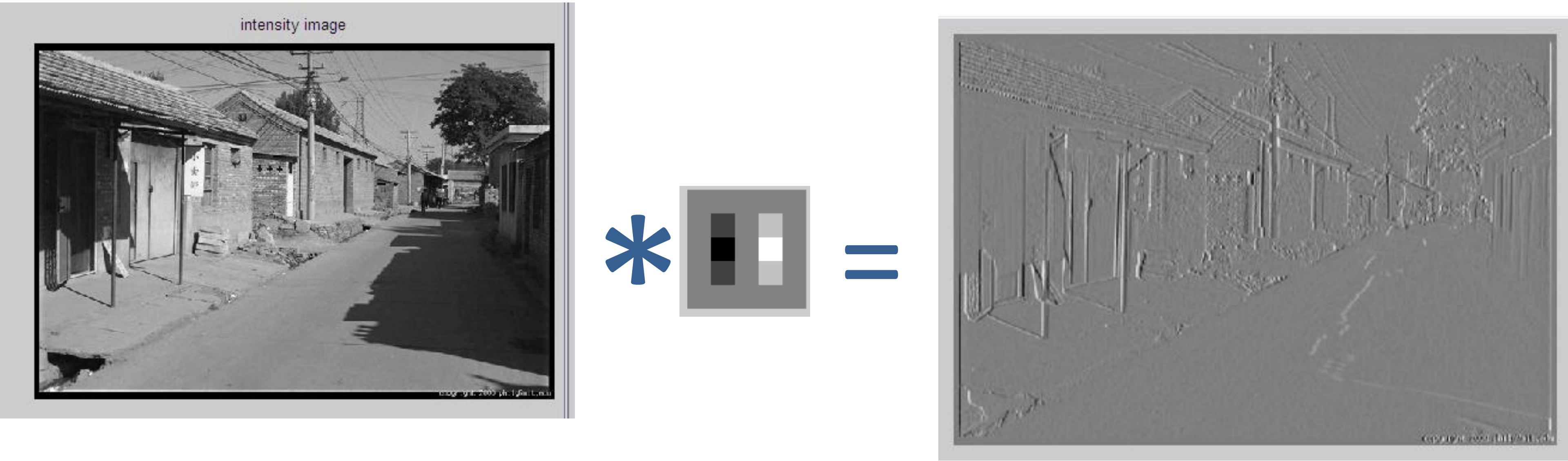
- The energy of the signal is the same as the energy of its Fourier transform

Source: James Hays, Brown

See Szeliski Book (3.4)

