Ensemble Learning

CS 550: Machine Learning

Ensemble Learning

- Problem: Given M base learners $\{L_1, L_2, \ldots, L_M\}$, find a combined (meta) learner with better performance
  - Very effective in many applications
  - Usually easy to implement

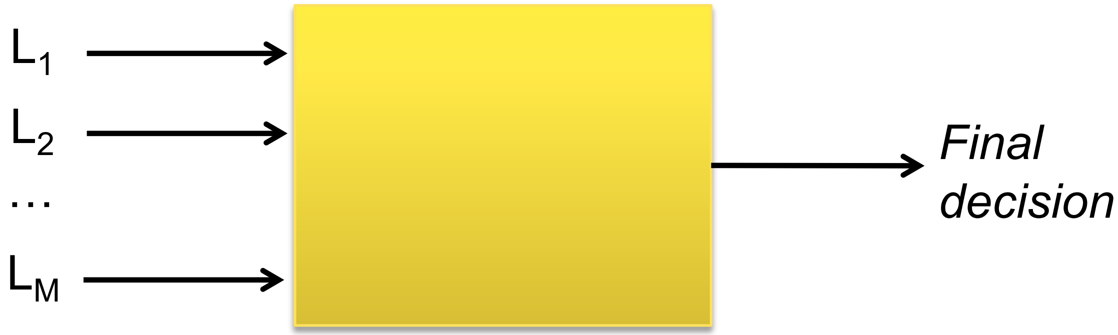
1. How to generate the base learners?
2. How to combine them?

How to Generate Base Learners?

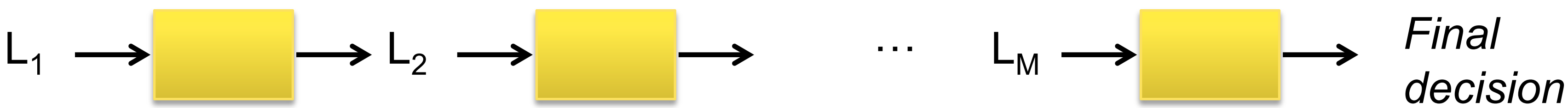
- Ensemble techniques usually work well when base learners are "reasonably" accurate (but not too much) and diverse
- Base learners can be generated using
  - Different learning algorithms
  - Same algorithm with different parameters
  - Different representations of the same input
    - Sensor fusion at the data level, feature level or decision level
  - Different training sets
    - Bagging (samples are randomly drawn)
    - Boosting (samples are drawn to generate complementary learners)

How to Combine Base Learners?

- We combine the base learners after training them in parallel



- We combine them while training them in serial



Voting

- Simplest ensemble method
- Suppose that learner L_j has prediction d_j with weight W_j

Final output $y = \sum_{j=1}^{M} W_j d_j$, $W_j \geq 0$ and $\sum_{j=1}^{M} W_j = 1$

- Simple voting (majority voting in classification) $W_j = \frac{1}{M}$
- Weighted voting
  - For example, use posteriors as weights and select the class for which y_i is the maximum $y_i = \sum_{j=1}^{M} P(C_i \mid x, L_j)$
  - You can also consider the whole procedure as a Bayesian model $y_i \equiv P(C_i \mid x) = \sum_{j=1}^{M} P(C_i \mid x, L_j)\ P(L_j)$

Bagging (**B**ootstrap **AG**gregating)

- Generate L base learners from the same training set $\mathcal{D}$
  - For each learner, use a separate training set that is generated by drawing N samples randomly from $\mathcal{D}$ by replacement (each training set may have duplicate samples)
  - For a given new sample, combine the decisions of all learners (for example by simple voting)

Boosting

- Generate L base learners from the same training set $\mathcal{D}$
  - For the first classifier, generate a training set similar to bagging
  - Then, for the next classifier, generate a training set that more likely contains samples misclassified by the previous classifiers
  - The most famous boosting algorithm is called AdaBoost (by Freund and Schapire, 1996), which has many variants

AdaBoost – Training

There are M learners, L_j, each of which will be trained on $\mathcal{D}_j$ generated from the training set $\mathcal{D} = \{x^t\}_{t=1}^{T}$

p_j^t: probability of selecting sample x^t for the training set $\mathcal{D}_j$ of the learner L_j

initialize $p_1^t = 1/T$

for $j = 1$ to M

    construct $\mathcal{D}_j$ by drawing N samples from $\mathcal{D}$ according to p_j^t

    train L_j on $\mathcal{D}_j$

    calculate the class of each sample using L_j and compute the error rate ϵ_j

    if $\epsilon_j > 0.5$ stop

    $\beta_j = \frac{\epsilon_j}{1-\epsilon_j}$ ($\beta_j < 1$ when $\epsilon_j < 0.5$)

