

CS476: Automata Theory and Formal Languages

Homework 2

Assigned: 19/03/2012
Due: 02/04/2012, 17:00

Questions

- (16pts) Give a PDA for each of the following languages (with $k, l, m \geq 0$). Explain how each PDA works and why it is correct.
 - $L = \{a^k b^l c^m : k = |l - m|\}$
 - $L = \{w \in \{a, b, c\}^* : |w| = 3n_a(w)\}$
- (24pts) Give context free grammars for the languages in Question 1. Prove that the grammar you gave generates the desired language correctly.
- (32pts) Let $\Sigma = \{a, b, c\}$. Which of the following languages are context free?
 - $L = \{a^k b^l c^m : m > l, m > k\}$
 - $L = \{a^k b^l c^m : k = l \text{ or } l \neq m\}$
 - $L = \{w \in \{a, b, c\}^* : n_b(w) + n_c(w) \neq n_a(w)\}$
 - $L = \{w \in \{a, b, c\}^* : n_a(w) < n_b(w) < n_c(w)\}$

If the language is not context free, prove this by the pumping lemma. If it is context free, give a grammar or draw a PDA, and argue why your answer gives the correct language. (A formal proof is not required in this case.)

- (13pts) Consider the following grammar: (Apply the following procedures using the algorithms discussed in class)
 $S \rightarrow AB \mid aB \mid aC \mid bD \quad A \rightarrow aab \mid \epsilon \quad B \rightarrow bbA \quad C \rightarrow aD \mid bD \quad D \rightarrow Ca \mid Cb$
 - Eliminate any useless symbols.
 - Eliminate any ϵ productions in the resulting grammar.
 - Eliminate any unit productions in the resulting grammar.
 - Put the resulting grammar into Chomsky Normal Form.
- (15pts) Consider the following grammar:

$$\begin{aligned} S &\rightarrow aB \mid bA \mid \epsilon \\ A &\rightarrow aS \mid bAA \\ B &\rightarrow bS \mid aBB \end{aligned}$$

- Prove that the grammar is ambiguous.
- Give an unambiguous grammar for the language.
- Explain that the grammar you gave in (c) is unambiguous and it generates the language correctly.

Bonus Question

Consider the language $L = (a+b)^* - \{(a^i b^i)^i \mid i \geq 1\}$. Show that this language is context-free by carefully describing in English how a PDA that accepts this language would work. Be sure to explain why your PDA accepts exactly this language.