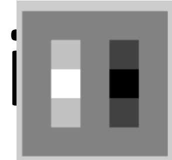
Filtering in spatial dom

1	0	-1
2	0	-2
1	0	-1

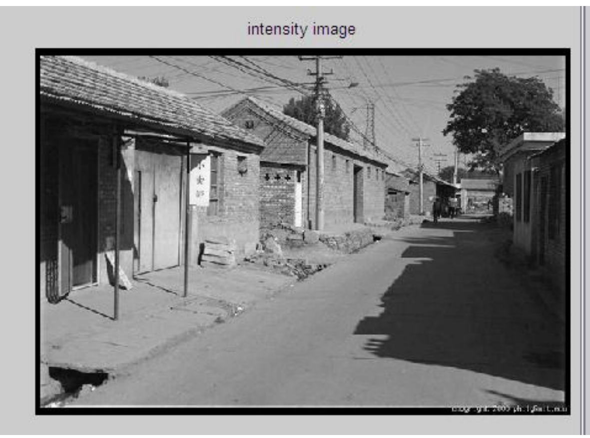


Source: James Hays, Brown

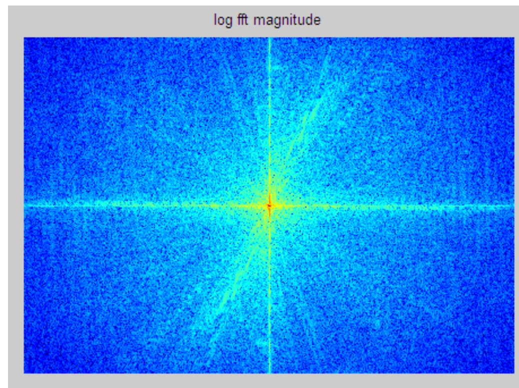
Filtering in frequency domai

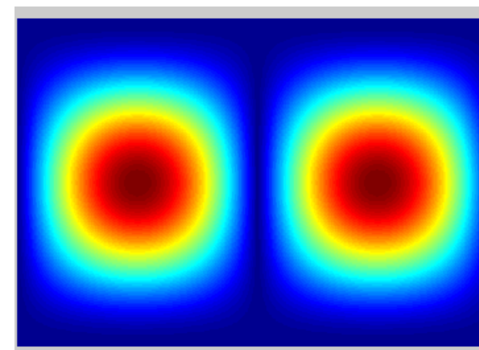


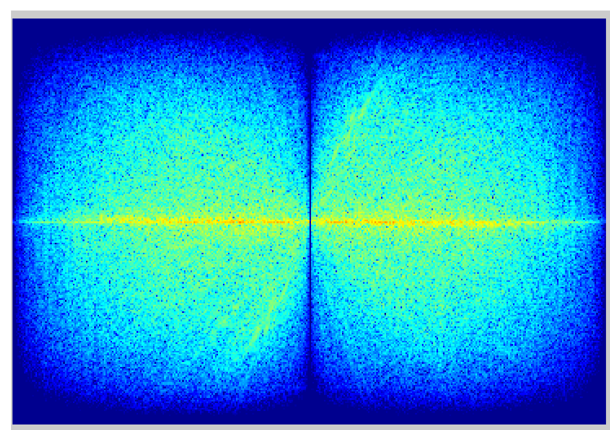
FFT



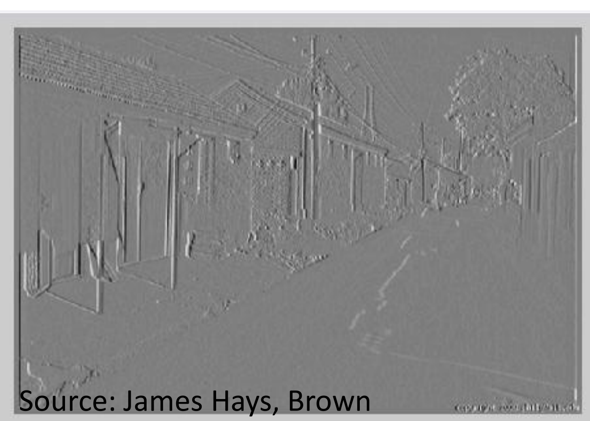
FFT







Inverse FFT

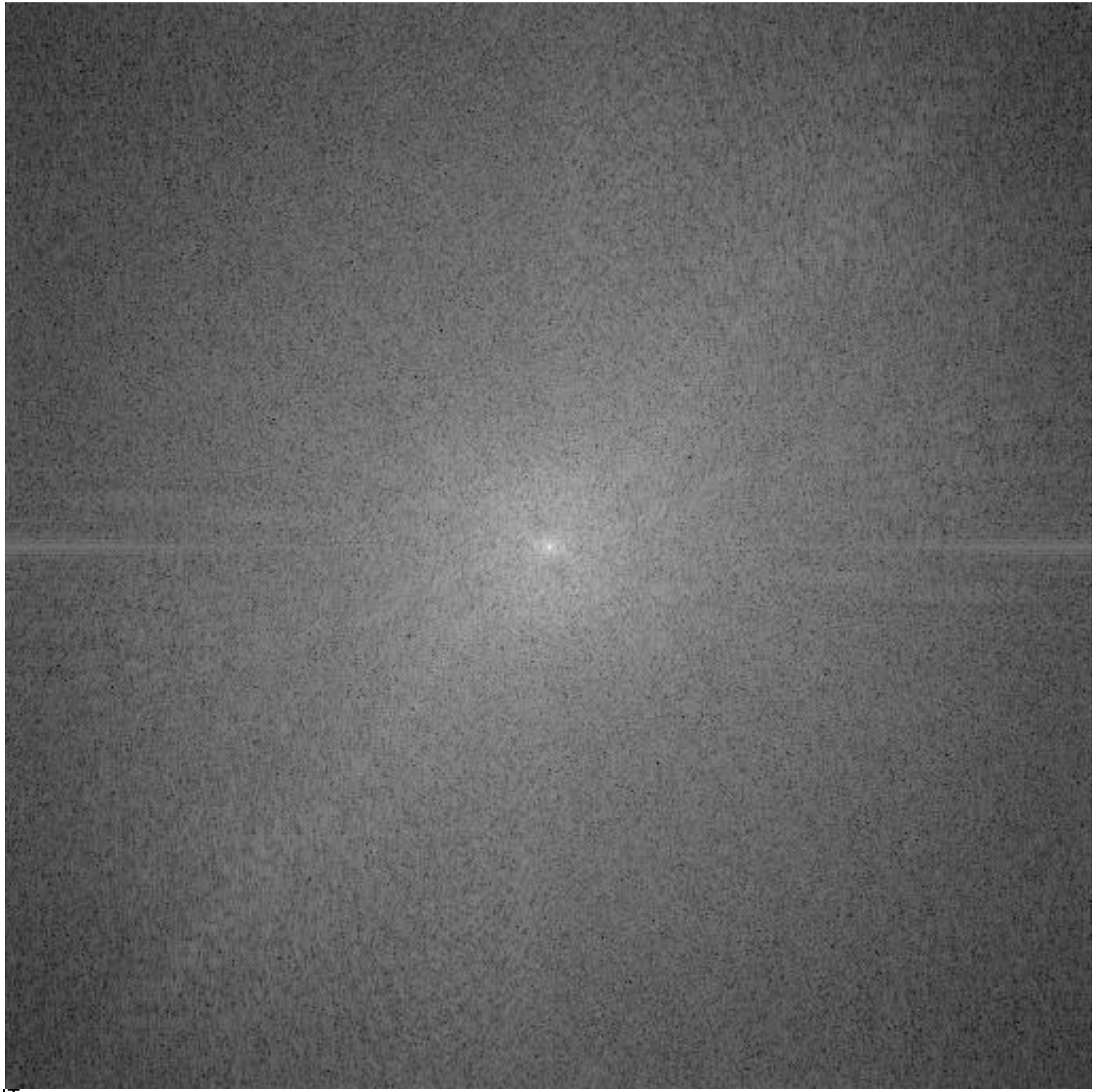


Source: James Hays, Brown

Slide: Hoiem

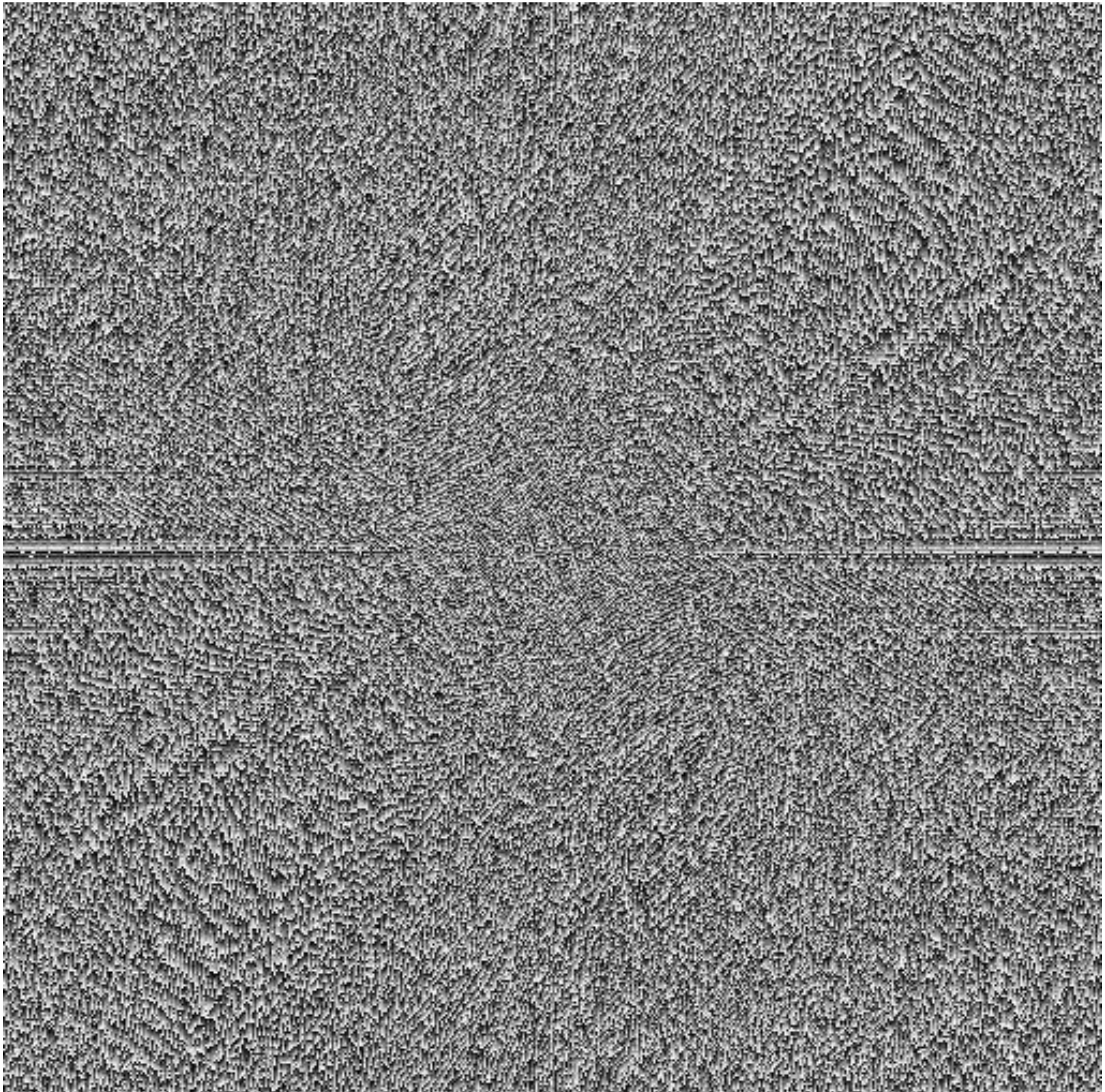
Source: Torralba, MIT

This is the magnitude transform of the cheetah pic



Source: Torralba, MIT

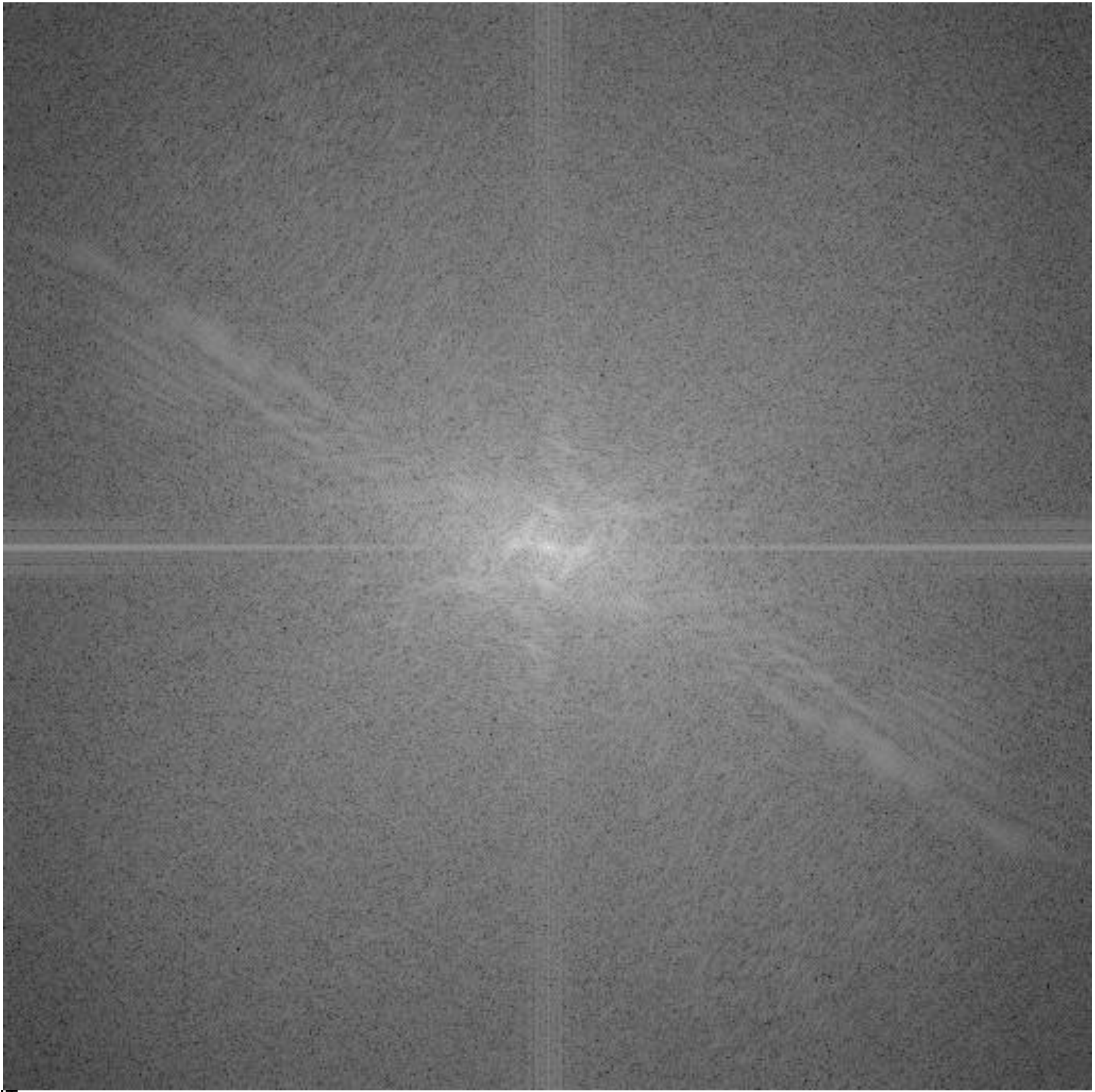
This is the phase transform of the cheetah pic