    for each sample x^t

        if x^t is correctly classified

            $p_{j+1}^t = \beta_j \cdot p_j^t$ (decrease its probability)

        else

            $p_{j+1}^t = p_j^t$

    normalize probabilities by $p_{j+1}^t = \frac{p_{j+1}^t}{\sum_u p_{j+1}^u}$

AdaBoost – Classifying

for a given x, calculate $P(C_i|x, L_j)$ using each classifier L_j

for each class i

$$y_i = \sum_{j=1}^{M} log(\frac{1}{\beta_j}) \cdot P(C_i|x, L_j)$$

select the class with the maximum y_i

Random Forests

- Construct many classification trees (diversity is important) and combine their decisions (for example by voting)
- Each tree may be grown
  - Using a different training set (e.g., draw N samples from the original set with replacement)
  - Randomly selecting k features out of d features and considering only the splits on the selected features
  - Using a different training set without any pruning

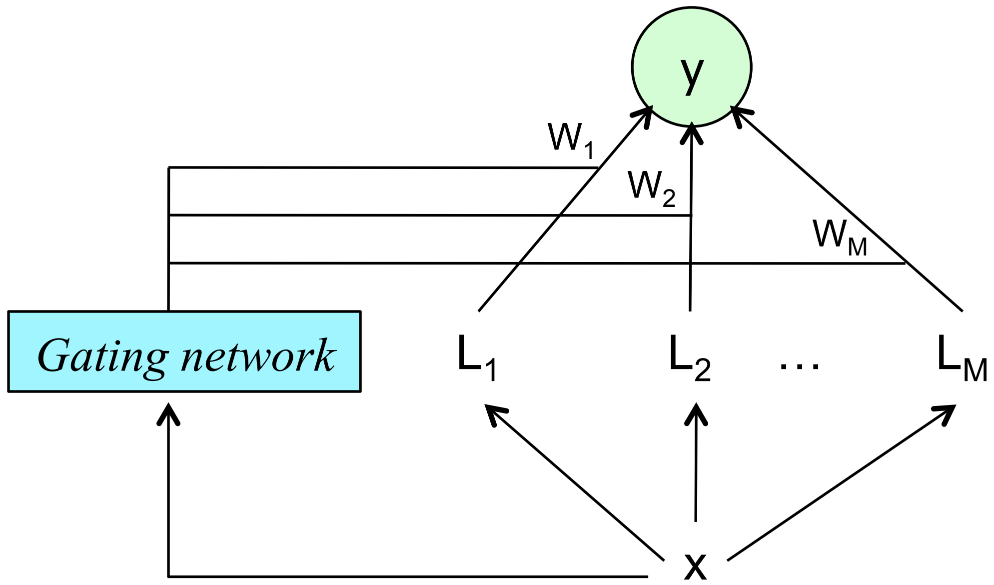
Mixture of Experts

- Each base learner is considered as an expert
- There is a gating network that outputs the weight of each expert for a given sample x

Voting

$$y = \sum_{j=1}^{M} W_j \, d_j(x)$$

same for all instances



Mixture of experts

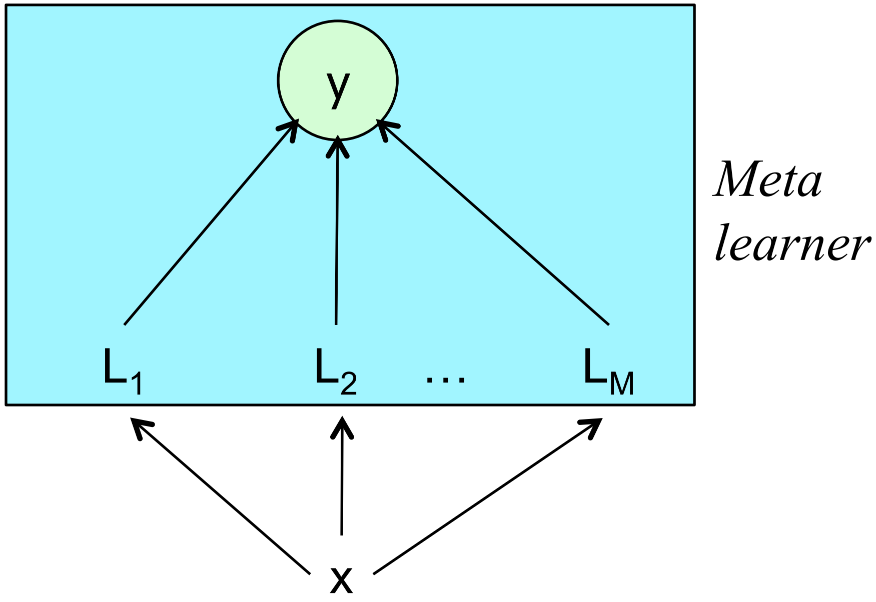
$$y = \sum_{j=1}^{M} W_j(x) \, d_j(x)$$

determined for each sample separately by the gating network

How do you learn the gating network?

