Submission Instructions

Please return a hard copy of your answers to the following problem before the class meeting on Wednesday, October 11, 2017. You can submit your assignment as a team of two students, provided that you have not collaborated with your partner in two previous assignments. Submissions written in LaTeX will receive 5 bonus points.

Problem 1

The diameter of an undirected, unweighted graph is defined as the largest of all pairwise shortest path distances (measured as the number of edges on the path) in the graph. A tree $T = (V, E)$ is an undirected graph that is connected, but does not contain any cycles (so there is exactly one path between any pair of vertices and $|E| = |V| - 1$).

[20 pts.] (a) Devise an algorithm that uses Breadth-First-Search to compute the diameter of a tree in $O(V)$ time.

[15 pts.] (b) Prove that your algorithm is correct.

Problem 2

There are $n$ basketball teams in the world. The ranking of these teams from the previous year is available. This year, some of these $n$ teams played against each other and the winner of each game was determined. There were $m$ games in total. The International Basketball Association wants to introduce a new performance criterion, called “domination factor”, defined as follows: Team $i$ is said to “dominate” team $j$ if we can find a chain of games such that $j$ was beaten by a team that was beaten by a team ... that was beaten by $i$ (observe that, according to this definition, domination can be bi-directional, i.e., $i$ and $j$ can dominate each other). Then, for each team $i$, the domination factor $z_i$ is defined as the rank of the best team (that is, the highest ranked team according to last year’s rankings) that is dominated by team $i$.

[20 pts] (a) Use DFS to devise an $O(m + n)$ time algorithm to compute the domination factor for all the $n$ teams.

[15 pts.] (b) Prove that your algorithm is correct.

Problem 3

Prove or disprove:
(a) Let $G = (V, E)$ be a directed graph. For any $uv \in E$, if some run of DFS on $G$ results in $v.f > u.f$, then $uv$ must be on a cycle.

(b) Prove or disprove: Consider any run of DFS on a directed graph $G = (V, E)$. For any edge $uv \in E$, if there is a path from $v$ to $u$ in $G$, then $uv$ cannot be a cross edge.