CS476: Automata Theory and Formal Languages

Homework 3

Assigned: 27/04/2011

Due: 11/05/2011 **17:00**

Questions

1. (10pts) State whether the following statements are true or not. You must give a BRIEF explanation or show a counter example to receive full credit.

   (a) If languages L_1 and $L_1 \cup L_2$ are decidable then L_2 must also be decidable.

   (b) If a language L is Turing decidable, then L is in NP.

   (c) If a language L is in NP, then L is Turing decidable.

   (d) There exist reductions both from Boolean Satisfiability Problem (SAT) to Vertex Cover (VC) and from VC to SAT.

2. (15pts) Suppose a Turing machine model where transitions can possibly depend on three consecutive cells under the head, the one on the left and the one on the right, instead of depending on only the one under the head. Prove that this model of Turing machines is equivalent to the standard Turing machine model.

3. (30pts) Disprove (by reduction) or prove that the following languages are decidable.

   (a) $BITFLIP_{DFA} = \{\langle A \rangle : A \text{ is a DFA and } L(A) = L'(A)\}$, where $L'(A) = \{\overline{w} : w \in L(A)\}$

   (b) $BITFLIP_{TM} = \{\langle A \rangle : A \text{ is a TM and } L(A) = L'(A)\}$, where $L'(A)$ is defined as in (a).

   (c) $EACHOTHER_{TM} = \{\langle M_1, M_2 \rangle : M_1$ and M_2 are TMs such that M_1 accepts $\langle M_2 \rangle$ and M_2 accepts $\langle M_1 \rangle$.

4. (15pts) Let us define *Glue* operation on languages as follows:

$$Glue(L_1, L_2) = \{w_1 v w_2 : |v| \geq 1; w_1 v \in L_1; v w_2 \in L_2; v, w_1, w_2 \in \Sigma^*\}$$

   (a) Show that the class of decidable languages are closed under *Glue* operation.

   (b) Show that the class of recognizable languages are closed under *Glue* operation.

5. (30pts) Prove that the following problems are NP-complete.

   (a) For a directed graph G, a *core* of G is defined as a set of vertices such that:

      i. There are no arcs between any two vertices in the core, and

      ii. Every vertex outside the core has to have an arc into it from the core.

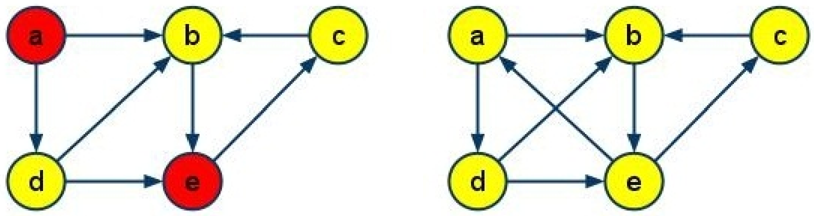
   Note that a directed graph may not have a core at all. (See Figure 1.)

   We define the `Core` problem as follows:

`Core` Problem:
Instance: Directed graph $G = (V, E)$.
Question: Does G contain a core?

   Prove that `Core` is an NP-complete problem.

   [Hint: Reduce from 3SAT. Have a vertex for each clause and a vertex for each literal. Also note that a cycle of length two or three may have only one vertex in a core.]



(a) $\{a, e\}$ is a core of this graph. Actually, it is the only core. (Note that b and d are covered by a and c is covered by e.)

(b) There are no cores in this graph.

Figure 1: Two example graphs with and without a core.

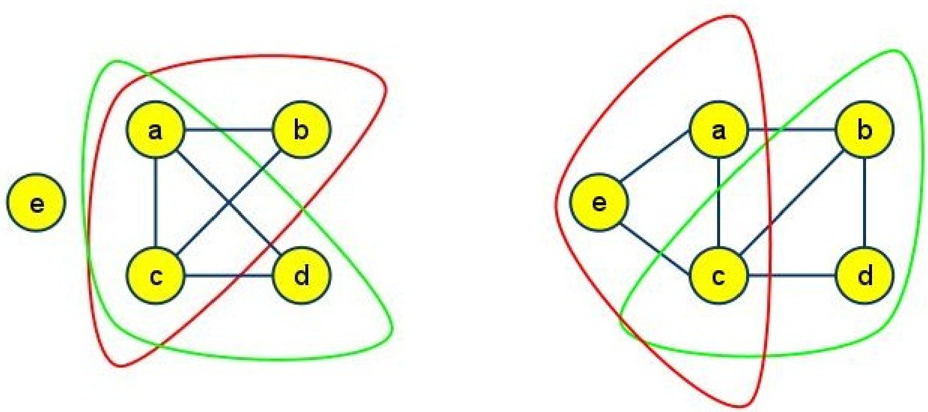
(b) We define the *Edge Clique Cover* (`ECC`) and *Vertex Clique Cover* (`VCC`) problems as follows:

`ECC` Problem:
Instance: An undirected graph $G = (V, E)$, a positive integer p.
Question: Do there exist *at most* p different cliques in G that together *cover* all <u>edges</u>? (i.e. every edge is contained in at least one of the selected cliques).

`VCC` Problem:
Instance: An undirected graph $G = (V, E)$, a positive integer q.
Question: Do there exist *at most* q different cliques in G that together *cover* all <u>vertices</u>? (i.e. every vertex is contained in at least one of the selected cliques).

Given that `ECC` problem is NP-complete, prove that `VCC` problem is NP-complete.

[Hint: Given a graph G of `ECC` instance, consider constructing the graph G' of `VCC` instance, where for every edge in G, there is a vertex in G'.]



(a) Cliques $\{a, b, c\}$ and $\{a, c, d\}$ together cover all the edges. So, there is an edge clique cover of 2 cliques for the given graph, i.e., there is an `ECC` solution for $p = 2$. Whereas, there is no clique that covers all edges by itself, i.e., there is no `ECC` solution for $p = 1$.

(b) Cliques $\{a, c, e\}$ and $\{b, c, d\}$ together cover all the vertices. So, there is a vertex clique cover of 2 cliques for the given graph, i.e., there is a `VCC` solution for $q = 2$. Whereas there is no clique that covers all vertices by itself, i.e., there is no `VCC` solution for $q = 1$.

Figure 2: Example edge and vertex clique covers.