Source: Torralba, MIT

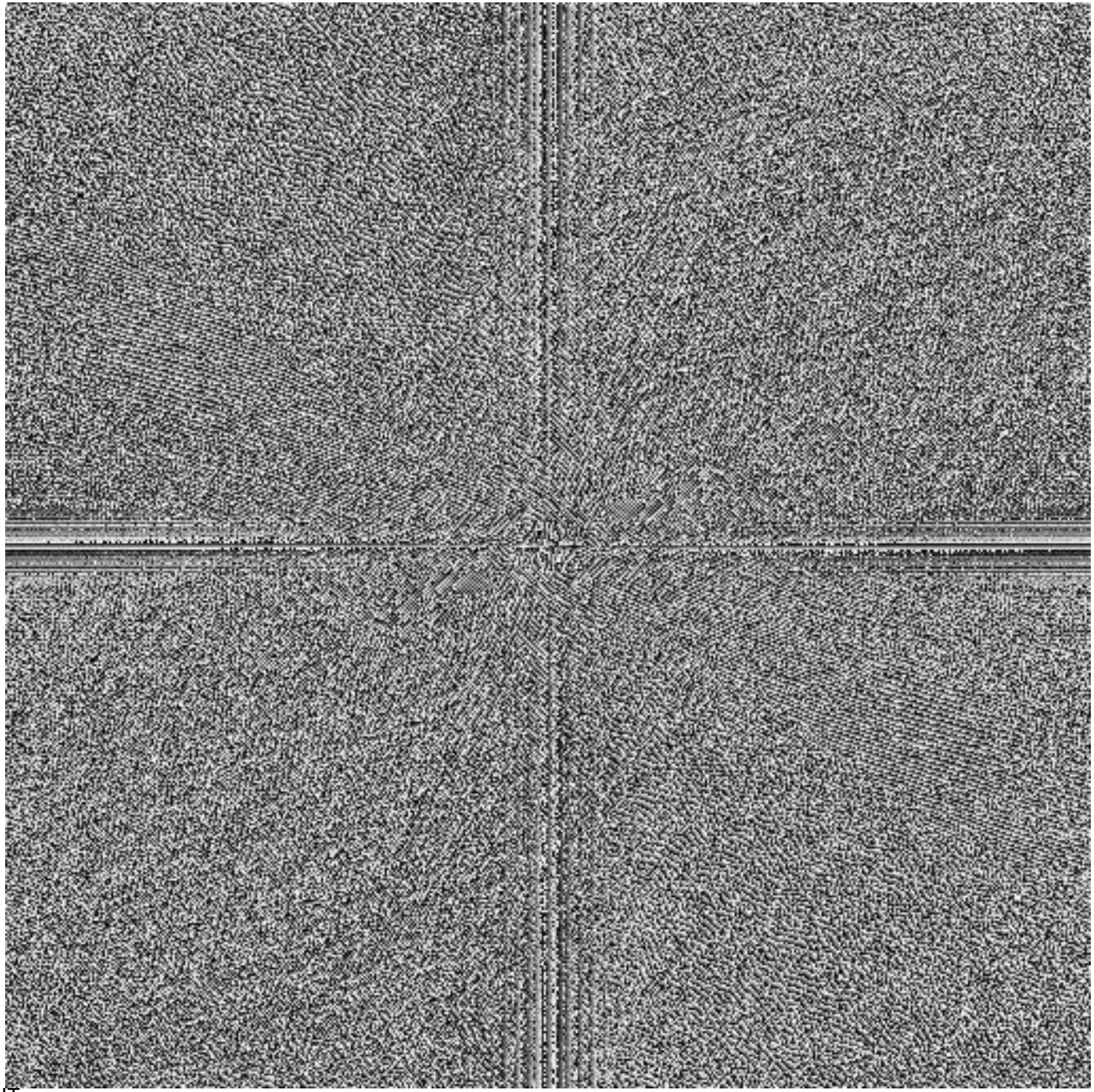
Source: Torralba, MIT

This is the magnitude transform of the zebra pic



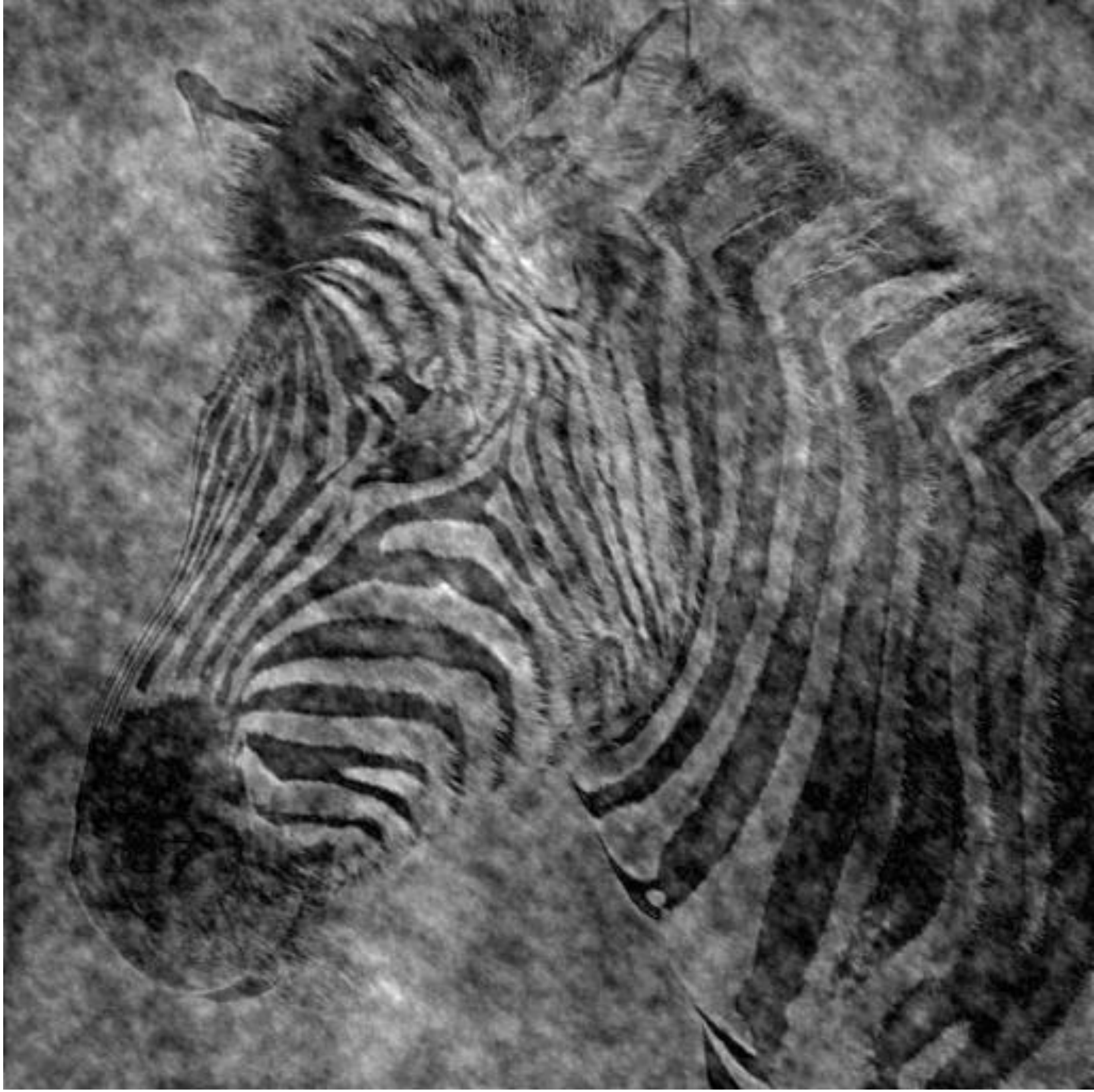
Source: Torralba, MIT

This is the phase transform of the zebra pic



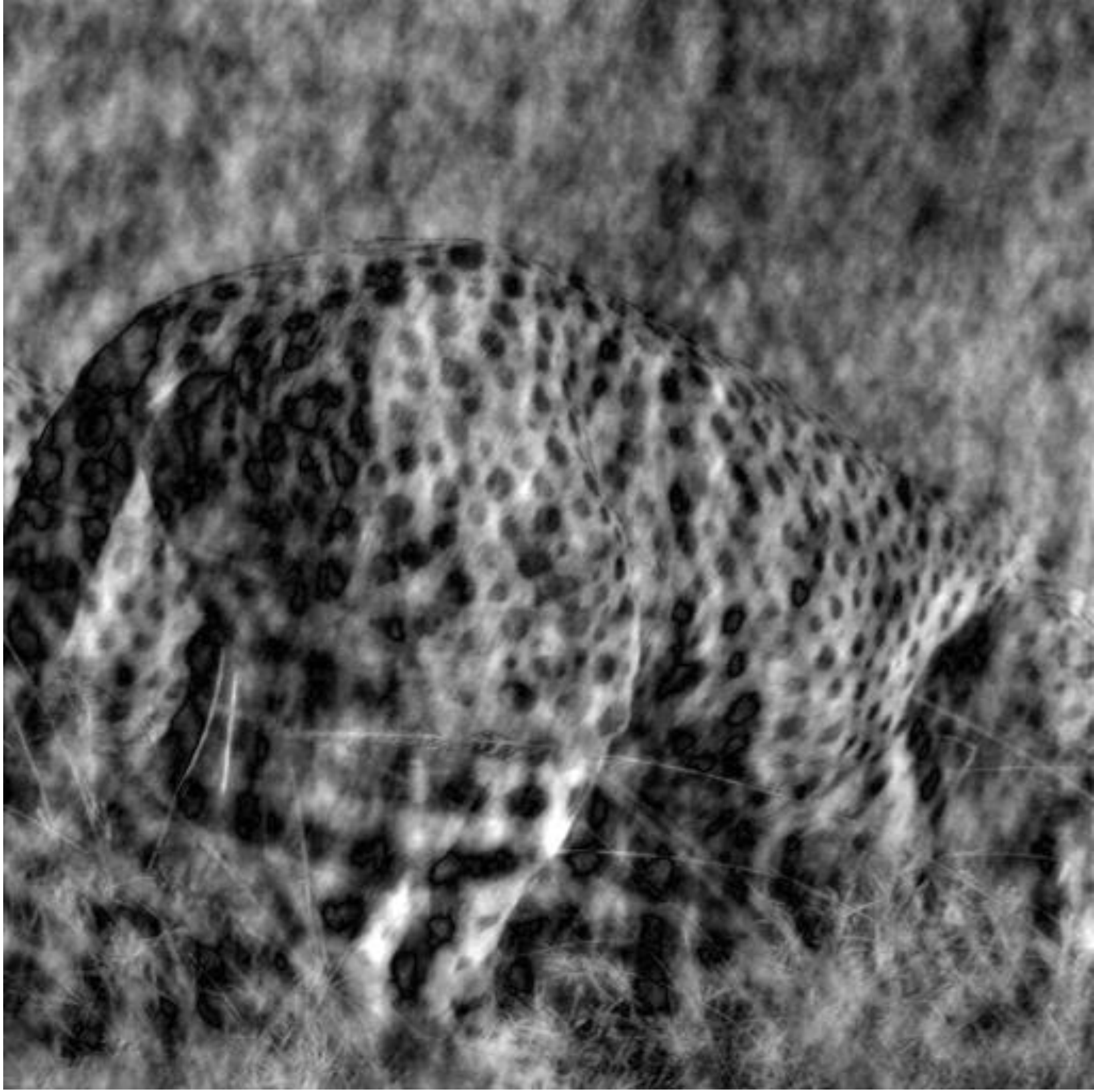
Source: Torralba, MIT

Reconstruction with zebra phase, cheetah magnitude



Source: Torralba, MIT

Reconstruction with cheetah phase, zebra magnitude



Source: Torralba, MIT

Phase and Magnitude

Image with cheetah phase
(and zebra magnitude)

