

Homework #2

Due 17.00, April 2, 2013

1. (16pts) Give a PDA for each of the following languages. Explain how each PDA works and why it is correct.
 - (a) $L = \{w \in \{a, b\}^* \mid n_a(w) \leq n_b(w)\}$.
 - (b) $L = \{w \in \{0, 1\}^* \mid \text{the length of } w \text{ is odd and it contains the same symbol in the beginning and middle positions}\}$ (e.g. $110 \in L, 010 \notin L$).
2. (24pts) Give a CFG for each of the languages in Question 1. Prove that each grammar generates the desired language correctly.
3. (20pts) Which of the following languages are context free and which are not? Justify your answer by either giving CFG or PDA, or using pumping lemma.
 - (a) $L = \{a^i b^j c^k \mid k = ij\}$.
 - (b) $L = \{a^n b^n a^m b^m \mid n, m \geq 0\}$.
 - (c) $L = \{0^{i_1} 1^{i_2} 0^{i_3} 1^{i_4} \dots 0^{i_{2n-1}} 1^{i_{2n}} 0^{i_{2n+1}} \mid n \geq 1; i_{2m} \geq i_{2m-1} + i_{2m+1}, 1 \leq m \leq n\}$. i.e. The language in which any number of consecutive 1s is at least as much as the total number of consecutive 0s around it.
4. (20pts) Consider the grammar below and apply the following procedures using the algorithms discussed in class.

$$S \rightarrow BAb \mid aAB \mid BAB \quad A \rightarrow Aaa \mid AB \mid a \quad B \rightarrow Bb \mid BA \mid \epsilon \quad C \rightarrow CB \mid CA \mid cC$$

- (a) Eliminate any ϵ productions.
 - (b) Eliminate any unit productions in the resulting grammar.
 - (c) Eliminate any useless symbols in the resulting grammar.
 - (d) Put the resulting grammar into Chomsky Normal Form.
5. (20pts) Let $b(n)$ denote the binary representation of $n \geq 1$, leading zeros omitted. For example, $b(7) = 111$ and $b(10) = 1010$. Let the alphabet $\Sigma = \{0, 1, \#\}$. Consider the following languages:
 - (i) $L_1 = \{b(n) \# b(n+1) \mid n \geq 1\}$.
 - (ii) $L_2 = \{b(n)^R \# b(n+1) \mid n \geq 1\}$, where x^R is the reverse of binary string x .

One of these languages is context free and the other one is not.

- (a) Determine which one is context free and give a CFG for that language. Prove that the grammar you gave generates the language correctly.
- (b) Prove that the other language is not context free.

Bonus Question

Consider the language $L = (a + b)^* - \{(a^i b^i)^i \mid i \geq 1\}$. Show that this language is context-free by building a PDA that accepts this language. Make sure that you justify your answer by carefully explaining why your PDA is correct (that it accepts *exactly* L).