CS473-Algorithms I

Lecture 15

Graph Searching:

Depth-First Search and Topological Sort

Depth-First Search

- Graph G=(V,E) directed or undirected
- Adjacency list representation
- Goal: Systematically explore every vertex and every edge
- Idea: search deeper whenever possible
  - Using a LIFO queue (Stack; FIFO queue used in BFS)

Depth-First Search

- Maintains several fields for each $v \in V$
- Like BFS, colors the vertices to indicate their states. Each vertex is
  - Initially white,
  - grayed when discovered,
  - blackened when finished
- Like BFS, records discovery of a white v during scanning Adj[u] by $\pi[v] \leftarrow u$

Depth-First Search

- Unlike BFS, predecessor graph G_π produced by DFS forms spanning forest
- $G_\pi = (V, E_\pi)$ where

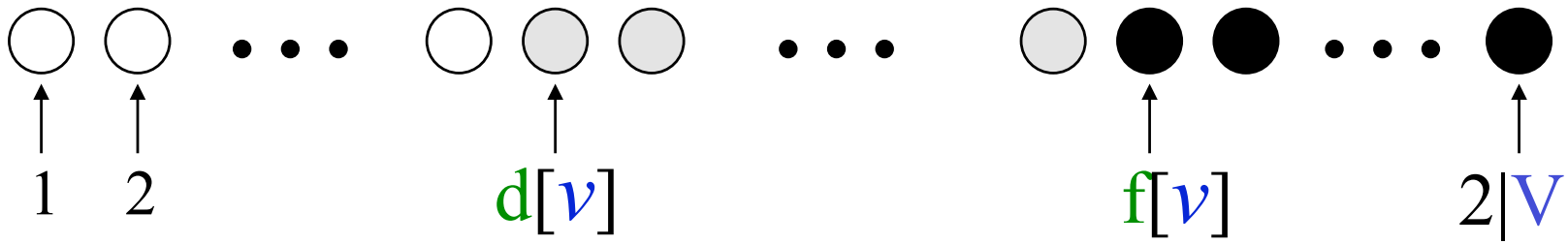
$$E_\pi = \{(\pi[v], v) : v \in V \text{ and } \pi[v] \neq \text{NIL}\}$$

- G_π = depth-first forest (DFF) is composed of disjoint depth-first trees (DFTs)

Depth-First Search

- DFS also timestamps each vertex with two timestamps
- d[v]: records when v is first discovered and grayed
- f[v]: records when v is finished and blackened
- Since there is only one discovery event and finishing event for each vertex we have $1 \leq d[v] < f[v] \leq 2|V|$

Time axis for the color of a vertex



Depth-First Search

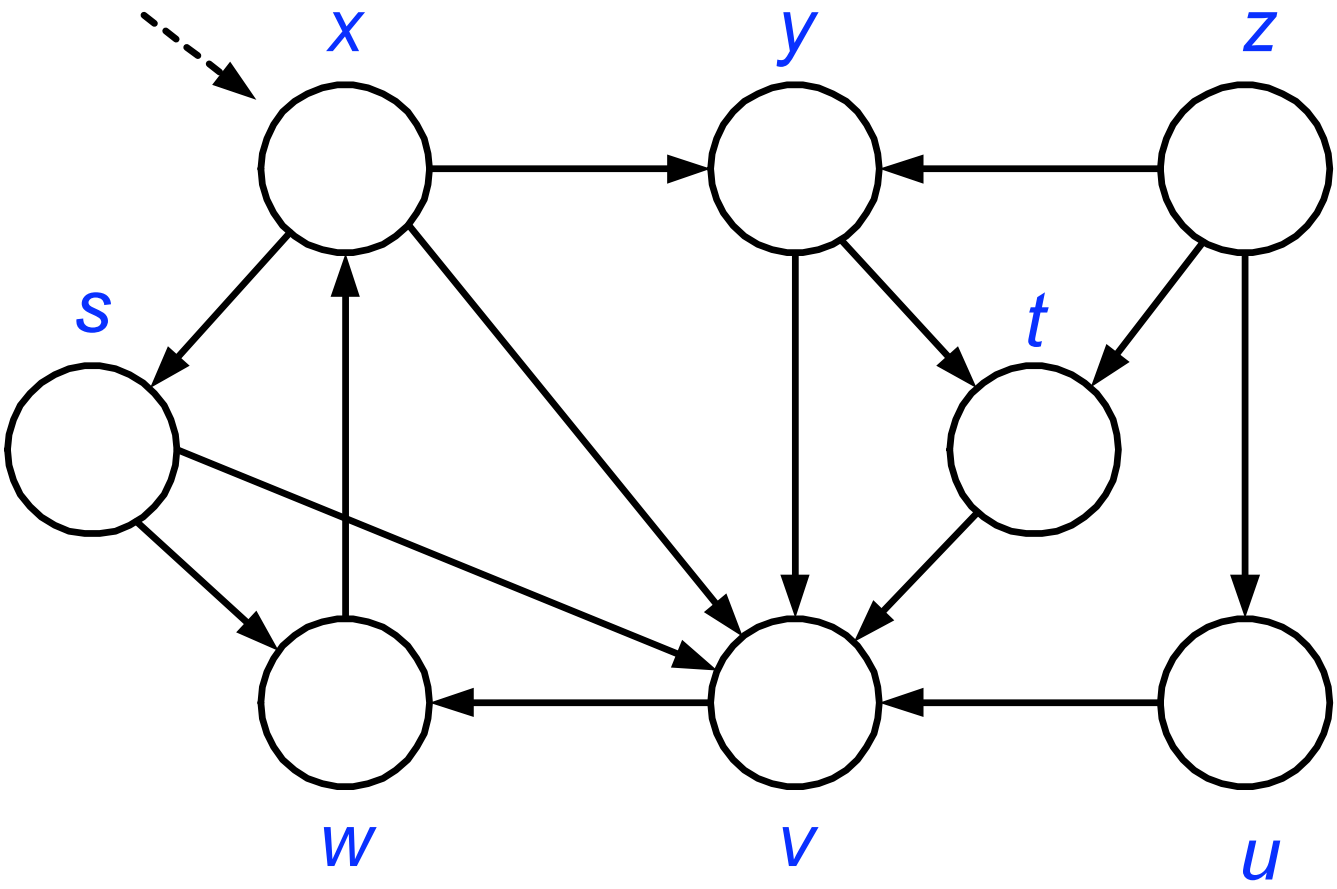
```
DFS(G)
    for each u∈V do
        color[u]← white
        π[u] ← NIL
    time ← 0
    for each u∈V do
        if color[u] = white then
            DFS-VISIT(G, u)
```

```
DFS-VISIT(G, u)
    color[u]← gray
    d[u]← time ← time +1
    for each v∈ Adj[u] do
        if color[v] = white then
            π[v] ← u
            DFS-VISIT(G, v)
    color[u]← black
    f[u]← time ← time +1
```