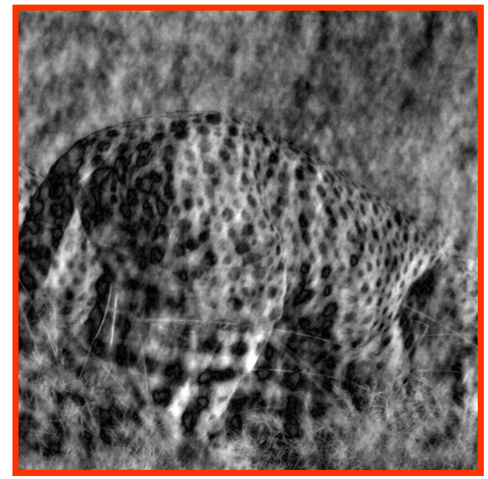
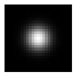


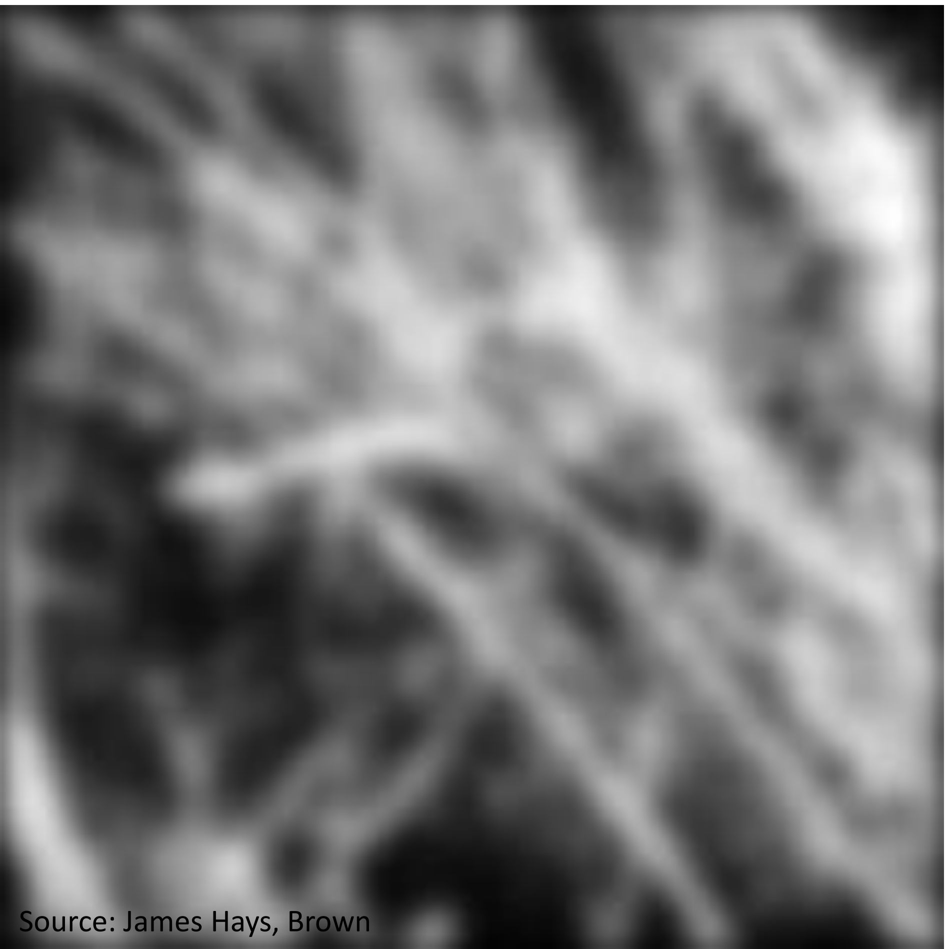
Image with zebra phase
(and cheetah magnitude)

Source: Torralba, MIT

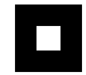
Filtering

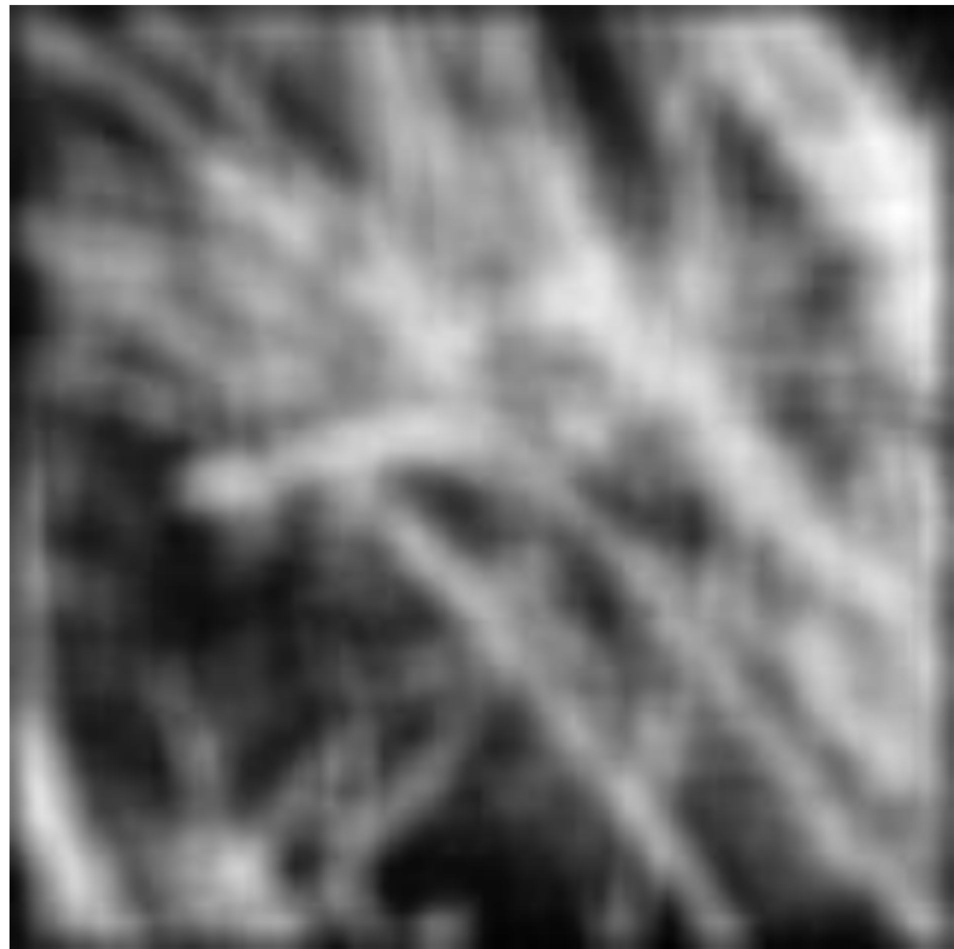
Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian 

