Title: Implementation of a Parallel Delaunay Triangulation Algorithm

Goal: To successfully implement and demonstrate a Parallel Delaunay Triangulation Algorithm in MATLAB

Via the steps of the algorithm detailed in the work of Blelloch, Hardwick, Miller, and Talmor [1], this project will implement Delaunay triangulation using a specialized parallel programming software (MATLAB: Parallel Computing Toolbox). This implementation will be attempted to run on $O(n \log_2 n)$ time, as this was demonstrated to be possible by Blelloch et al [1]. The usage of MATLAB is tentative.

Here is a brief summary of the algorithm [1]: As told by Blelloch et al., the Delaunay triangulation is able to be transformed into a 3d-convex hull where each point that is to be triangulated $p = (x, y)$ is mapped to the point $p' = (x, y, \alpha |p|^2)$, which lies on the paraboloid $z = \alpha x^2 + \alpha y^2$, where $\alpha$ is a constant. The 3D convex hull of all points $p'$ is then projected onto the vertical plane that passes through the line that is parallel to the x-axis and passes through the median of all internal points to the 2D convex hull of points p. This newest projection is then converted into new Delaunay edges. A divide-and-conquer recursive algorithm that follow these steps eventually produces almost all Delaunay edges, with the last few edges requiring careful consideration.

The data structures that will be used to represent all PSLGs will be DCELs. 3D straight-line graphs that arrive from computing the 3D convex hull derived from the transformation from the flat horizontal xy-plane to a vertical paraboloid will also be represented by special forms of DCELs if needed, where “clockwise” and “counterclockwise” will be defined on the surface of the paraboloid.

The algorithm will be demonstrated on four different randomly generated datasets of 2D points $(x, y)$ with the following distributions [1]:

- Uniform distribution: $(x, y)$ are independent random variables uniformly distributed on the interval $[0, 1]$.
- Normal distribution: $(x, y)$ are independent random variables with standard Gaussian distribution of zero mean and unit variance.
- Line singularity: $(x, y) = \left(\frac{u}{u-bu+b}, v\right)$ where $(u, v)$ are independent random variables uniformly distributed on the interval $[0, 1]$ and $b$ is a predetermined constant.
- Kuzmin distribution: The points in this one are generated by their polar coordinates $(r, \theta)$, where $\theta$ is a uniform random variable on the interval $[0, 2\pi)$ and $r = \frac{\sqrt{2X-X^2}}{|X-1|}$, where $X$ is a uniform random variable on the interval $(0, 1)$. $\theta$ and $X$ are independent.

References: