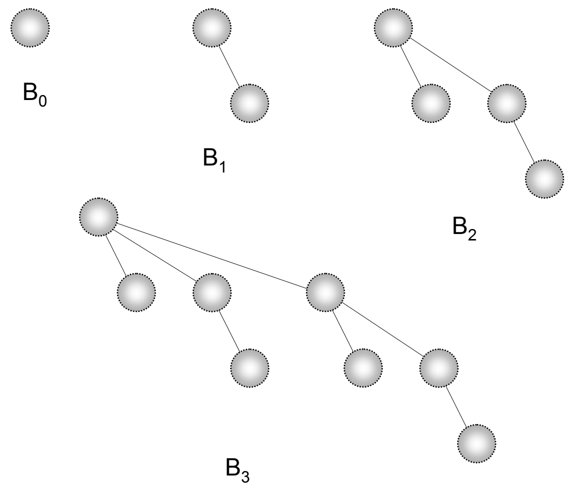
Priority Queues - 2

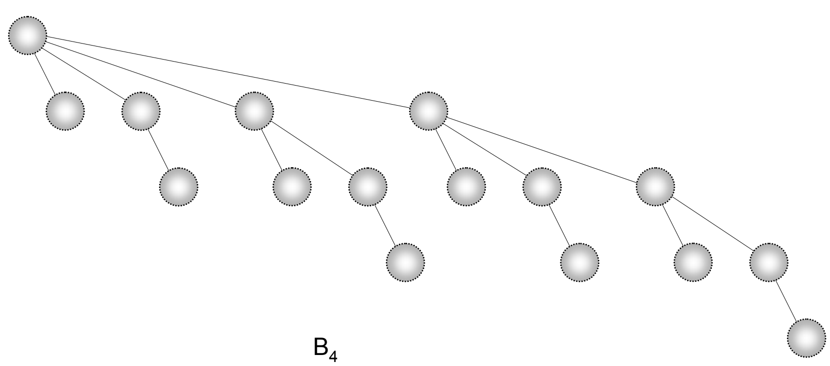
CS 202 – Fundamental Structures of Computer Science II
Bilkent University
Computer Engineering Department

Binomial Queues

- Binamial Queue
  - A collection of heap-ordered trees (a forest).
  - A tree in the forest is a *binomial tree*.
  - There is *at most one* binomial tree of *every height* in the forest.
- A binomial tree of height 0 has one node.
- A binomial tree of height k has 2^k nodes.
- A binomial tree of height k, B_k, is formed by attaching a binomial tree of height k-1, B_{k-1}, to another binomial tree of height k-1, B_{k-1}.

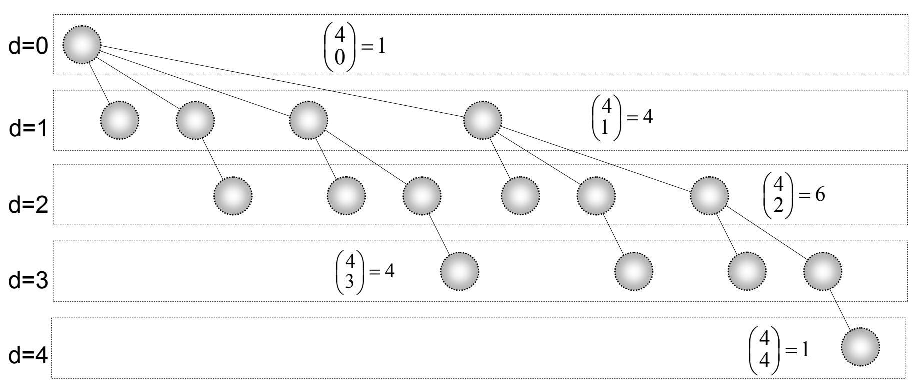
Binomial Trees





- A binomial tree B_k consists of root with children B_0, B_1, …, B_{k-1}
- Binomial trees of height **k** have exactly 2^k nodes.
- The number of nodes at depth *d* is binomial coefficient $\binom{k}{d}$

k = 4

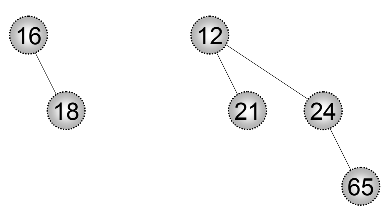


B_4

Heaps and binomial trees

- Assume all binary trees in a forest are in heap order.
- Assume we have at most one binomial tree of any height.
- Then we can represent a heap of any size uniquely by a binomial tree forest.
- Example: heap of size 13 could be represented by forest: B_3, B_2, B_0,
- We call this heap ordered binomial forest a binomial queue.

Example



H_1

A binomial queue H_1 of size 6 is shown above

Can be represented as: 110

Binomial Queue Operations

- FindMin
- Merge
- Insert
- DeleteMin

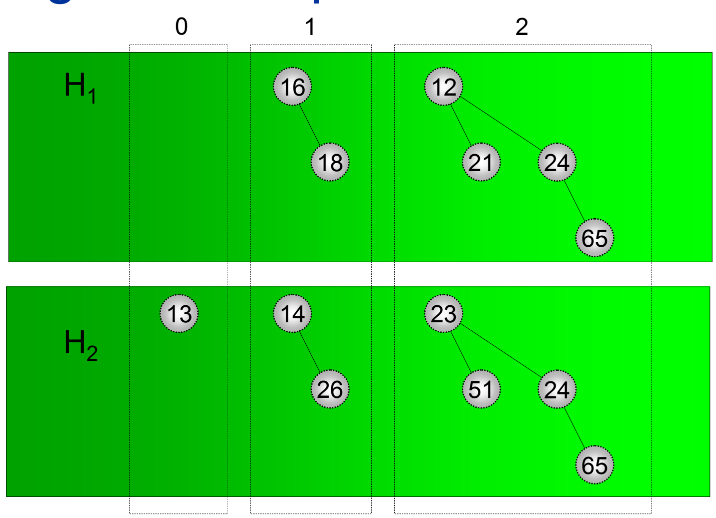
FindMin

- The minimum element in the binomial queue can be found by scanning the roots of all trees in the forest.
- There are at most logN different trees
- The cost of findMin is therefore O(logN) in the worst case.

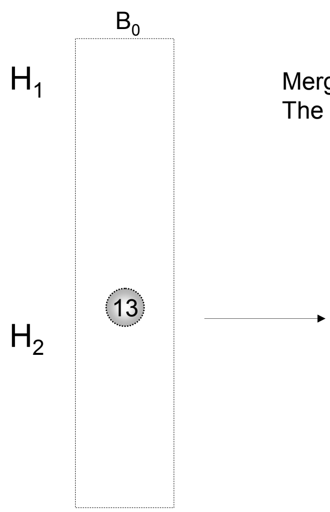
Merge

- Merging two binomial queues is conceptually simple.
- Merge H_1 and H_2
- Just add them in binary.
- Assume H_1 has 6 nodes. H_2 has 7 nodes.
  - H_3 = node (H_1) + nodes(H_3) = 13.
  - H_1: 0110
  - H_2: 0111
  - H_3: 1101 (this is the resulting bin. heap).

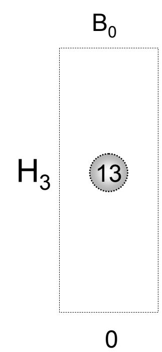
Merge - example



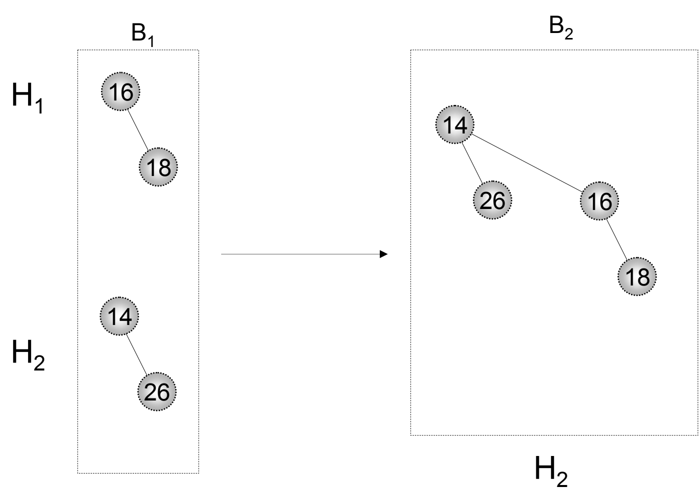
Merge - example - steps

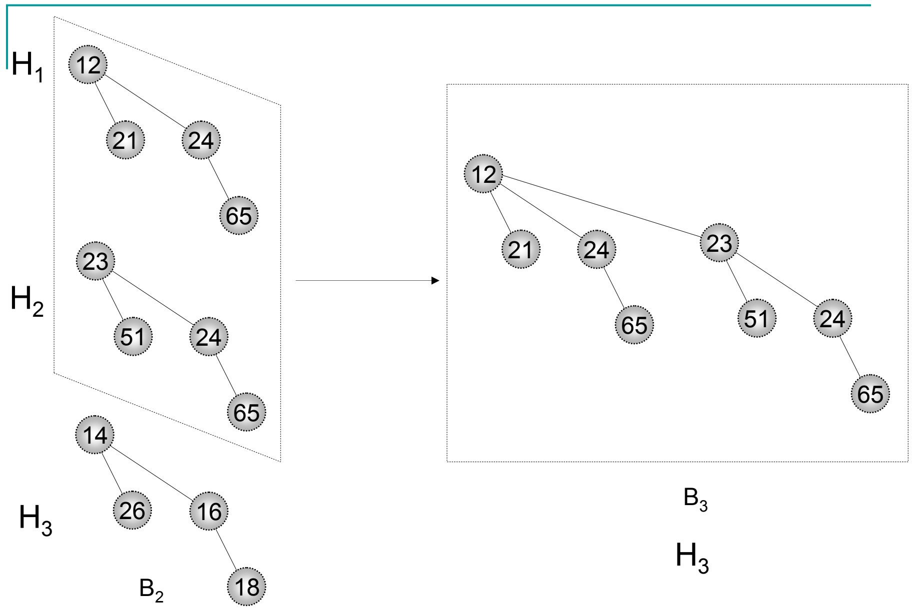


Merge 1 node tree with empty tree.
The result is the one node tree itself.

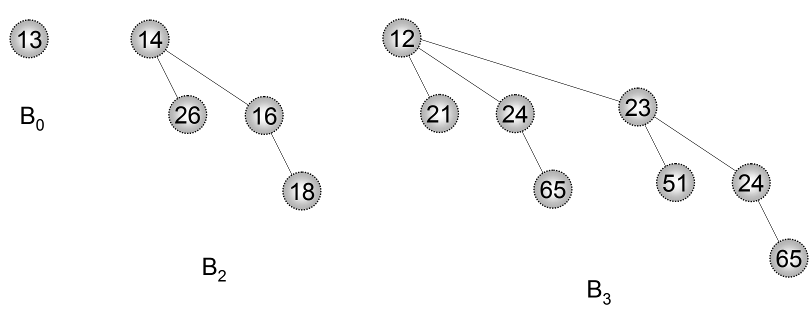


Merge - example - steps





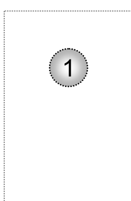
Final Binomial Queue H_3



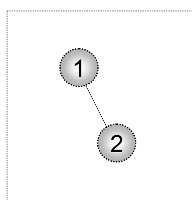
$H_3 = 1101$

Insert

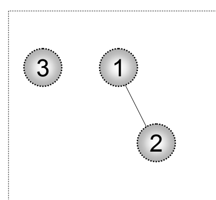
- Insertion is a special case of merge
- Insert an item to a binomial heap H_1
- Make a new heap, H_2, of one node (item to be inserted)
- Merge H_1 and H_2.
- Example
  - Insert items 1, 2, 3, 4, 5, 6, 7, in the given order into an empty binomial heap.