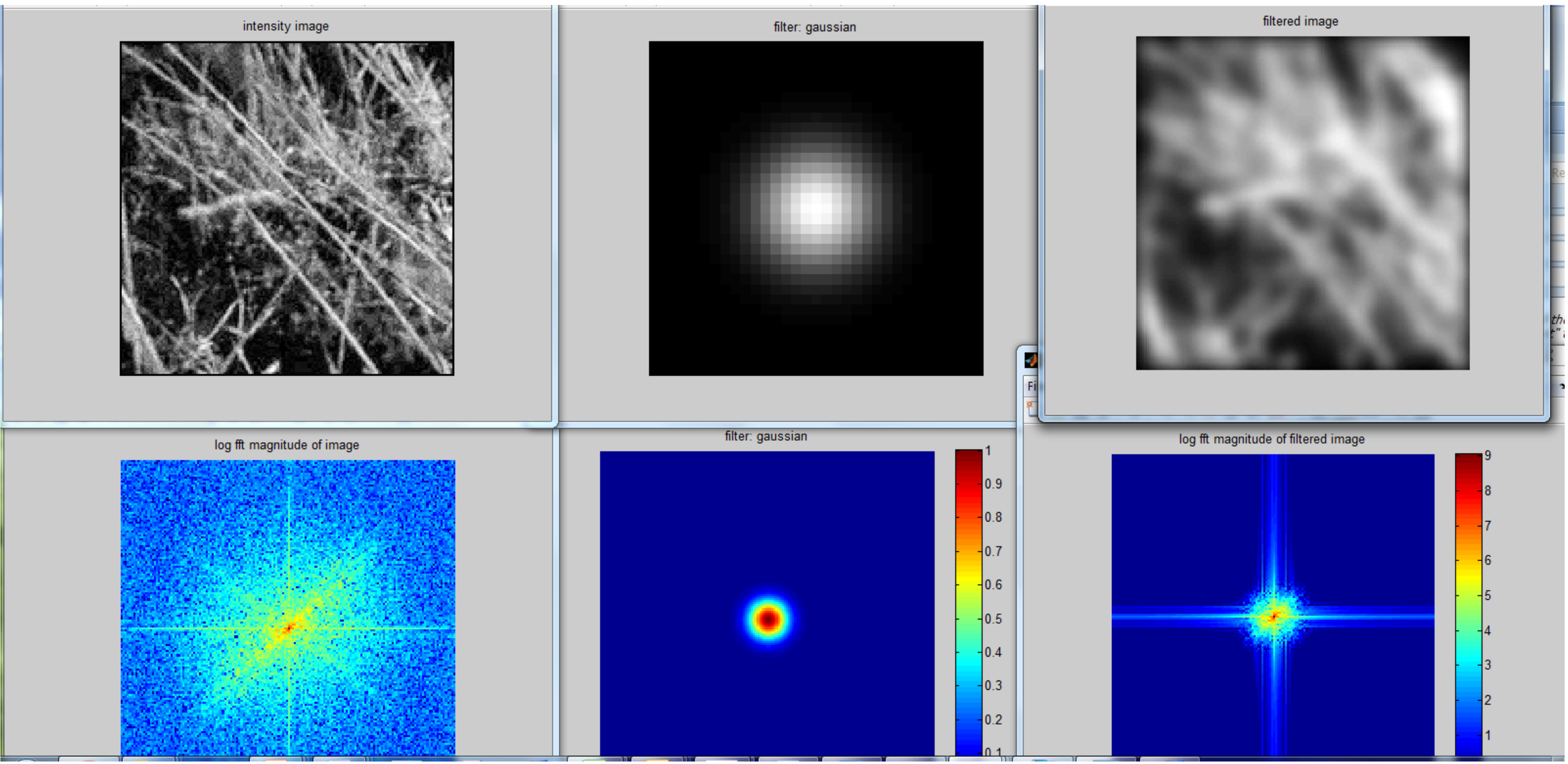


Source: James Hays, Brown

Box filter 

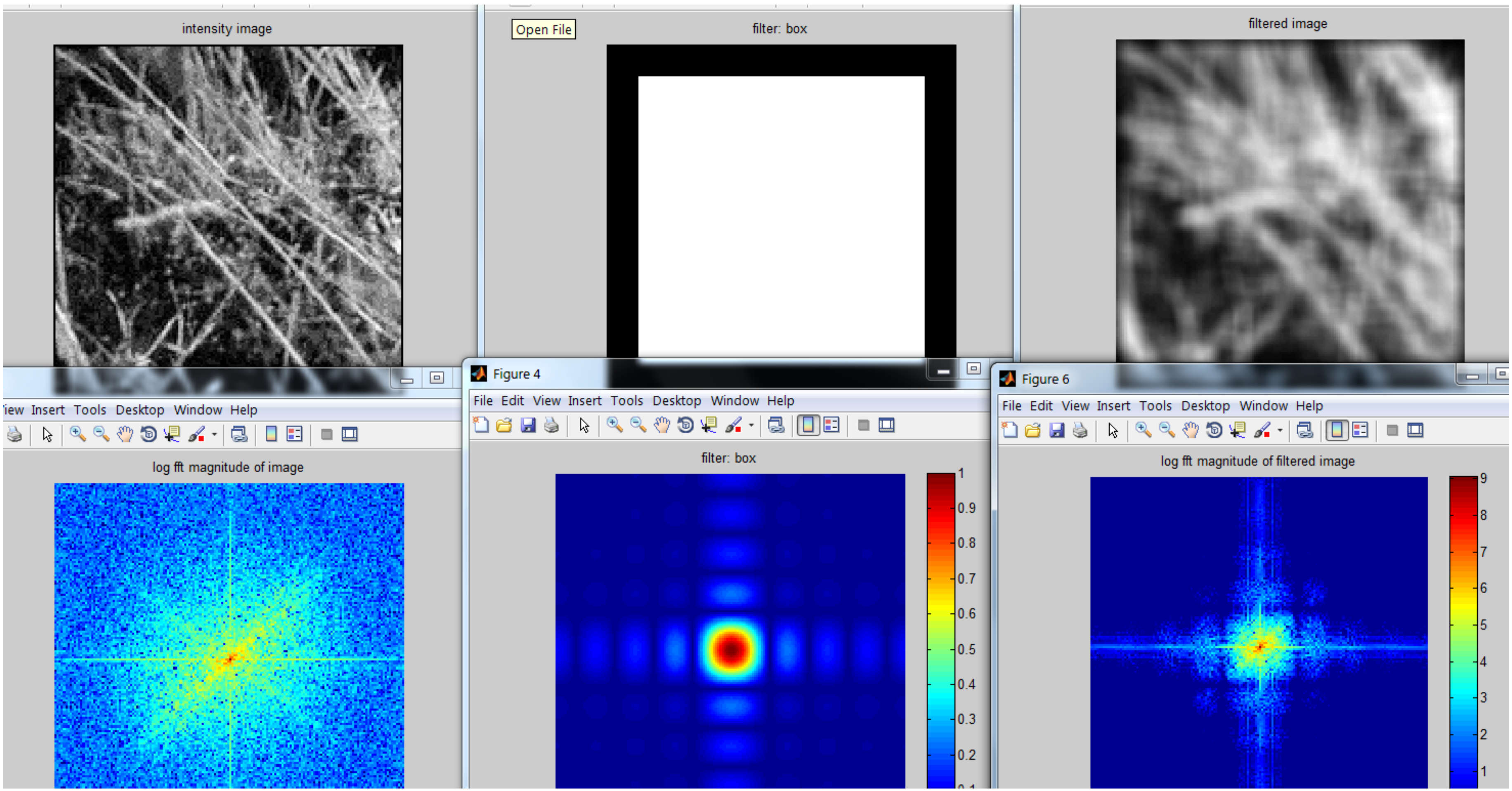


Gaussian



Source: James Hays, Brown

Box Filter



Source: James Hays, Brown

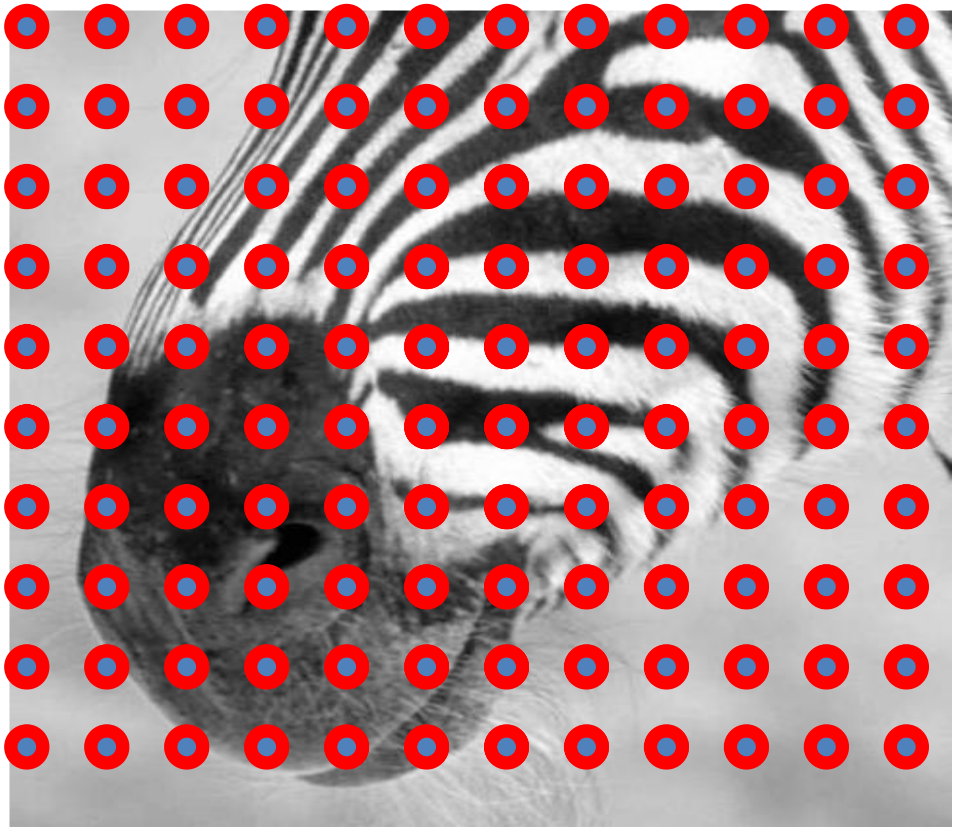
Sampling

Why does a lower resolution image still make sense to us? What do we lose?

Source: James Hays, Brown

Image: http://www.flickr.com/photos/igorms/136916757/

Subsampling by a factor of 2

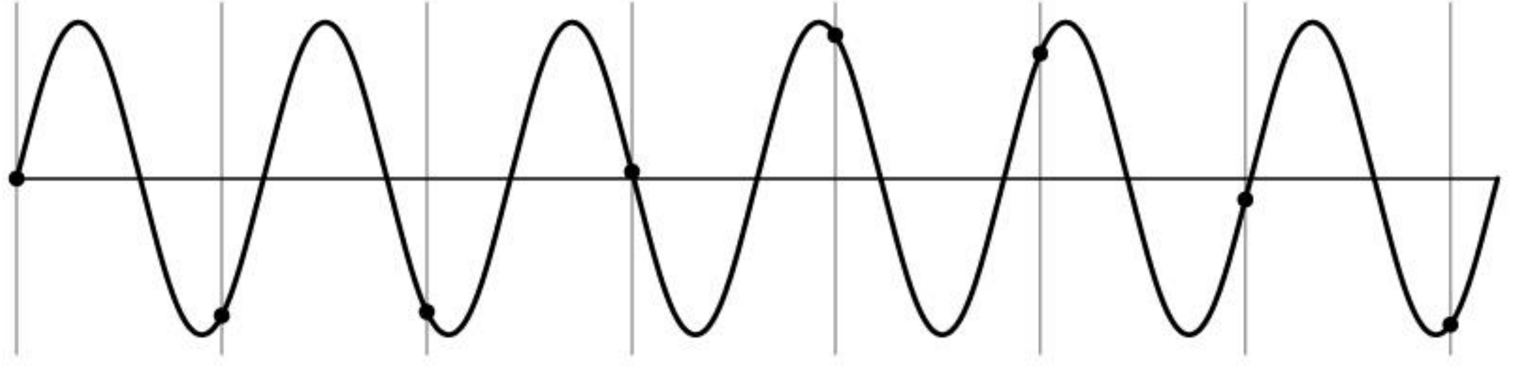


Throw away every other row and column to create a 1/2 size image

Source: James Hays, Brown

Aliasing problem

- 1D example (sinewave):

