Filters
(cont.)

CS 554 – Computer Vision
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Today’s topics

• Image Formation

• Image filters in spatial domain
  – Filter is a mathematical operation of a grid of numbers
  – Smoothing, sharpening, measuring texture

• Image filters in the frequency domain
  – Filtering is a way to modify the frequencies of images
  – Denoising, sampling, image compression

• Templates and Image Pyramids
  – Filtering is a way to match a template to the image
  – Detection, coarse-to-fine registration
Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Source: James Hays, Brown
Why does a lower resolution image still make sense to us? What do we lose?
Thinking in terms of frequency

Source: James Hays, Brown
Jean Baptiste Joseph Fourier (1768-1830)

had crazy idea (1807):

Any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.

• Don’t believe it?
  – Neither did Lagrange, Laplace, Poisson and other big wigs
  – Not translated into English until 1878!

• But it’s (mostly) true!
  – called Fourier Series
  – there are some subtle restrictions

...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.

Source: James Hays, Brown
A sum of sines

Our building block:

$$A \sin(\omega x + \phi)$$

Add enough of them to get any signal $f(x)$ you want!
Frequency Spectra

- example: \( g(t) = \sin(2\pi f t) + \frac{1}{3}\sin(2\pi(3f) t) \)
Frequency Spectra

Source: James Hays, Brown
Frequency Spectra

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Frequency Spectra

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Frequency Spectra

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Frequency Spectra

Source: James Hays, Brown
Frequency Spectra

\[ A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kt) \]

Source: James Hays, Brown
Example: Music

- We think of music in terms of frequencies at different magnitudes

Source: James Hays, Brown
Other signals

• We can also think of all kinds of other signals the same way
Fourier analysis in images

Intensity Image

Fourier Image

Source: James Hays, Brown

http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering
Signals can be composed

\[ \text{Signal 1} + \text{Signal 2} = \text{Result} \]

Source: James Hays, Brown

More: http://www.cs.unm.edu/~brayer/vision/fourier.html

http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering
Fourier Transform

- Fourier transform stores the magnitude and phase at each frequency
  - Magnitude encodes how much signal there is at a particular frequency
  - Phase encodes spatial information (indirectly)
  - For mathematical convenience, this is often notated in terms of real and complex numbers

Amplitude: \[ A = \pm \sqrt{R(\omega)^2 + I(\omega)^2} \]

Phase: \[ \phi = \tan^{-1} \left( \frac{I(\omega)}{R(\omega)} \right) \]

Source: James Hays, Brown
The Convolution Theorem

• The Fourier transform of the convolution of two functions is the product of their Fourier transforms

\[ F[g \ast h] = F[g]F[h] \]

• The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

\[ F^{-1}[gh] = F^{-1}[g] \ast F^{-1}[h] \]

• **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

Source: James Hays, Brown
Properties of Fourier Transforms

• Linearity \( F[ax(t) + by(t)] = aF[x(t)] + bF[y(t)] \)

• Fourier transform of a real signal is symmetric about the origin

• The energy of the signal is the same as the energy of its Fourier transform

Source: James Hays, Brown

See Szeliski Book (3.4)
Filtering in spatial domain

\[
\begin{bmatrix}
1 & 0 & -1 \\
2 & 0 & -2 \\
1 & 0 & -1 \\
\end{bmatrix}
\]

Source: James Hays, Brown
Filtering in frequency domain

1. Intensity image
2. FFT
3. Log FFT magnitude
4. Filtering
5. Inverse FFT

Source: James Hays, Brown

Slide: Hoiem
This is the magnitude transform of the cheetah pic

Source: Torralba, MIT
This is the phase transform of the cheetah pic

Source: Torralba, MIT
This is the magnitude transform of the zebra pic

Source: Torralba, MIT
This is the phase transform of the zebra pic

Source: Torralba, MIT
Reconstruction with zebra phase, cheetah magnitude

Source: Torralba, MIT
Reconstruction with cheetah phase, zebra magnitude

Source: Torralba, MIT
Phase and Magnitude

Image with cheetah phase (and zebra magnitude)

Image with zebra phase (and cheetah magnitude)

Source: Torralba, MIT
Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian

[Image of Gaussian filter output]

Box filter

[Image of Box filter output]

Source: James Hays, Brown
Gaussian

Source: James Hays, Brown
Box Filter

Source: James Hays, Brown
Sampling

Why does a lower resolution image still make sense to us? What do we lose?

Source: James Hays, Brown

Image: http://www.flickr.com/photos/igorms/136916757/
Subsampling by a factor of 2

Throw away every other row and column to create a 1/2 size image

Source: James Hays, Brown
Aliasing problem

• 1D example (sinewave):

Source: James Hays, Brown

Source: S. Marschner
Aliasing problem

• 1D example (sinewave):

Source: James Hays, Brown

Source: S. Marschner
Aliasing problem

• Sub-sampling may be dangerous....
• Characteristic errors may appear:
  – “Wagon wheels rolling the wrong way in movies”
  – “Checkerboards disintegrate in ray tracing”
  – “Striped shirts look funny on color television”
Aliasing in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what’s happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)
Aliasing in graphics

Disintegrating textures
Sampling and aliasing

Source: James Hays, Brown
Nyquist-Shannon Sampling Theorem
• When sampling a signal at discrete intervals, the sampling frequency must be $\geq 2 \times f_{\text{max}}$
• $f_{\text{max}} = \text{max frequency of the input signal}$
• This will allow to reconstruct the original perfectly from the sampled version

Source: James Hays, Brown
Anti-aliasing

Solutions:

- Sample more often

- Get rid of all frequencies that are greater than half the new sampling frequency
  - Will lose information
  - But it’s better than aliasing
  - Apply a smoothing filter

Source: James Hays, Brown
Algorithm for downsampling by factor of 2

1. Start with image(h, w)
2. Apply low-pass filter
   \[ \text{im\_blur} = \text{imfilter}(	ext{image}, \text{fspecial('gaussian', 7, 1)}) \]
3. Sample every other pixel
   \[ \text{im\_small} = \text{im\_blur}(1:2:end, 1:2:end); \]

Source: James Hays, Brown
Anti-aliasing

Source: James Hays, Brown

Forsyth and Ponce 2002
Subsampling without pre-filtering

1/2
1/4 (2x zoom)
1/8 (4x zoom)

Source: James Hays, Brown
Subsampling with Gaussian pre-filtering

Gaussian 1/2  G 1/4  G 1/8

Source: James Hays, Brown
Salvador Dali invented Hybrid Images?

Salvador Dali
“Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln”, 1976

Source: James Hays, Brown
Clues from Human Perception

- Early processing in humans filters for various orientations and scales of frequency
- Perceptual cues in the mid-high frequencies dominate perception
- When we see an image from far away, we are effectively subsampling it

Source: James Hays, Brown
Campbell-Robson contrast sensitivity curve

Source: James Hays, Brown
Hybrid Image in FFT

Hybrid Image

Low-passed Image + High-passed Image

Source: James Hays, Brown