



CS 564 Computational Geometry
Project Proposal

Facial Expression Recognition via Geometric Delaunay Triangulation

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Proposed method

The project will construct Delaunay triangulations from detected 3d facial landmarks for each face image. These landmarks serve as vertices, forming a topological mesh¹ that captures the spatial relationships between facial features. From this mesh, handcrafted geometric features will be extracted that are invariant to scaling and rotation, including:

- (1) *triangle aspect ratios* to capture local stretching,
- (2) *internal angle vectors* to describe the "sharpness" or deformation of the mesh,
- (3) *area ratios* (individual triangle areas normalized by the total face area), and
- (4) *barycentric connectivity*, measuring the relative distances between triangle centroids.

A lightweight neural network or SVM classifier will then be trained for facial expression classification (e.g., happy, sad, angry, surprised).

Related Works

Recent research has increasingly leveraged facial landmarks for expression recognition. While [1] utilizes 2D Delaunay triangulation, it relies on raw coordinate values that lack invariance to scale and rotation. Similarly, [2] employs quantum-based nearest neighbor classifiers but remains limited to landmark positions as primary features. More complex architectures, such as Graph Neural Networks [3] and spatio-temporal models [4], integrate psychological Action Units (AUs) and 3D landmark data to improve accuracy. In the realm of spatial partitioning, [5] utilizes both Delaunay triangulation and Voronoi diagrams, though primarily for segmenting specific Regions of Interest (ROI).

Unlike these approaches, our work focuses on the geometric properties of the 3D mesh topology without relying on any priori knowledge on psychological Action Units. By extracting rotation- and scale-invariant features, we aim to capture the underlying deformation of the facial surface more robustly than raw coordinate-based methods.

Experiments

Experiments will be conducted on publicly available datasets such as *CK+* [6] or *FER2013* [7]. The project will compare two setups: a baseline classifier trained on raw landmark coordinates versus the proposed pipeline utilizing Delaunay-based geometric features to evaluate the descriptive power of mesh-based spatial partitioning. Evaluation will be performed using classification accuracy and confusion matrices.

Deliverables

include:

- (1) a complete processing pipeline
 {landmark → Delaunay → feature extraction → classifier};
- (2) quantitative results comparing methods,
- (3) visualizations of the Delaunay mesh overlaid on faces,
- (4) and an interactive application visualizing outputs of each step in the pipeline.

¹ To ensure uniform feature extraction, we need *consistent* Delaunay triangulation of landmarks across all faces. Thus, we will derive a reference triangulation from an average facial shape and project this static mesh onto all detected landmark sets.

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Technologies

The implementation will be done in Python, using MediaPipe [8] for facial landmark detection, SciPy for Delaunay triangulation construction, and PyTorch/scikit-learn for training the classifier. For visualization and the interactive application we will use Streamlit together with Plotly and OpenCV. Additional libraries that may be employed include Matplotlib/Seaborn for static plots, Bokeh or Dash for alternative dashboards, Shapely/trimesh for geometry utilities.

References

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- [4] Abedi, Ali, and Shehroz S. Khan. "Engagement measurement based on facial landmarks and spatial-temporal graph convolutional networks." *International Conference on Pattern Recognition*. Cham: Springer Nature Switzerland, 2024.
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- [7] Dumitru, et al. *Challenges in Representation Learning: Facial Expression Recognition Challenge*. 2013, kaggle.com/competitions/challenges-in-representation-learning-facial-expression-recognition-challenge.
- [8] Lugaesi, Camillo, et al. "Mediapipe: A framework for perceiving and processing reality." *Third workshop on computer vision for AR/VR at IEEE computer vision and pattern recognition (CVPR)*. Vol. 2019. Long Beach, CA, 2019.