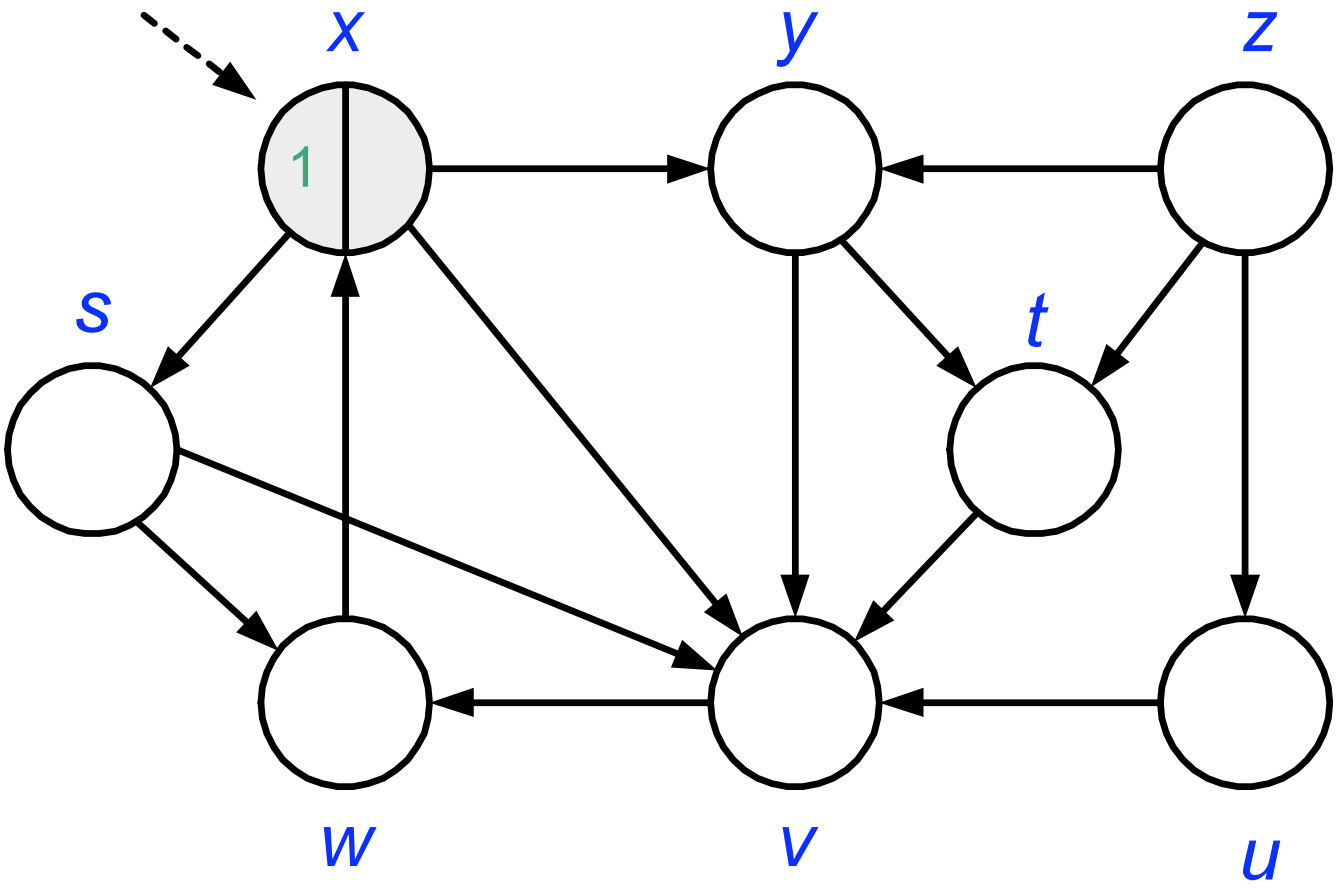
Depth-First Search

- Running time: $\Theta(V+E)$
- Initialization loop in **DFS** : $\Theta(V)$
- Main loop in **DFS**: $\Theta(V)$ exclusive of time to execute calls to **DFS-VISIT**
- **DFS-VISIT** is called exactly once for each $v \in V$ since
  - **DFS-VISIT** is invoked only on white vertices and
  - **DFS-VISIT(G, *u*)** immediately colors u as gray
- For loop of **DFS-VISIT(G, *u*)** is executed $|Adj[u]|$ time
- Since $\Sigma\ |Adj[u]| = E$, total cost of executing loop of **DFS-VISIT** is $\Theta(E)$

Depth-First Search: Example



Depth-First Search: Example



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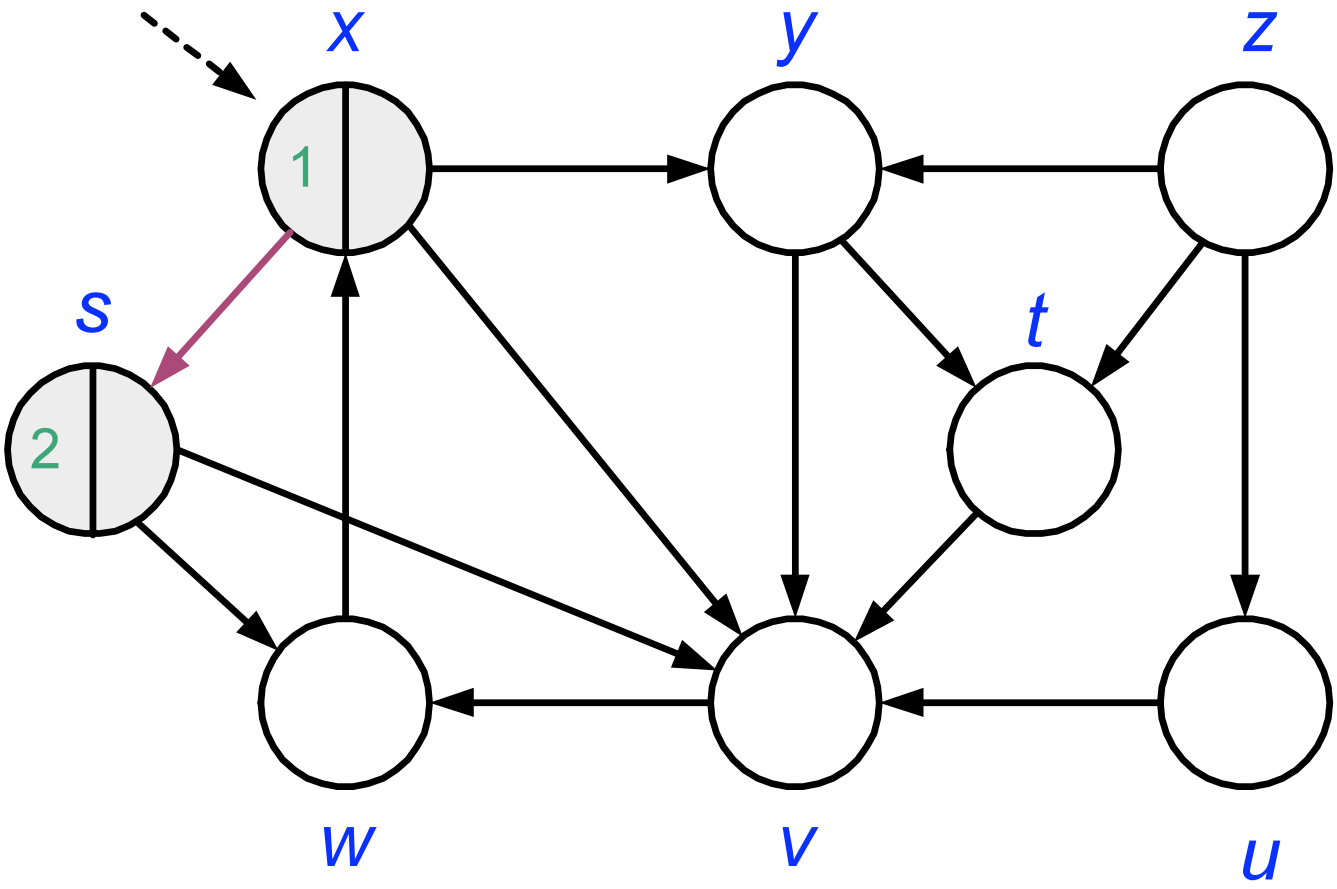