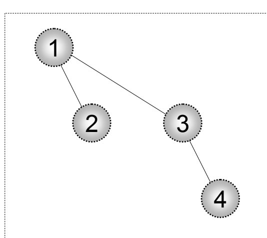
After 1 is inserted



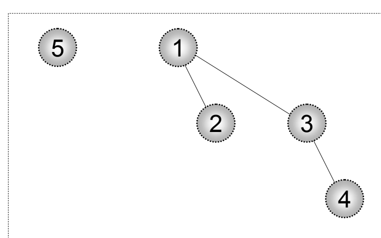
After 2 is inserted



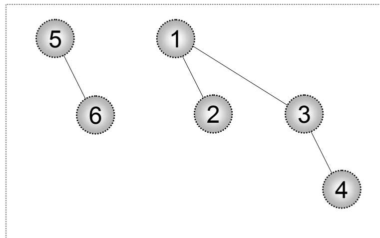
After 3 is inserted



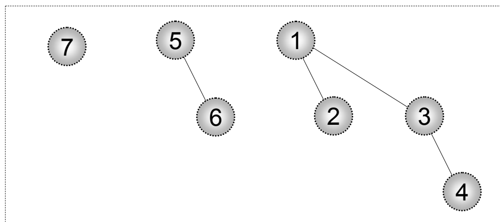
After 4 is inserted



After 5 is inserted



After 6 is inserted

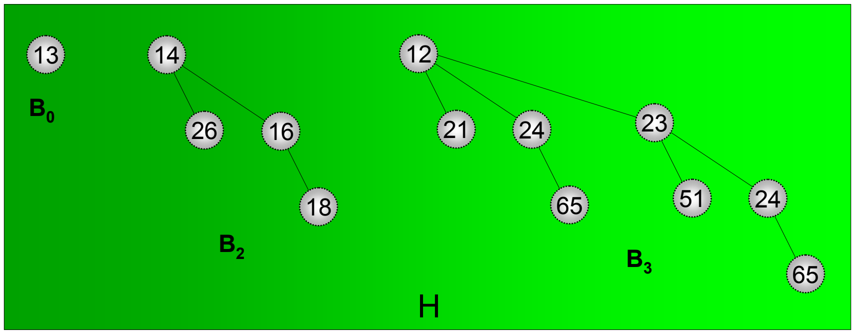


After 7 is inserted

DeleteMin

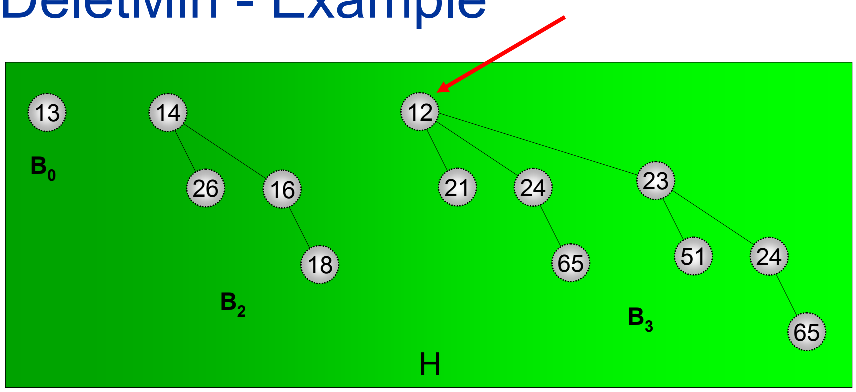
- Sketch of the Algorithm
- Assume we want to delete minimum item from binomial queue H.
  - Find the binomial tree in H that has the minimum root. Let say this is B_k in H.
  - Take tree B_k from H. Let the remaining trees in H make a new binomial queue H_1
  - Remove the root from B_k (root is the minimum that you should return as a result of algorithm).
    - The children of root of B_k make a new heap H_2 that can consists of tree B_0 through B_{k-1}
  - Merge H_1 and H_2.

DeletMin - Example

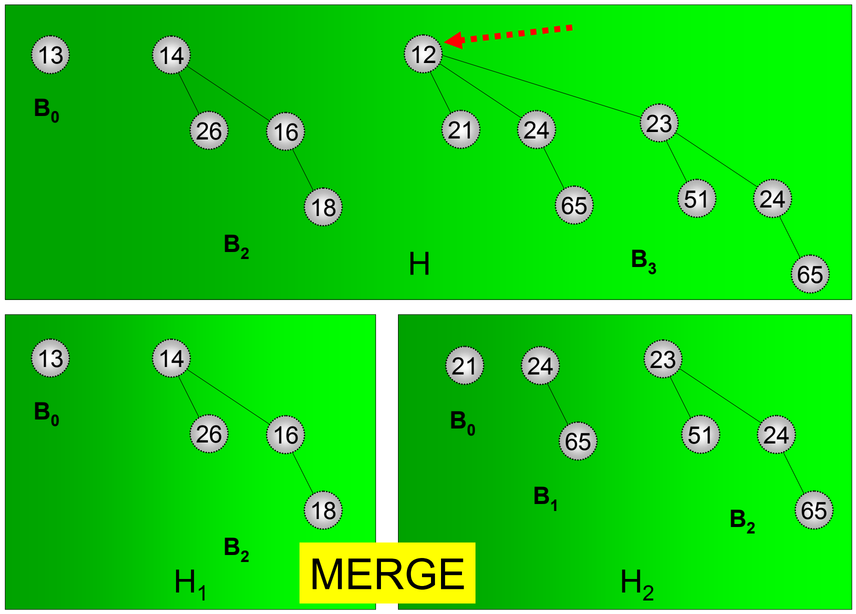


Delete the minimum item from the binomial queue above.

