CS476: Automata Theory and Formal Languages

Homework 3

Assigned: May 10, 2012

Due: May 14, 2012 **17:00**

Questions

1. (10pts) State whether the following statements are true or not. You must give a BRIEF explanation or show a counter example to receive full credit.

   (a) If a language L is decidable then the language $\bar{L}$ is recognizable.

   (b) If languages L_1 and $L_1 \cap L_2$ are decidable then L_2 must also be decidable.

   (c) If a language L is decidable, then L is in NP.

   (d) Any undecidable problem is NP-complete.

2. (10pts) A one-change Turing machine is a TM that cannot overwrite on the tape. In other words, it can write to a previously empty cell but cannot write to a already filled cell. Show that such a TM model is equivalent to the standard TM model.

3. (30pts) Disprove (by reduction) or prove that the following languages are decidable.

   (a) $MIRROR_{DFA} = \{\langle A \rangle : A$ is a DFA and it accepts w^r iff it accepts w $\}$ (w^r denotes the inverse of w. For example $abcc^r = ccba$)

   (b) $MIRROR_{TM} = \{\langle A \rangle : A$ is a TM and it accepts w^r iff it accepts w $\}$

   (c) $\text{INTERSECT}_{TM} = \{\langle M, M', k \rangle : M, M'$ are TMs and $|L(M) \cap L(M')| \geq k$ $\}$

4. (20pts) Let us define $Cycle$ operation on languages as follows:

$$Cycle(L) = \{w : rotate_i(w) \in L \text{ for some } i\}$$

   where $rotate_i(w)$ rotates the string i times to the right. e.g. $rotate_2(abcde) = deabc$.

   (a) Show that the class of decidable languages are closed under $Cycle$ operation.

   (b) Show that the class of recognizable languages are closed under $Cycle$ operation.

5. (30pts) Prove that the following problems are NP-complete.

   (a) For a directed graph G, a *core* of G is defined as a set of vertices such that:

      i. There are no arcs between any two vertices in the core, and

      ii. Every vertex outside the core has to have an arc into it from the core.

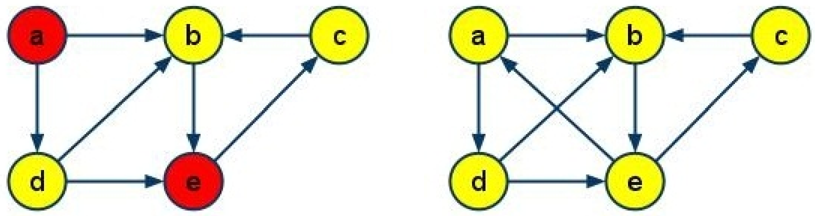
   Note that a directed graph may not have a core at all. (See Figure 1.)

   We define the `Core` problem as follows:

`Core` Problem:
Instance: Directed graph $G = (V, E)$.
Question: Does G contain a core?

   Prove that `Core` is an NP-complete problem.

   [Hint: Reduce from 3SAT. Have a vertex for each clause and a vertex for each literal. Also note that a cycle of length two or three may have only one vertex in a core.]



(a) $\{a, e\}$ is a core of this graph. Actually, it is the only core. (Note that b and d are covered by a and c is covered by e.)

(b) There are no cores in this graph.

Figure 1: Two example graphs with and without a core.

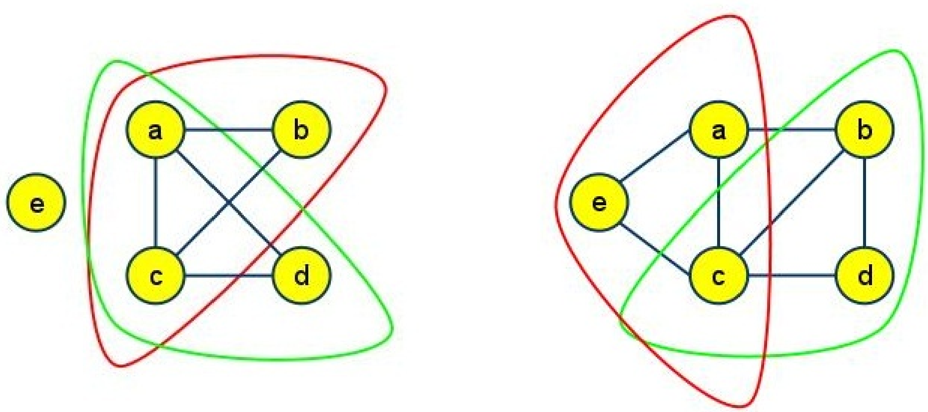
(b) We define the *Edge Clique Cover* (`ECC`) and *Vertex Clique Cover* (`VCC`) problems as follows:

`ECC` Problem:
Instance: An undirected graph $G = (V, E)$, a positive integer p.
Question: Do there exist *at most* p different cliques in G that together *cover* all edges? (i.e. every edge is contained in at least one of the selected cliques).

`VCC` Problem:
Instance: An undirected graph $G = (V, E)$, a positive integer q.
Question: Do there exist *at most* q different cliques in G that together *cover* all vertices? (i.e. every vertex is contained in at least one of the selected cliques).

Given that `ECC` problem is NP-complete, prove that `VCC` problem is NP-complete.

[Hint: Given a graph G of `ECC` instance, consider constructing the graph G' of `VCC` instance, where for every edge in G, there is a vertex in G'.]



(a) Cliques $\{a, b, c\}$ and $\{a, c, d\}$ together cover all the edges. So, there is an edge clique cover of 2 cliques for the given graph, i.e., there is an `ECC` solution for $p = 2$. Whereas, there is no clique that covers all edges by itself, i.e., there is no `ECC` solution for $p = 1$.

(b) Cliques $\{a, c, e\}$ and $\{b, c, d\}$ together cover all the vertices. So, there is a vertex clique cover of 2 cliques for the given graph, i.e., there is a `VCC` solution for $q = 2$. Whereas there is no clique that covers all vertices by itself, i.e., there is no `VCC` solution for $q = 1$.

Figure 2: Example edge and vertex clique covers.