Stacking

- There is a meta learner that learns the output of a sample from the outputs of the base learners (not directly from the inputs of the sample)

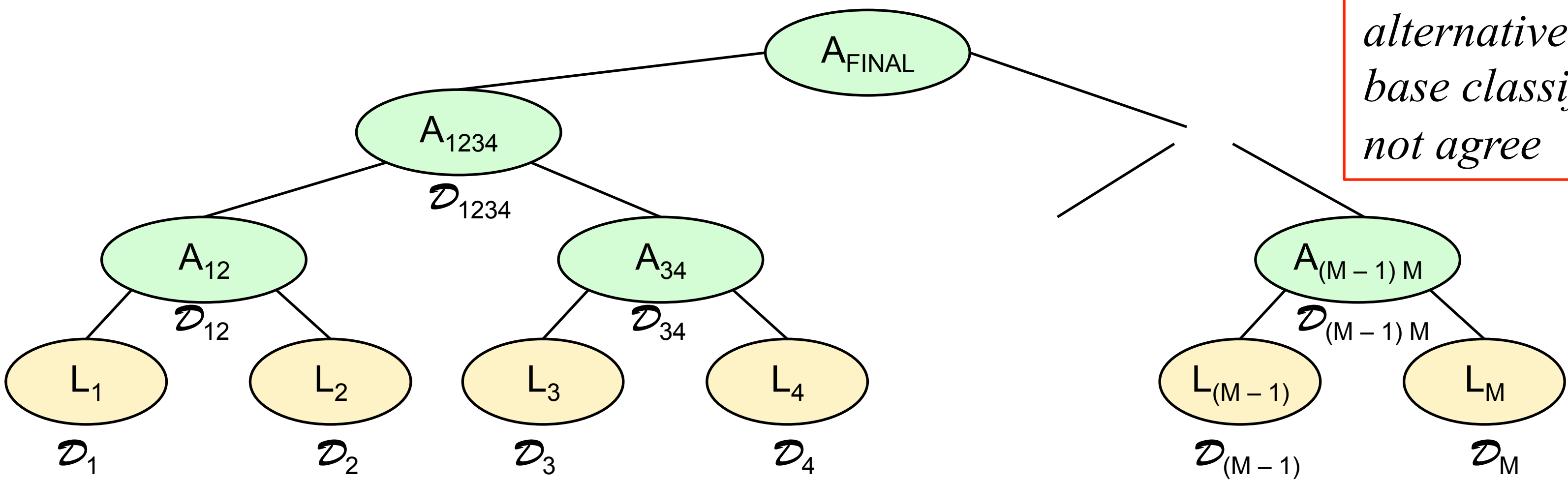


How do you learn the meta learner?

Arbiter Trees

- Base learners are trained on disjoint subsets of training data
- $\mathcal{D}_{ij}$ can be formed
  1. Considering samples on which base classifiers disagree
  2. Item 1 + incorrectly classified samples
  3. Item 2 + some (or all) correctly classified samples
- To classify an unseen sample, one may
  - Use the arbiter if there exists disagreement
  - Combine its decision with those of the base learners
  - Use your own technique

Arbiter provides an alternative decision if base classifiers do not agree



Error-Correcting Output Codes

- Create many binary classifiers that distinguish one class from the others and then combine their decisions
- After training binary classifiers, classify a sample with each of them and select the class whose coding is the most similar to the coding of the sample
  - Sum of squared errors
  - Hamming distance

$$W = \underbrace{\left.\begin{bmatrix} & & & 1 & & & & \\ & & & 0 & & & & \\ & & & 0 & & & & \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ & & & 1 & & & & \\ & & & 0 & & & & \end{bmatrix}\right\} K\ classes}_{M\ binary\ base\ learners}$$

Binary classification by the 5th learner
gives label 0 for Class 1 and 5
gives label 1 for Class 2, 3, 4, and 6

Error-Correcting Output Codes

- How to construct a codebook? ← IMPORTANT CHALLENGE
  - Could be set a priori
  - Could be formed in a random manner
  - Could be designed to optimize accuracy

$$W = \underbrace{\left.\begin{bmatrix} & & & 1 & & & & \\ & & & 0 & & & & \\ & & & 0 & & & & \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ & & & 1 & & & & \\ & & & 0 & & & & \end{bmatrix}\right\} \textit{K classes}}_{\textit{M binary base learners}}$$

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