Genetic Algorithm + Simulated Annealing = GENOCOP II: A Tool for Nonlinear Programming

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Abstract

In this paper we discuss a construction of a hybrid optimization system Genocop II for general nonlinear programming problems. We present the first experimental results of the system on five test cases. These include a variety of objective functions with nonlinear constraints. The first results are more than encouraging.

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1 Introduction

The general nonlinear programming problem $\mathcal{NP}$ is to find $\overline{X}$ so as to

$$\text{optimize } f(\overline{X}),\ \overline{X} = (x_1, \ldots, x_q) \in R^q,$$

subject to $p \geq 0$ equations:

$$c_i(\overline{X}) = 0,\ i = 0, \ldots, p,$$

and $m - p \geq 0$ inequalities:

$$c_i(\overline{X}) \leq 0,\ i = p+1, \ldots, m.$$

In this paper we discuss a new hybrid system, Genocop II, to solve optimization problems in this class. The name of the system Genocop is taken from the expression "GEnetic algorithm for Numerical Optimization for COnstrainted Problems". The first version of the Genocop system, Genocop I, handled only linear constraints [16], [18], [19], whereas the current version, Genocop II, handles any set of constraints.

The concept of the presented system is based on the ideas taken from the recent developments in area of optimization [2] combined together with iterative execution of the Genocop I; these executions are controlled by a temperature of the system (which can be also interpreted as a variable penalty coefficient). This explains the equation used in the title of the paper.

The prototype of the new system Genocop II was run on several test-cases; the results of experiments are presented and compared with known optimas.

The paper is organized as follows. The next section summarizes one traditional approach to solve nonlinear programming problems, which was adopted as a basis for Genocop II. Section 3 introduces the system Genocop II, and the following section discusses several nonlinear test cases together with the experimental results of the Genocop II. The last section contains conclusions and some directions for future work.

2 A Traditional Calculus Based Method

Calculus based methods assume that the objective function $f(\overline{X})$ and all constraints are twice continuously differentiable functions of $\overline{X}$. The general approach in most methods is to transform the nonlinear problem $\mathcal{NP}$ into a sequence of solvable subproblems. The amount of work involved in a subproblem varies considerably among methods. These methods require explicit (or implicit) second derivative calculations of the objective (or transformed) function which in some methods can be ill-conditioned and causes the algorithm to fail.

During the last 30 years there has been considerable research directed toward the nonlinear optimization problems and progress has been made in theory and practice [11].

Several approaches have been developed in this area, among these are: the sequential quadratic penalty function [6], [2], recursive quadratic programming method [5], penalty trajectory methods [20], and the SOLVER method [9].

In this section, we discuss briefly one of these approaches, the sequential quadratic penalty function method.

The method replaces a problem $\mathcal{NP}$ by the problem $\mathcal{NP}'$:

$$\text{optimize } F(\overline{X}, r) = f(\overline{X}) + \tfrac{1}{2r}\overline{C}^T\overline{C},$$

where $r > 0$ and $\overline{C}$ is a vector of all active constraints $c_1, \ldots, c_\ell$.

Fiacco and McCormick [14] have shown that the solutions of $\mathcal{NP}$ and $\mathcal{NP}'$ were identical in the limit as $r \longrightarrow 0$. It was thought that $\mathcal{NP}'$ could then be solved simply by minimizing $F(\overline{X}, r)$ for a sequence of decreasing positive values of r by Newton's method then being developed [8]. This hope, however, was short-lived, because minimizing $F(\overline{X}, r)$ proved to be extremely inefficient for the smaller values of r; it was shown by Murray [20] that this was due to the Hessian matrix of $F(\overline{X}, r)$ becoming increasingly ill-conditioned as $r \longrightarrow 0$. Since there seemed to be no obvious way of overcoming this problem, the method gradually fell into disuse. More recently, Broyden and Attia [7], [6] gave a method of overcoming the numerical difficulties associated with the simple quadratic penalty function. In this method the computation of the search direction does not require the solution of any system of linear equations, and can thus be expected to require less work than in some other algorithms. The method also provides an automatic technique for calculating the initial value for the parameter r and its successive values [6].