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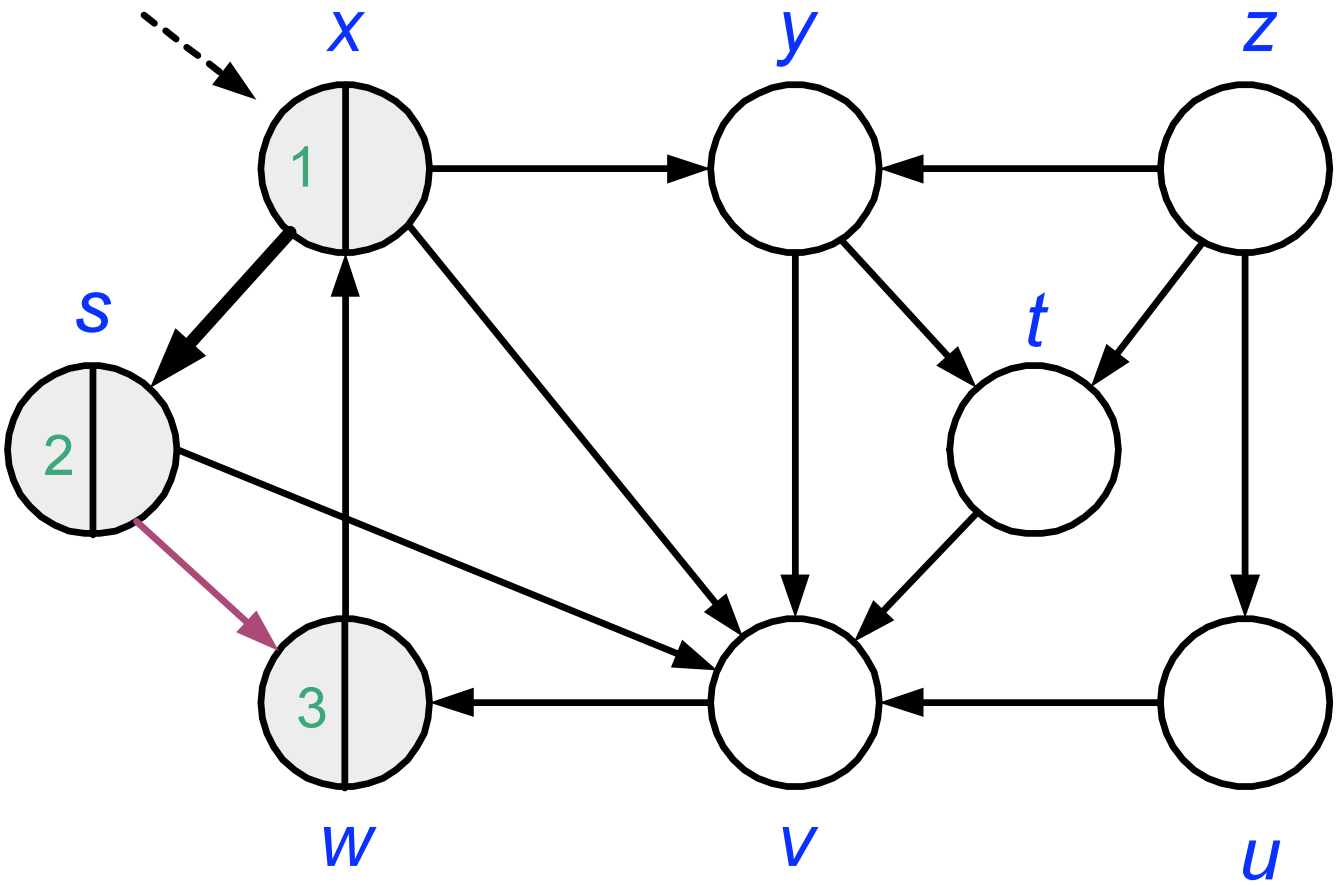

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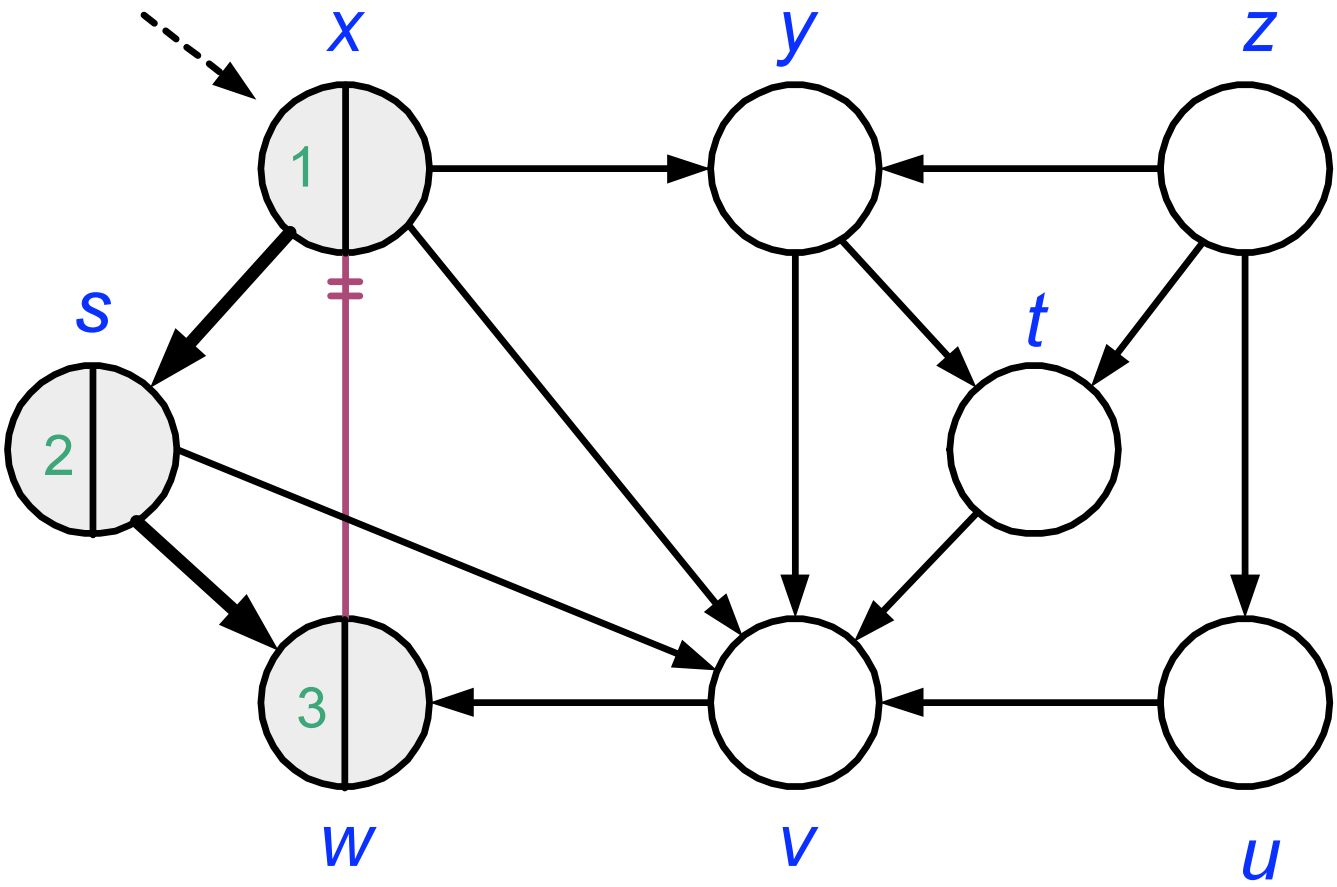