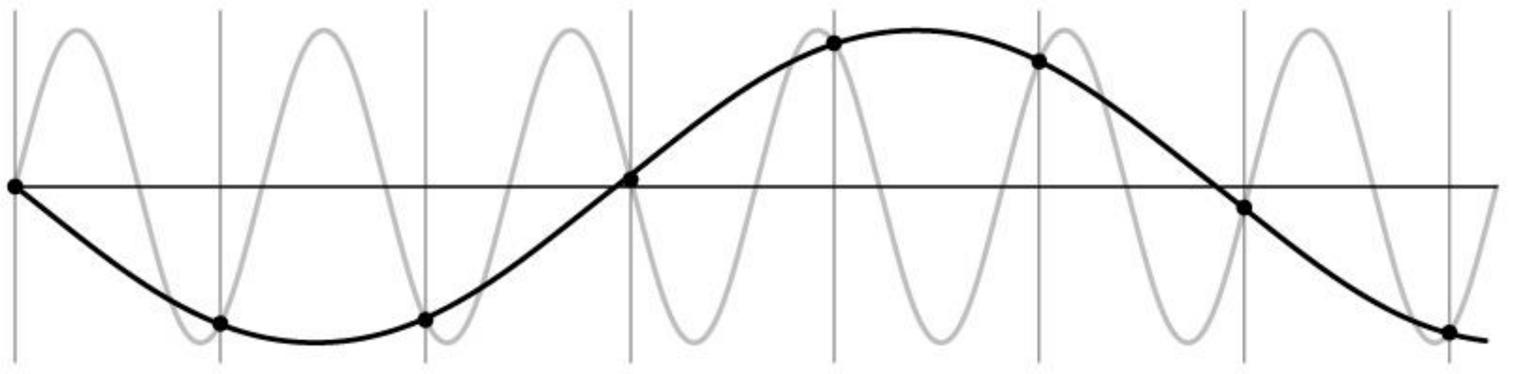


Source: S. Marschner

Source: James Hays, Brown

Aliasing problem

- 1D example (sinewave):



Source: S. Marschner

Source: James Hays, Brown

Aliasing problem

- Sub-sampling may be dangerous….
- Characteristic errors may appear:
  - “Wagon wheels rolling the wrong way in movies”
  - “Checkerboards disintegrate in ray tracing”
  - “Striped shirts look funny on color television”

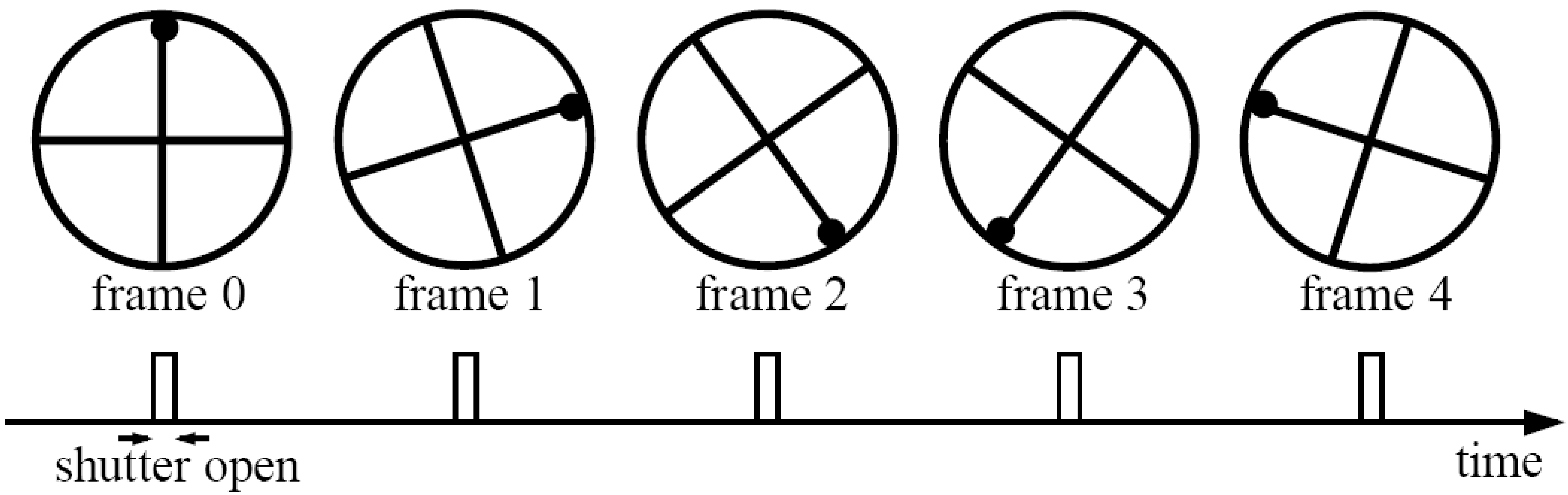
Source: James Hays, Brown

Source: D. Forsyth

Aliasing in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

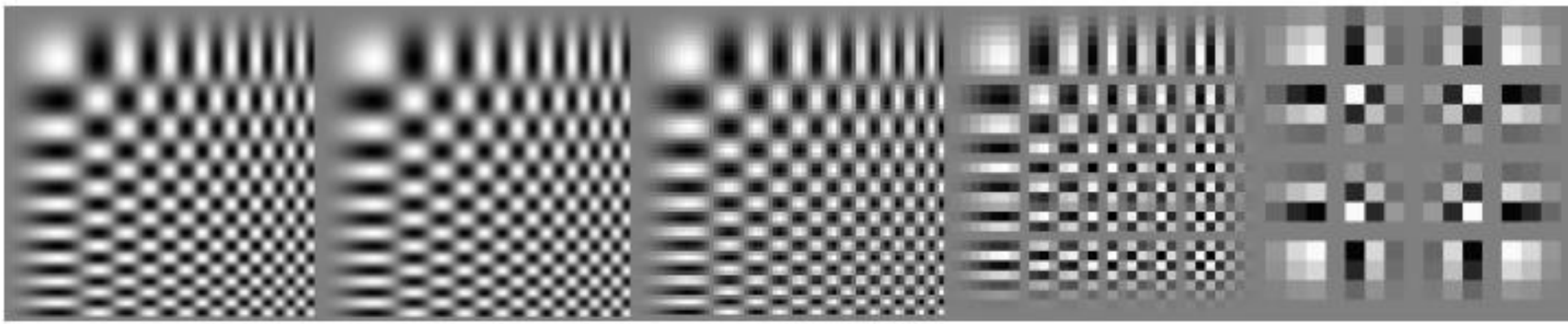
Aliasing in graphics

Source: James Hays, Brown

Source: A. Efros

Sampling and aliasing

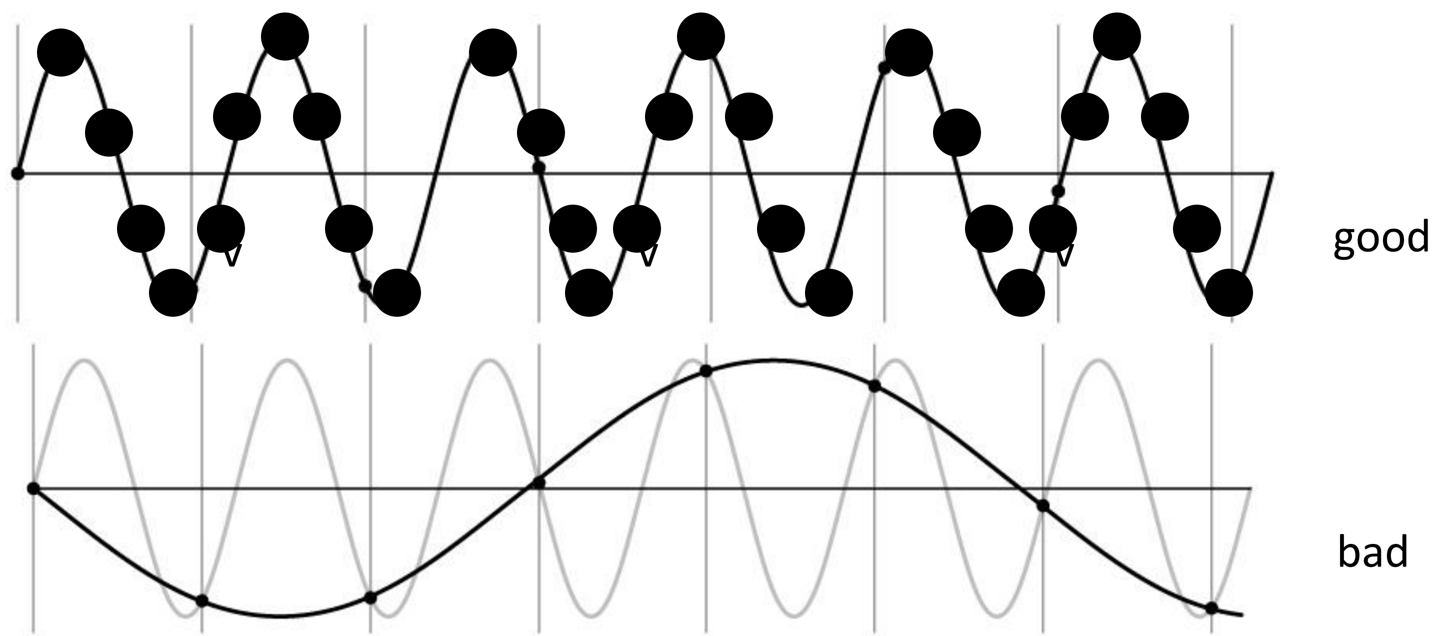
256x256 128x128 64x64 32x32 16x16



Source: James Hays, Brown

Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be $\geq 2 \times f_{max}$
- f_{max} = max frequency of the input signal
- This will allow to reconstruct the original perfectly from the sampled version



Source: James Hays, Brown

Anti-aliasing

Solutions:

- Sample more often

- Get rid of all frequencies that are greater than half the new sampling frequency
  - Will lose information
  - But it's better than aliasing
  - Apply a smoothing filter