DeletMin - Example



The root that has the minimum item is 12 and belongs to tree B_3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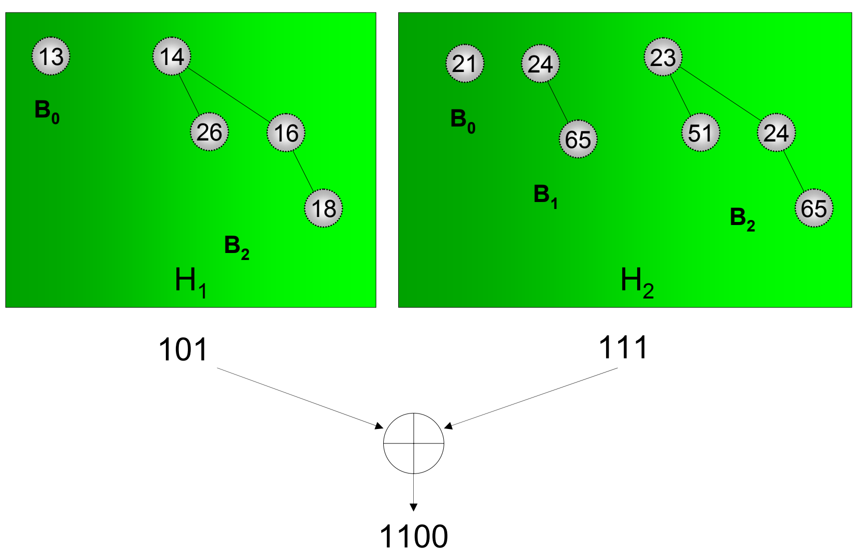
H2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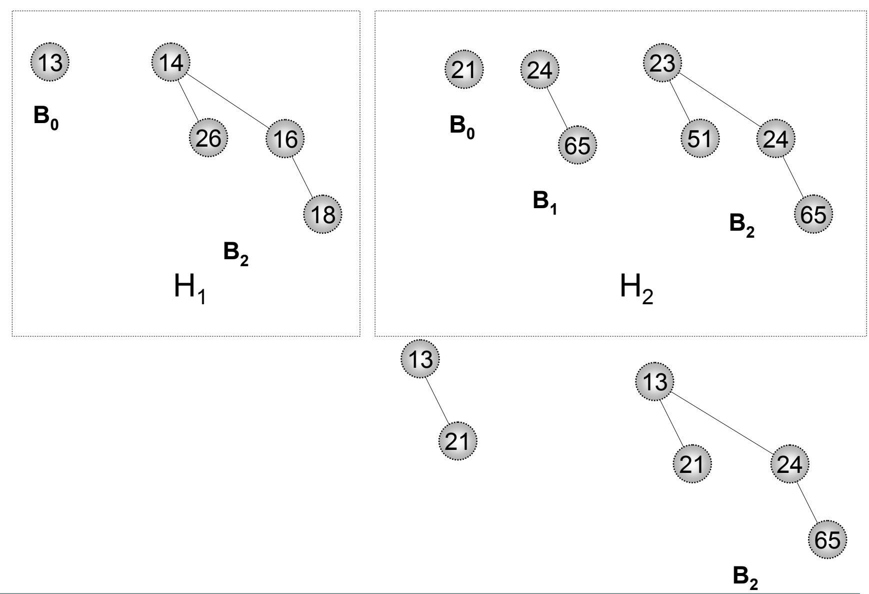
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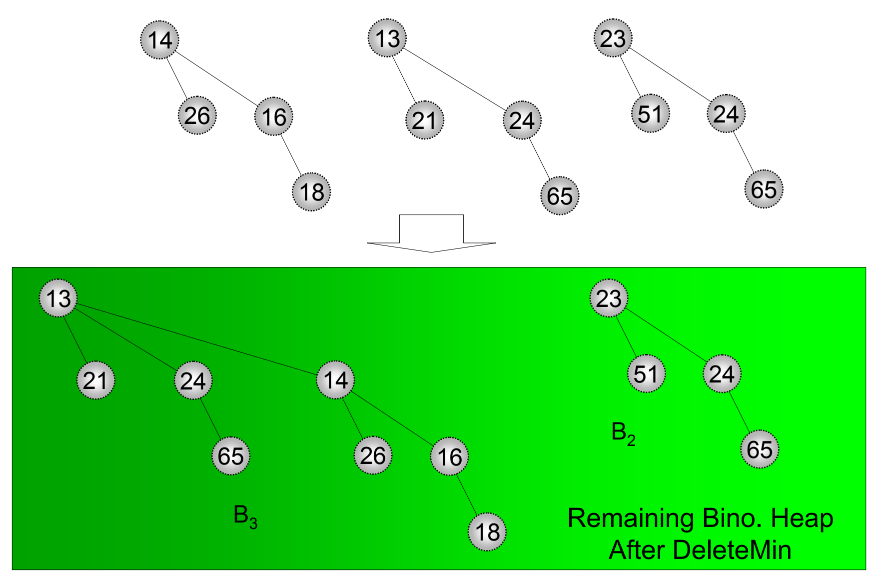
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Remaining Bino. Heap
After DeleteMin