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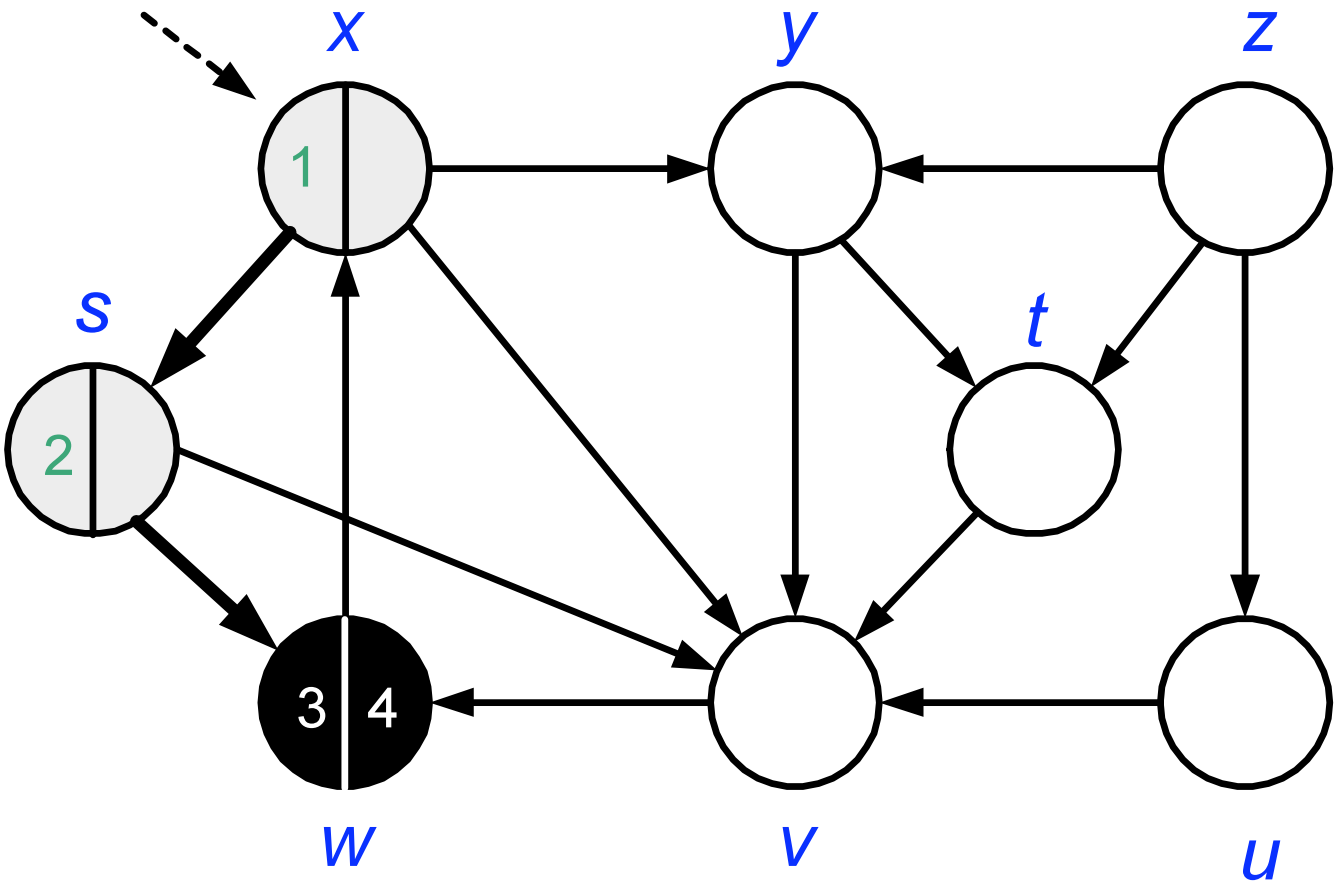

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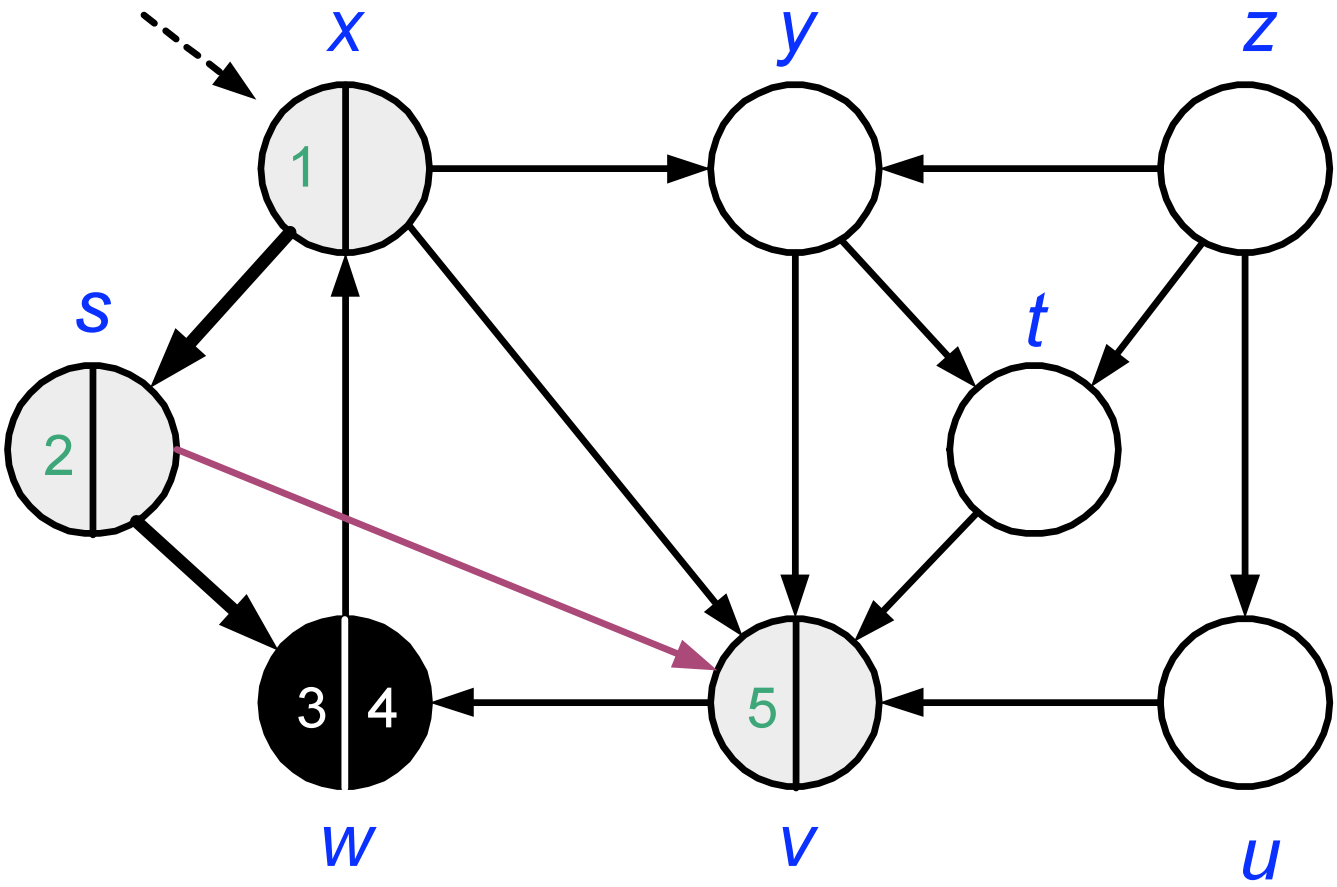