Source: James Hays, Brown

Algorithm for downsampling by factor of 2

1. Start with image(h, w)
2. Apply low-pass filter

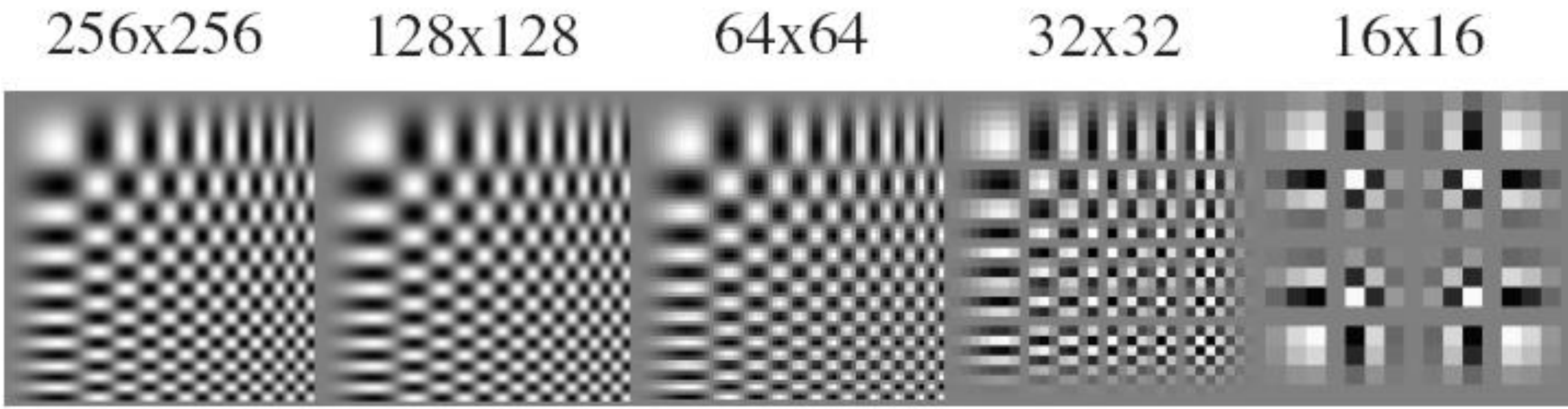
```
im_blur = imfilter(image, fspecial('gaussian', 7, 1))
```

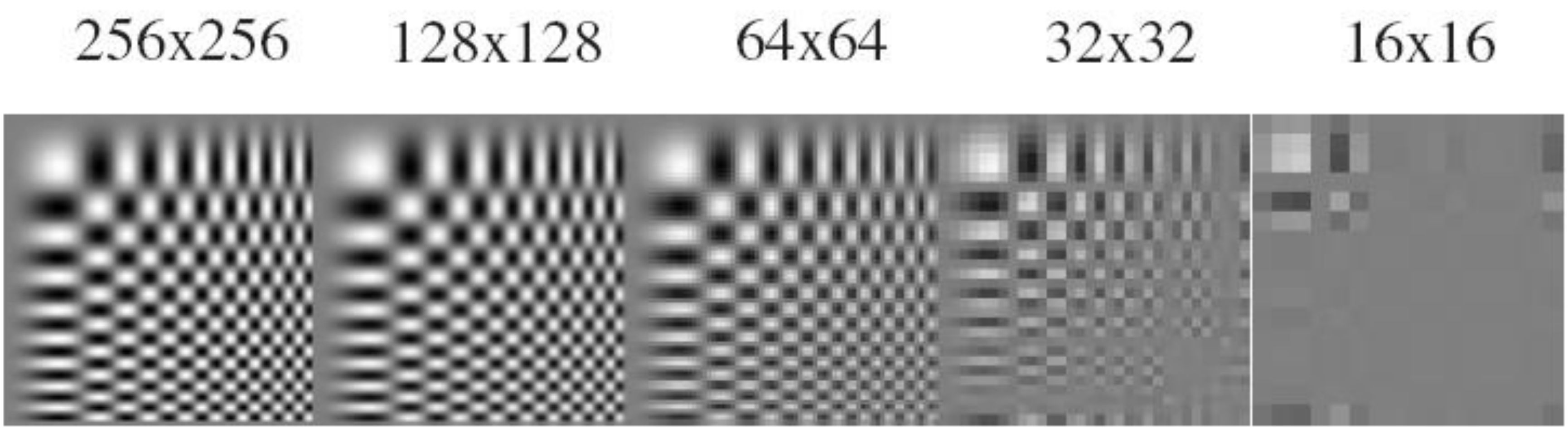
3. Sample every other pixel

```
im_small = im_blur(1:2:end, 1:2:end);
```

Source: James Hays, Brown

Anti-aliasing





Source: James Hays, Brown

Forsyth and Ponce 2002

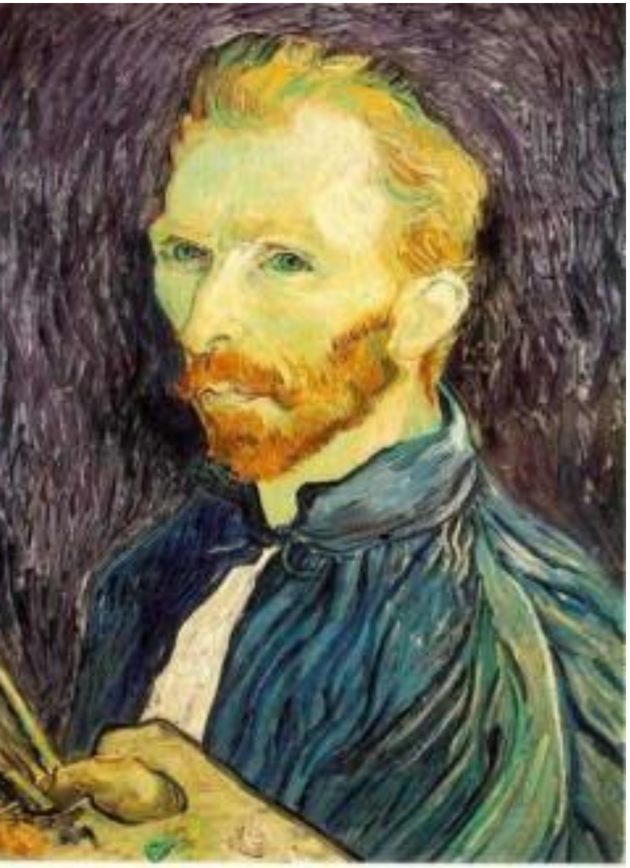
Subsampling without pre-filtering

1/2 1/4 (2x zoom) 1/8 (4x zoom)

Slide by Steve Seitz

Source: James Hays, Brown

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4

G 1/8

Slide by Steve Seitz

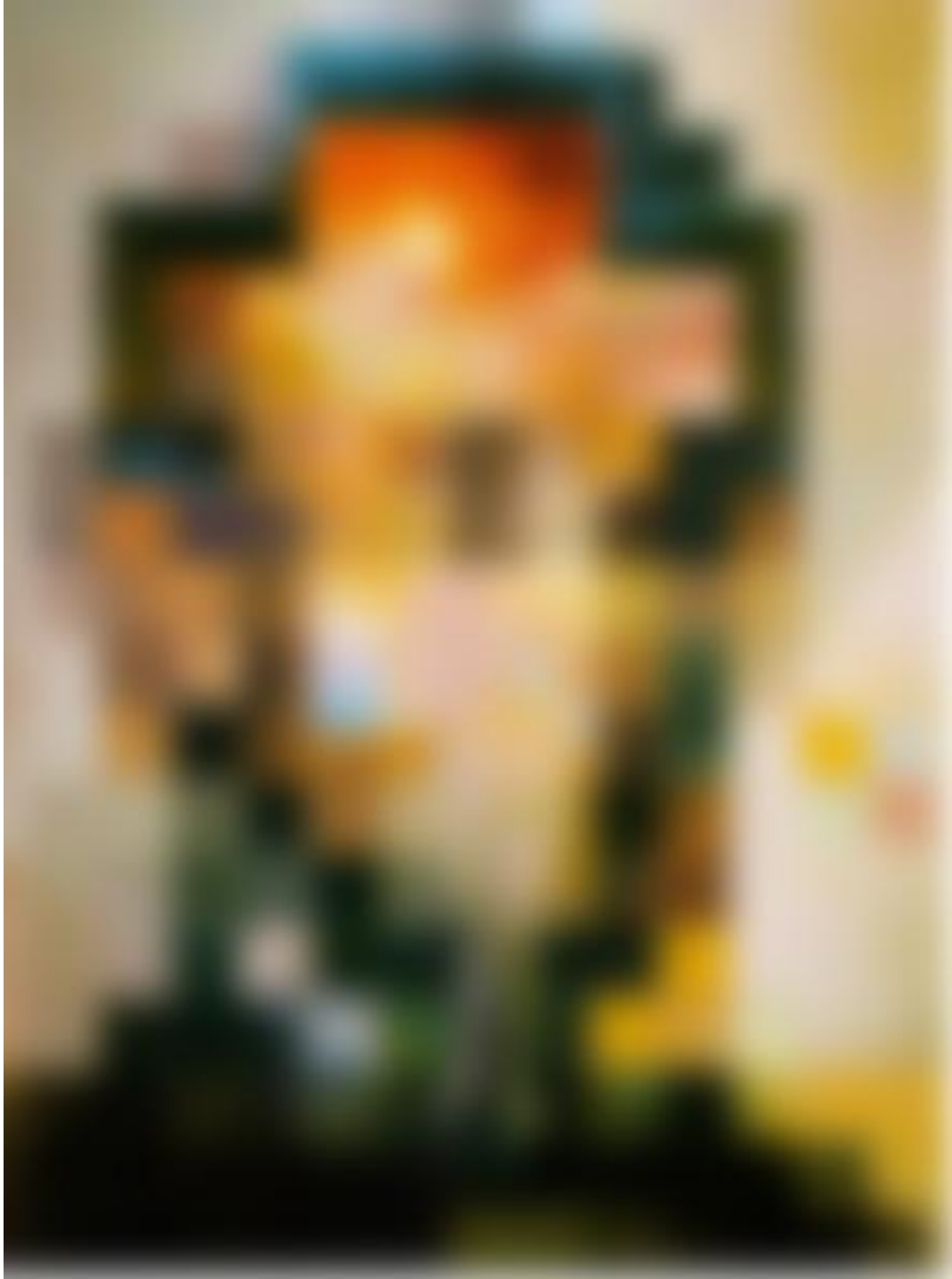
Source: James Hays, Brown

Salvador Dali invented Hybrid Images?