3 The System Genocop II

The technique discussed in the previous section, together with the existing system Genocop I, was used in construction of a new system, Genocop II. The structure of the Genocop II is given in Figure 1. We discuss the steps of this algorithm in the remaining part of this section.

There are several steps of the algorithm in the first phase of its execution (before it enters the *while* loop). The parameter t (which counts the number of iterations of the algorithm, i.e., the number of times the algorithm Genocop I is applied) is initialized to zero. The set of all constraints C is divided into three subsets: linear constraints L, nonlinear equations N_e and nonlinear inequalities N_i. A starting point $\overline{X}_s$ (which need not be feasible) for the following optimization process is selected (or a user is prompted for it). The set of active constraints A consists initially of elements of N_e and set $V \subseteq N_i$ of violated constraints from N_i. A constraint $c_j \in N_i$ is violated at point $\overline{X}$ iff $c_j(\overline{X}) > \delta$ ($j = p+1, \ldots, m$), where δ is a parameter of the method. Finally, the initial temperature of the system τ is set to τ_0 (a parameter of the method).

In the main loop of the algorithm we apply Genocop I to optimize a modified function

procedure **Genocop II**
begin
  $t \longleftarrow 0$
  split the set of constraints C into
    $C = L \cup N_e \cup N_i$
  select a starting point $\overline{X}_s$
  set the set of active constraints, A to
    $A \longleftarrow N_e \cup V$
  set temperature $\tau \longleftarrow \tau_0$
  while (**not** termination-condition) **do**
  begin
    $t \longleftarrow t + 1$
    execute Genocop I for the function
      $F(\overline{X}, \tau) = f(\overline{X}) + \frac{1}{2\tau}\overline{A}^T\overline{A}$
      with linear constraints L
      and the starting point $\overline{X}_s$
    save the best individual $\overline{X}^*$:
      $\overline{X}_s \longleftarrow \overline{X}^*$
    update A:
      $A \leftarrow A - S \cup V$,
    decrease temperature τ:
      $\tau \leftarrow g(\tau, t)$
  end
end

Figure 1: The structure of Genocop II

$$F(\overline{X}, \tau) = f(\overline{X}) + \tfrac{1}{2\tau}\overline{A}^T\overline{A}$$

with linear constraints L. Note that the initial population for Genocop I consists of *pop_size* identical copies (of the initial point for the first iteration and of the best saved point for subsequent ones); several mutation operators introduce diversity in the population at early stages of the process. When Genocop I converges, its best individual $\overline{X}^*$ is saved and used later as the starting point $\overline{X}_s$ for the next iteration. However, the next iteration is executed with a decreased value of the temperature parameter ($\tau \leftarrow g(\tau, t)$) and a new set of active constraints A:

$$A \leftarrow A - S \cup V,$$

where S and V are subsets of N_i satisfied and violated by $\overline{X}^*$, respectively.

The mechanism of the algorithm is illustrated on the following example. The problem is to

$$\text{minimize } f(\overline{X}) = x_1 \cdot x_2^2,$$

subject to one nonlinear constraint:

$$c_1 \ : \ \ 2 - x_1^2 - x_2^2 \geq 0.$$

The known global solution is $\overline{X}^* = (-0.816497, -1.154701)$, and $f(\overline{X}^*) = -1.088662$. The starting feasible point is $\overline{X}_0 = (-0.99, -0.99)$.

After the first iteration of Genocop II (A is empty) the system converged to $\overline{X}_1 = (-1.5, -1.5)$, $f(\overline{X}_1) = -3.375$. The point $\overline{X}_1$ violates the constraint c_1, which becomes active. The point $\overline{X}_1$ is used as the starting point for the second iteration.

The second iteration ($\tau = 10^{-1}$, $A = \{c_1\}$) resulted in $\overline{X}_2 = (-0.831595, -1.179690)$, $f(\overline{X}_2) = -1.122678$. The point $\overline{X}_2$ is used as the starting point for the third iteration.

The third iteration ($\tau = 10^{-2}$, $A = \{c_1\}$) resulted in $\overline{X}_3 = (-0.815862, -1.158801)$, $f(\overline{X}_3) = -1.09985$.

The sequence of points $\overline{X}_t$ (where $t = 4, 5, \ldots$ is the iteration number of the algorithm) approaches the optimum.

4 Test Cases

In order to evaluate the method of Genocop II, a set of test problems have been carefully selected to illustrate the performance of the algorithm and to indicate that it has been successful in practice. The five test cases include quadratic, nonlinear, and discontinuous functions with several nonlinear constraints.

All runs of the system were performed on SUN SPARC station 2. We used the following parameters for Genocop I for all experiments:

> $pop\_size = 70$, $k = 28$ (number of parents in each generation), $b = 2$ (coefficient for non-uniform mutation), $a = 0.25$ (parameter of arithmetical crossover), $\delta = 0.01$ (parameter which determines whether a constraint is active or not).

In most cases, the initial temperature τ_0 was set at 1 (i.e., $g(\tau, 0) = 1$); additionally, $g(\tau, t) = 10^{-1} \cdot g(\tau, t-1)$.

For each test case we run the Genocop II ten times. For most problems, the number of generations necessary for Genocop I to converge was 1000 (harder problems required larger number of iterations). We did not report the computational times for these test cases, since we do not have full implementation of Genocop II yet. The actions of the system were simulated by executing its the external loop in manual fashion: when Genocop I converges, the best point is incorporated as the starting point for the next iteration, the constraints are checked for their activity status, and the evaluation function is adjusted accordingly.

All test cases and the results of the Genocop II system are reported in the following subsections.

4.1 Test Case #1

The problem (taken from [13]) is

$$\text{minimize } f(\overline{X}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2,$$

subject to nonlinear constraints:

$$\begin{aligned} c_1 &: \quad x_1 + x_2^2 \geq 0, \\ c_2 &: \quad x_1^2 + x_2 \geq 0, \end{aligned}$$

and bounds:

$$-0.5 \leq x_1 \leq 0.5, \text{ and } x_2 \leq 1.0.$$

The known global solution is $\overline{X}^* = (0.5, 0.25)$, and $f(\overline{X}^*) = 0.25$. The starting feasible point is $\overline{X}_0 = (0, 0)$.

The Genocop II found the exact optimum in one iteration, since none of the nonlinear constraints are active at optimum.

4.2 Test Case #2

The problem (taken from [10]) is

$$\text{minimize } f(x, y) = -x - y,$$

subject to nonlinear constraints:

$$\begin{aligned} c_1 &: \quad y \leq 2x^4 - 8x^3 + 8x^2 + 2, \\ c_2 &: \quad y \leq 4x^4 - 32x^3 + 88x^2 - 96x + 36, \end{aligned}$$

and bounds:

$$0 \leq x \leq 3 \text{ and } 0 \leq y \leq 4.$$

The known global solution is $\overline{X}^* = (2.3295, 3.1783)$, and $f(\overline{X}^*) = -5.5079$. The starting feasible point is $\overline{X}_0 = (0, 0)$. The feasible region is almost disconnected.

The Genocop II approached the optimum very closely at the 4th iteration. The progress of the system is reported in the table 1.

Iteration number	The best point	Active constraints
0	(0,0)	none
1	(3,4)	c_2
2	(2.06, 3.98)	c_1, c_2
3	(2.3298, 3.1839)	c_1, c_2
4	(2.3295, 3.1790)	c_1, c_2

Table 1: Progress of Genocop II on test case #2; for iteration 0 the best point is the starting point

4.3 Test Case #3

The problem (taken from [10]) is

$$\text{minimize } f(\overline{X}) = (x_1 - 10)^3 + (x_2 - 20)^3,$$

subject to nonlinear constraints:

$$\begin{aligned} c_1 &: (x_1 - 5)^2 + (x_2 - 5)^2 - 100 \geq 0, \\ c_2 &: -(x_1 - 6)^2 - (x_2 - 5)^2 + 82.81 \geq 0, \end{aligned}$$

and bounds:

$$13 \leq x_1 \leq 100 \text{ and } 0 \leq x_2 \leq 100.$$

The known global solution is $\overline{X}^* = (14.095, 0.84296)$, and $f(\overline{X}^*) = -6961.81381$ (see figure 2). The starting not feasible point is $\overline{X}_0 = (20.1, 5.84)$.

The Genocop II approached the optimum very closely at the 12th iteration. The progress of the system is reported in the table 2.

4.4 Test Case #4

The problem (taken from [4]) is

$$\text{minimize } f(x_1, x_2) = 0.01x_1^2 + x_2^2,$$

subject to nonlinear constraints:

$$\begin{aligned} c_1 &: x_1 x_2 - 25 \geq 0, \\ c_2 &: x_1^2 + x_2^2 - 25 \geq 0, \end{aligned}$$

and bounds:

Figure 2: A feasible space for test case #3.

$2 \leq x_1 \leq 50$ and $0 \leq x_2 \leq 50$.

The global solution is $\overline{X}^* = (\sqrt{250}, \sqrt{2.5}) = (15.811388, 1.581139,$ and $f(\overline{X}^*) = 5.0$. The starting (not feasible) point is $\overline{X}_0 = (2, 2)$.

It is interesting to note that the standard cooling scheme (i.e., $g(\tau, t) = 10^{-1} \cdot g(\tau, t-1)$) did not produce good results, however, when the cooling process was slowed down (i.e., $g(\tau, 0) = 5$ and $g(\tau, t) = 2^{-1} \cdot g(\tau, t-1)$), the system approached optimum easily (table 3).

4.5 Test Case #5

The final test problem (taken from [13]) is

minimize $f(\overline{X}) = (x_1 - 2)^2 + (x_2 - 1)^2$,

subject to a nonlinear constraint:

$$c_1 : \; -x_1^2 + x_2 \geq 0,$$

and a linear constraint:

$$x_1 + x_2 \leq 2.$$

The global solution is $\overline{X}^* = (1, 1)$ and $f(\overline{X}^*) = 1$. The starting (feasible) point is $\overline{X}_0 = (0, 0)$.

The Genocop II approached the optimum very closely at the 6th iteration. The progress of the system is reported in the table 4.

Iteration number	The best point	Active constraints
0	(20.1, 5.84)	c_1, c_2
1	(13.0, 0.0)	c_1, c_2
2	(13.63, 0.0)	c_1, c_2
3	(13.63, 0.0)	c_1, c_2
4	(13.73, 0.16)	c_1, c_2
5	(13.92, 0.50)	c_1, c_2
6	(14.05, 0.75)	c_1, c_2
7	(14.05, 0.76)	c_1, c_2
8	(14.05, 0.76)	c_1, c_2
9	(14.10, 0.87)	c_1, c_2
10	(14.10, 0.86)	c_1, c_2
11	(14.10, 0.85)	c_1, c_2
12	(14.098, 0.849)	c_1, c_2

Table 2: Progress of Genocop II on test case #3; for iteration 0 the best point is the starting point

5 Conclusions

There are several interesting points connected with the above method. First of all, as any other GA-based method, it does not require any implicit (or explicit) calculations of any gradient or Hessian matrix of the objective function and constraints. Consequently, the method does not suffer from the ill-conditioned Hessian problem usually associated with some calculus based methods.

It should be noted that any genetic algorithm can be used in place of Genocop I for the inner loop for Genocop II. In such a case all constraints (linear and nonlinear) should be considered for placement in the set of active constraints A (the elements of L should be

Iteration number	The best point	Active constraints
0	(2,2)	c_1, c_2
1	(3.884181, 3.854748)	c_1
2	(15.805878, 1.581057)	c_1
3	(15.811537, 1.580996)	c_1

Table 3: Progress of Genocop II on test case #4; for iteration 0 the best point is the starting point

Iteration number	The best point	Active constraints
0	(0, 0)	c_1
1	(1.496072, 0.503928)	c_1
2	(1.020873, 0.979127)	c_1
3	(1.013524, 0.986476)	c_1
4	(1.002243, 0.997757)	c_1
5	(1.000217, 0.999442)	c_1
6	(1.000029, 0.999971)	c_1

Table 4: Progress of Genocop II on test case #5; for iteration 0 the best point is the starting point

distributed between N_e and N_i). However, such method is much slower and less effective: for efficiency reasons, it is much better to process linear constraints separately (as it is done in Genocop I).

Further research includes experimenting with (1) problems of higher dimensions, (2) different cooling schemes and (3) different values of parameter δ which decides whether a constraint is active or not.

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