Salvador Dali

"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln", 1976

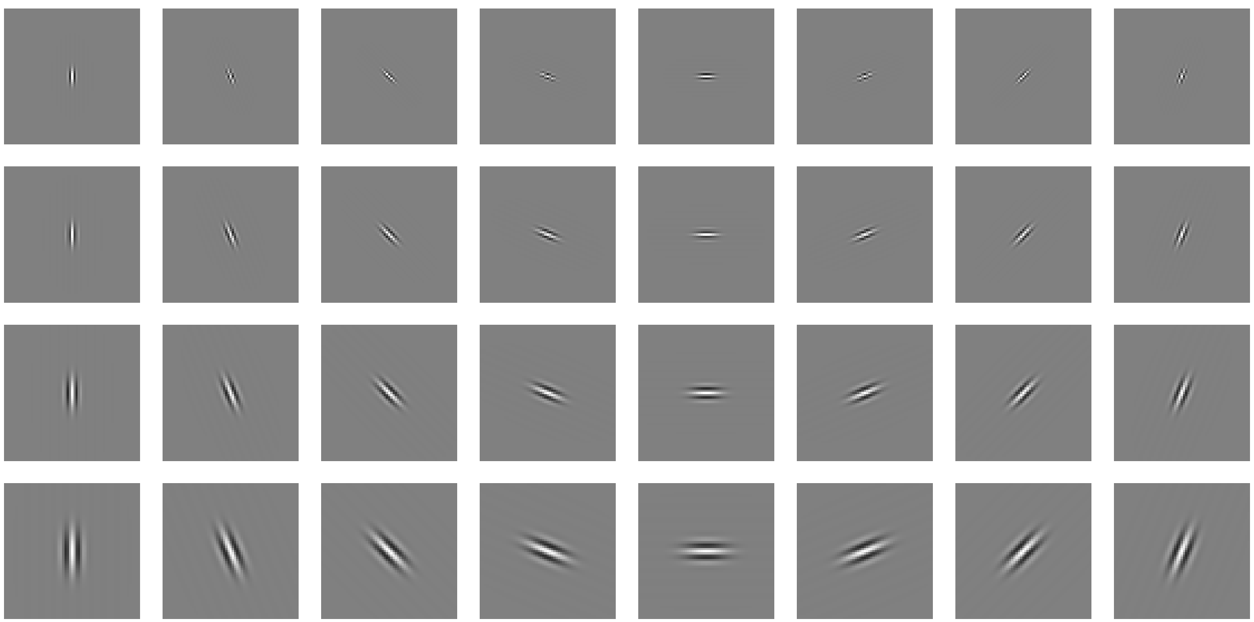
Source: James Hays, Brown



Source: James Hays, Brown

Clues from Human Perception

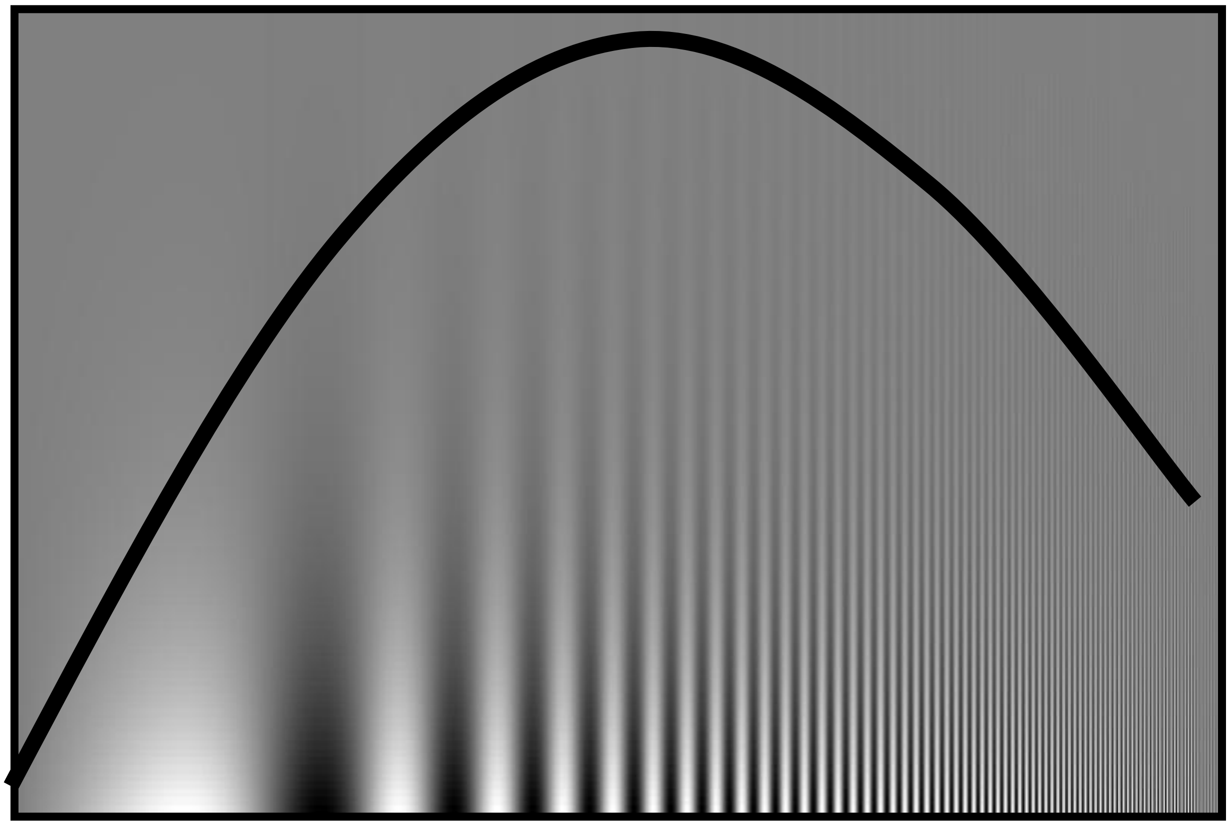
- Early processing in humans filters for various orientations and scales of frequency
- Perceptual cues in the mid-high frequencies dominate perception
- When we see an image from far away, we are effectively subsampling it



Early Visual Processing: Multi-scale edge and blob filters

Source: James Hays, Brown

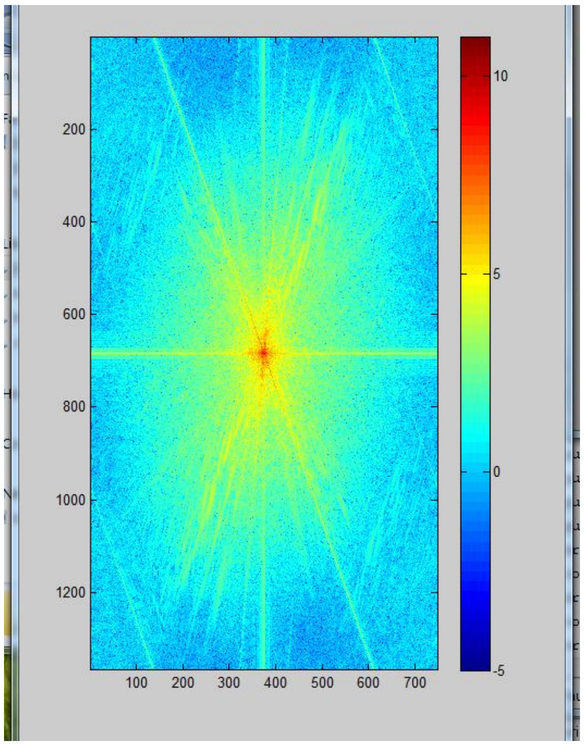
Campbell-Robson contrast sensitivity curve



Source: James Hays, Brown

Hybrid Image in FFT

Hybrid Image



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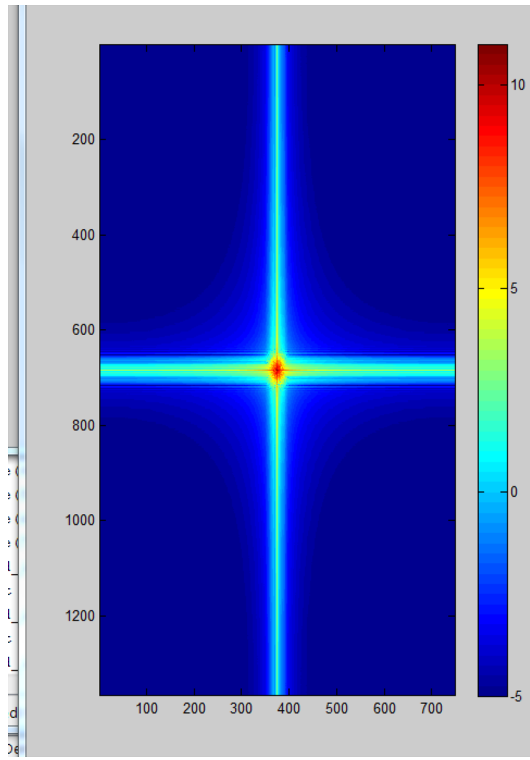
Low-passed Image



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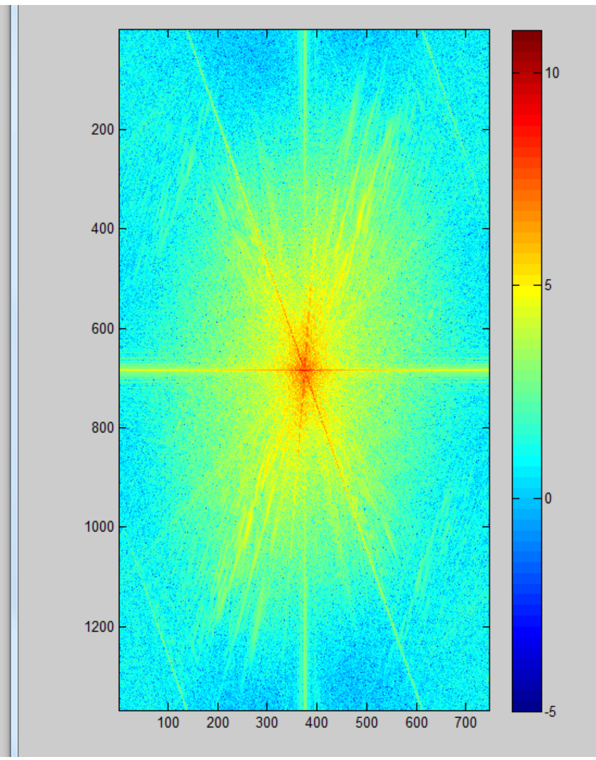
High-passed Image



Source: James Hays, Brown