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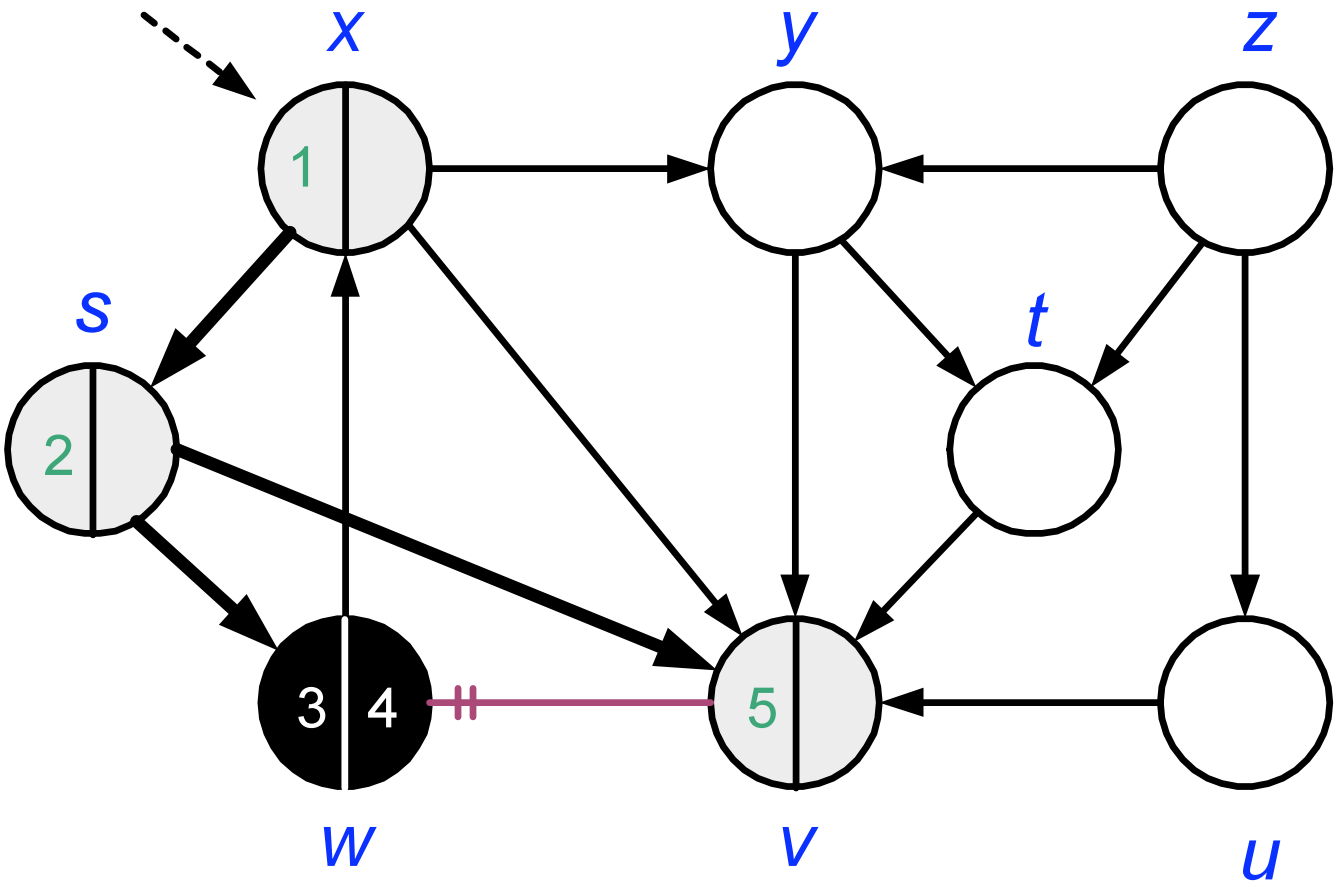

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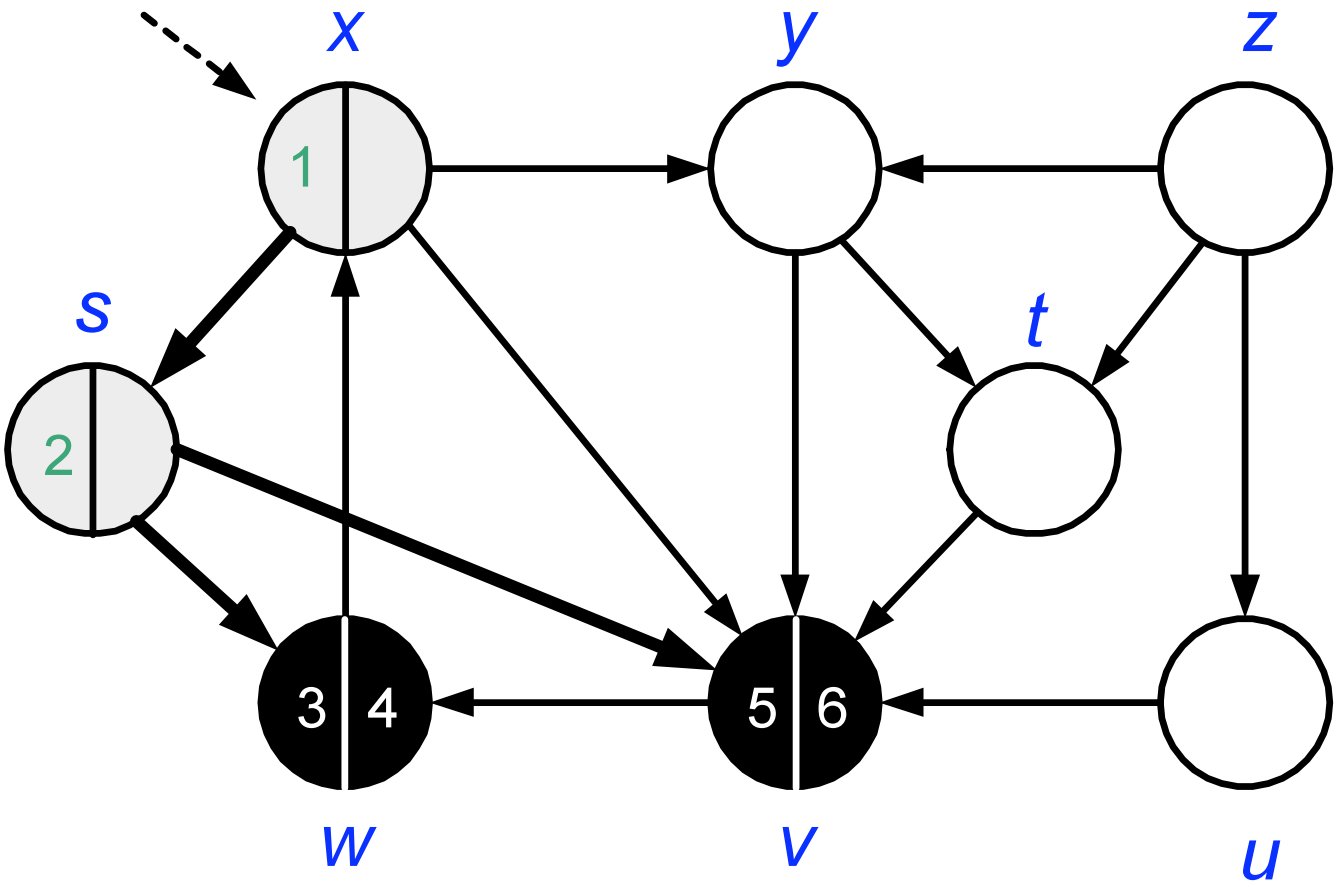

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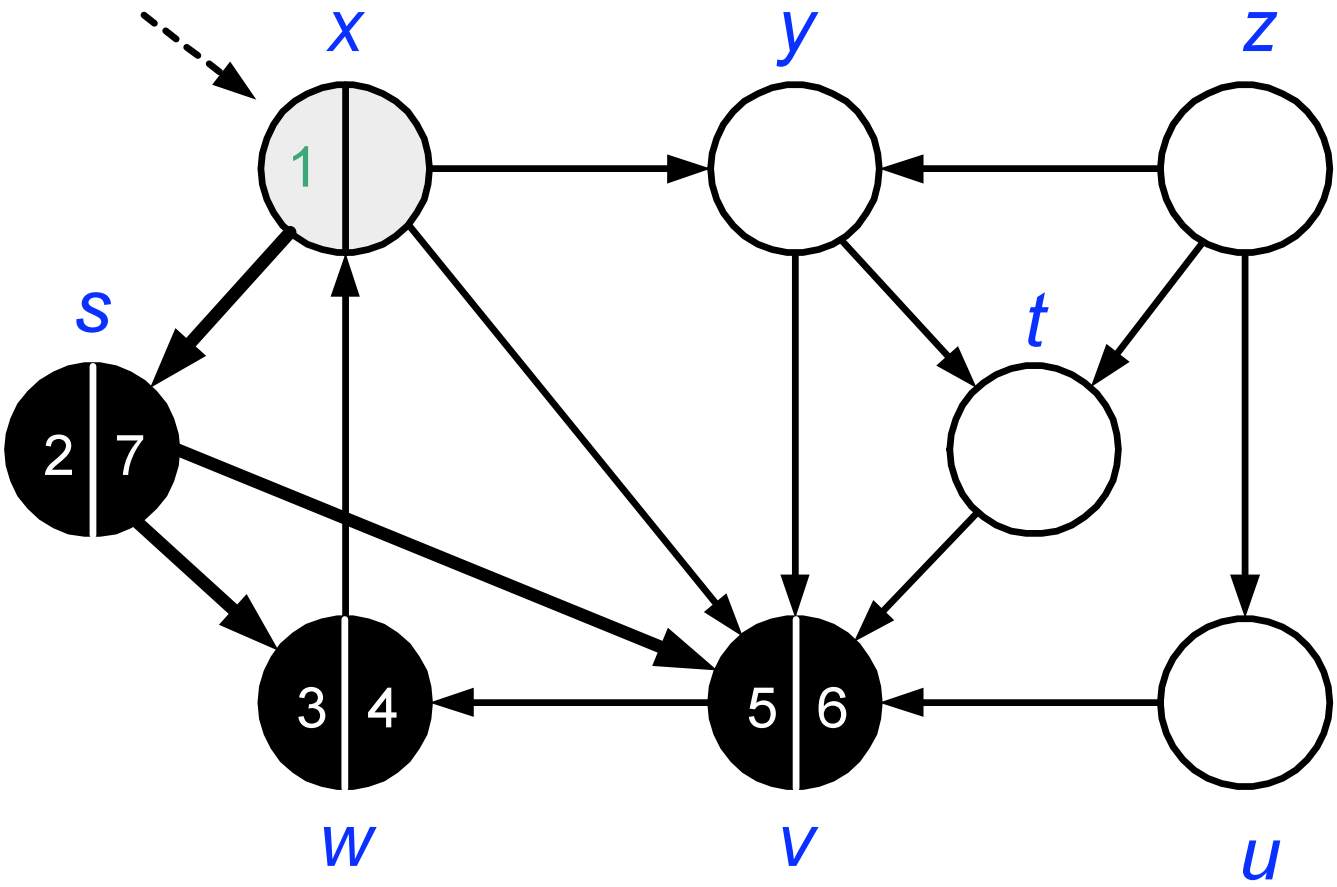

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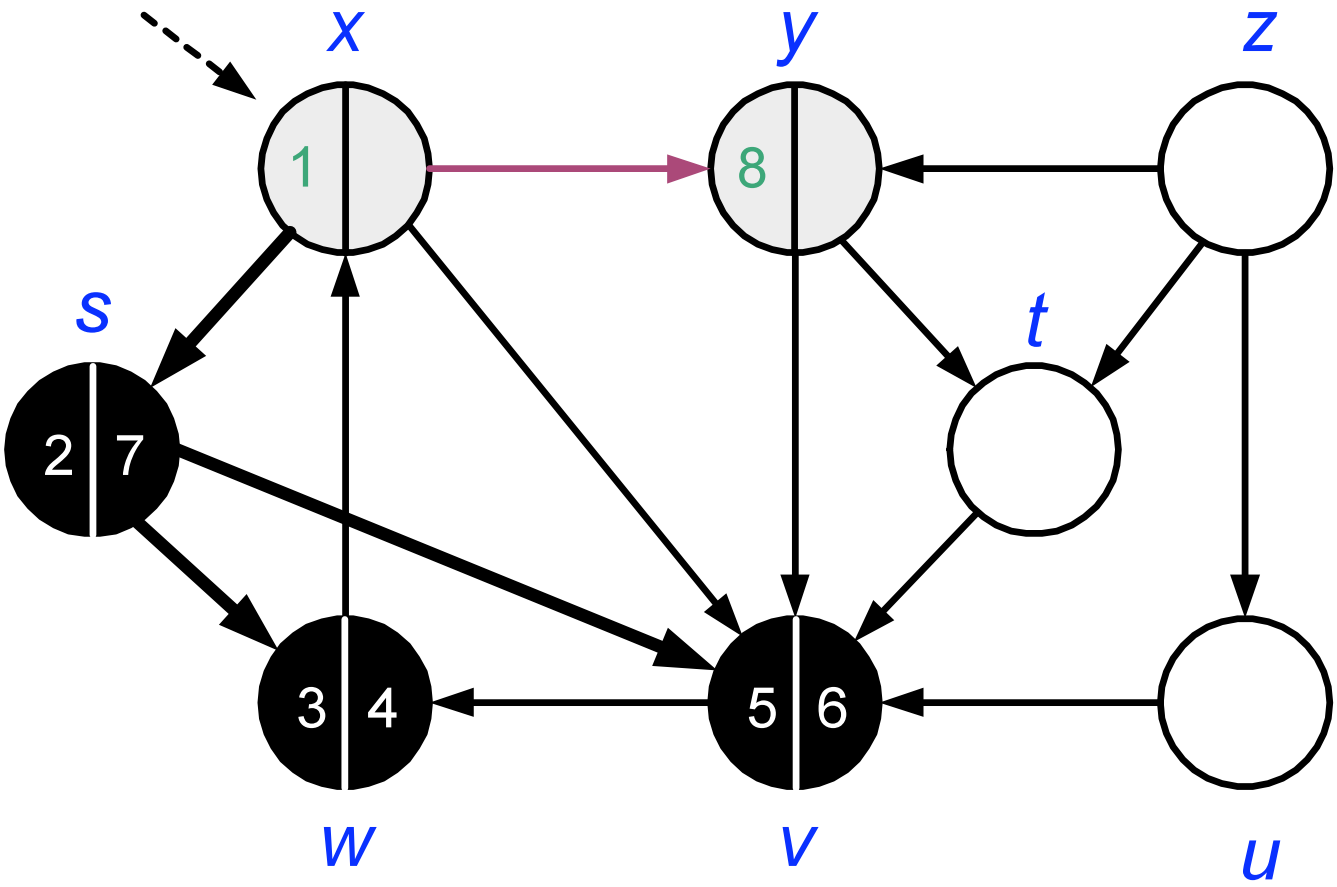

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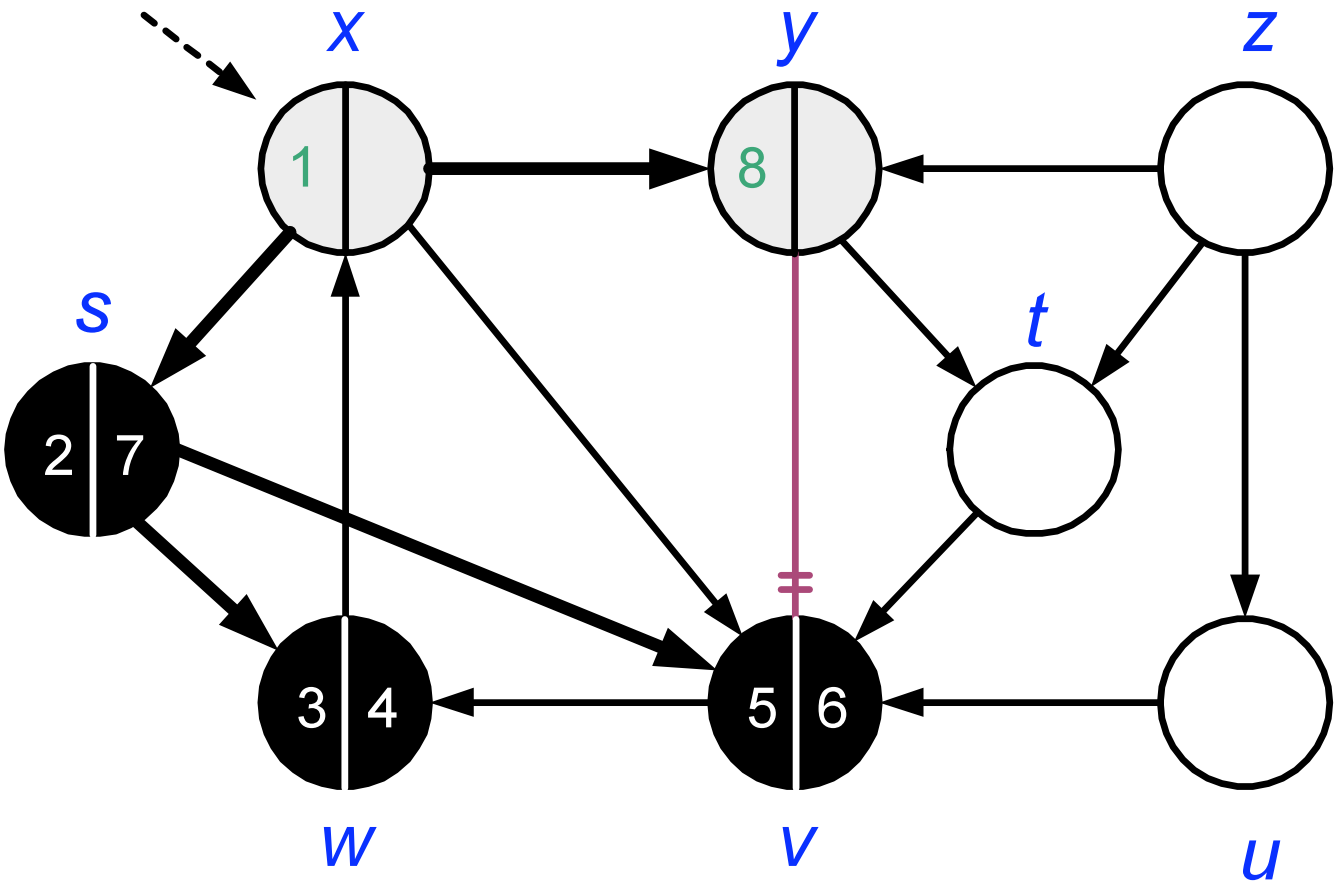

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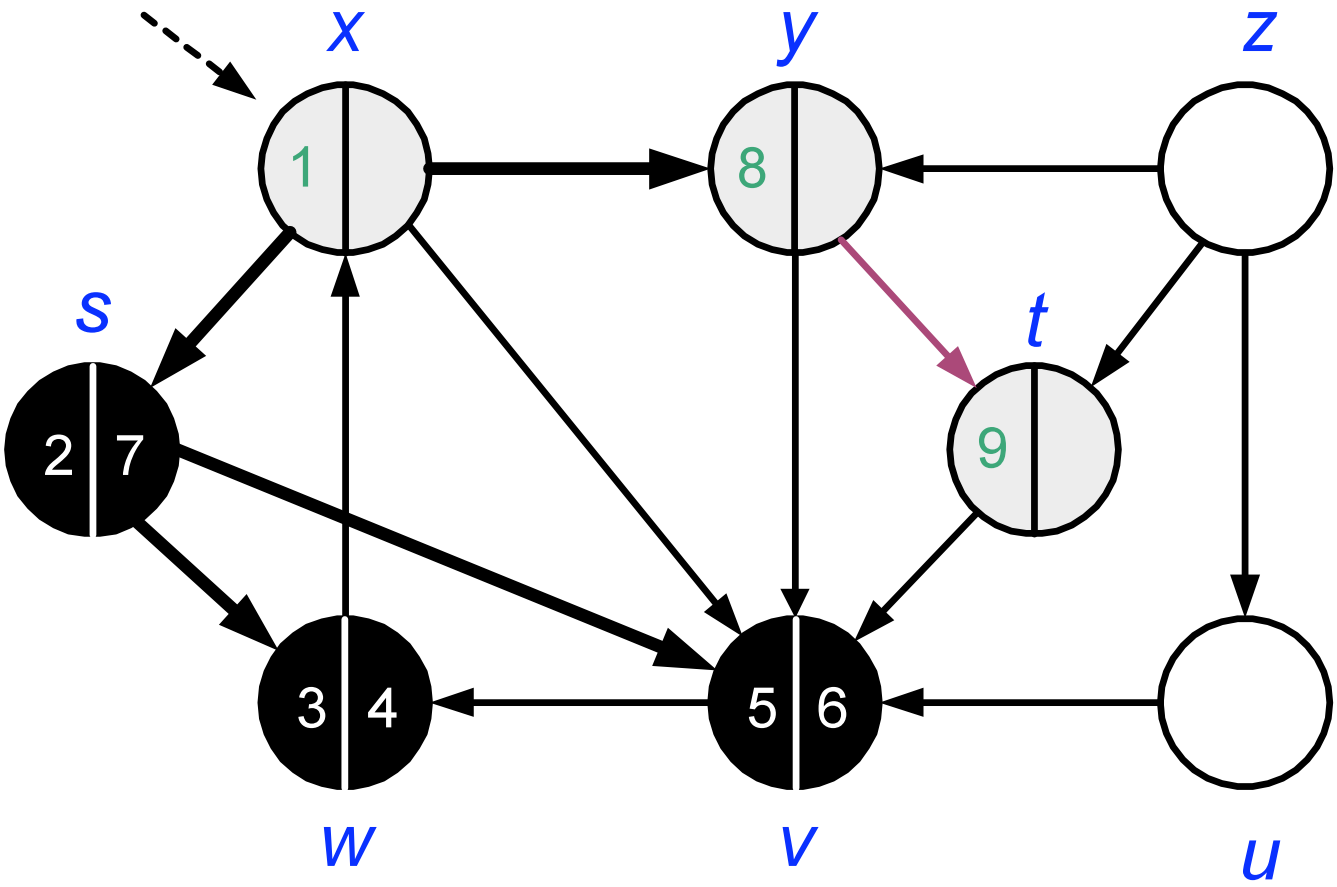

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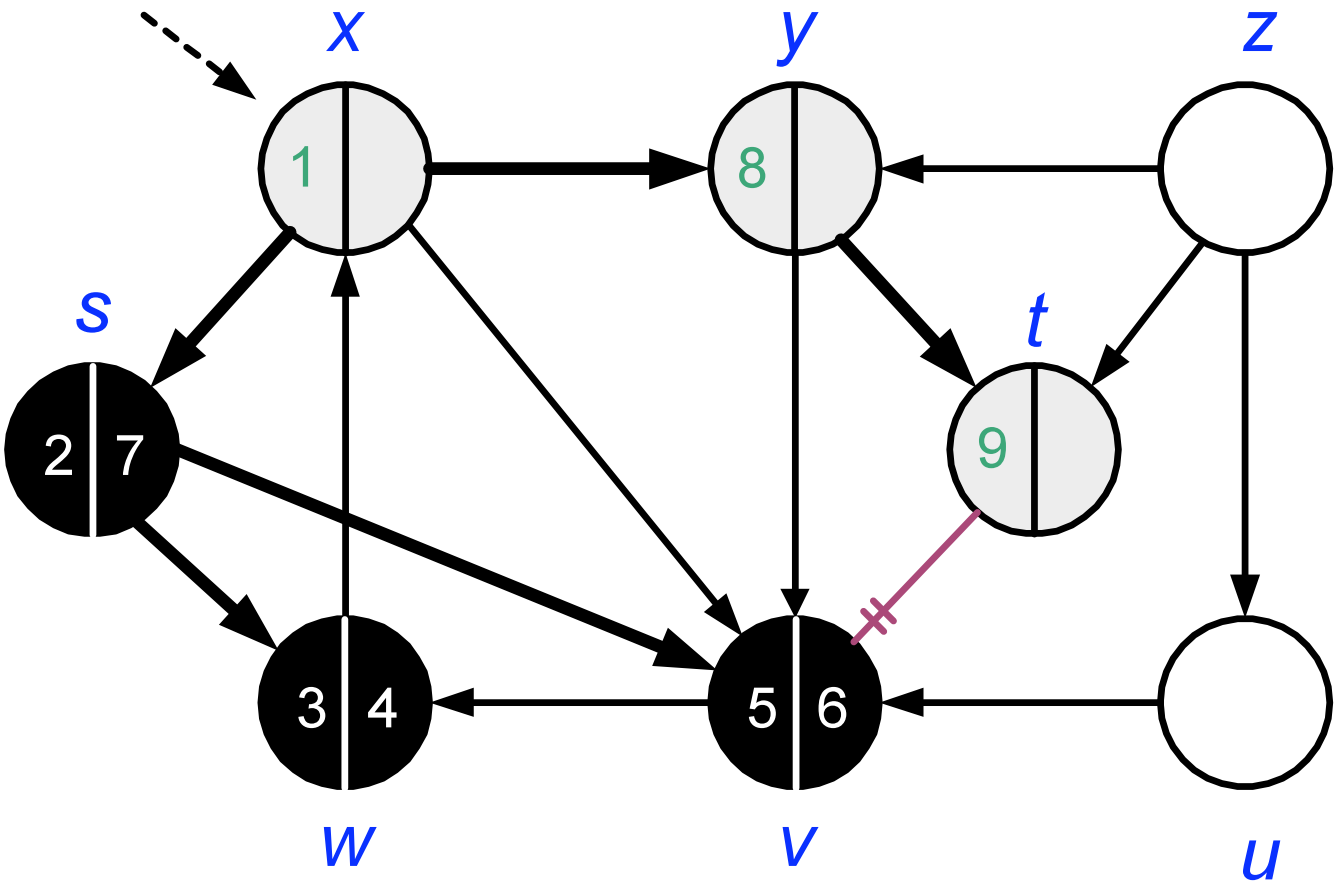

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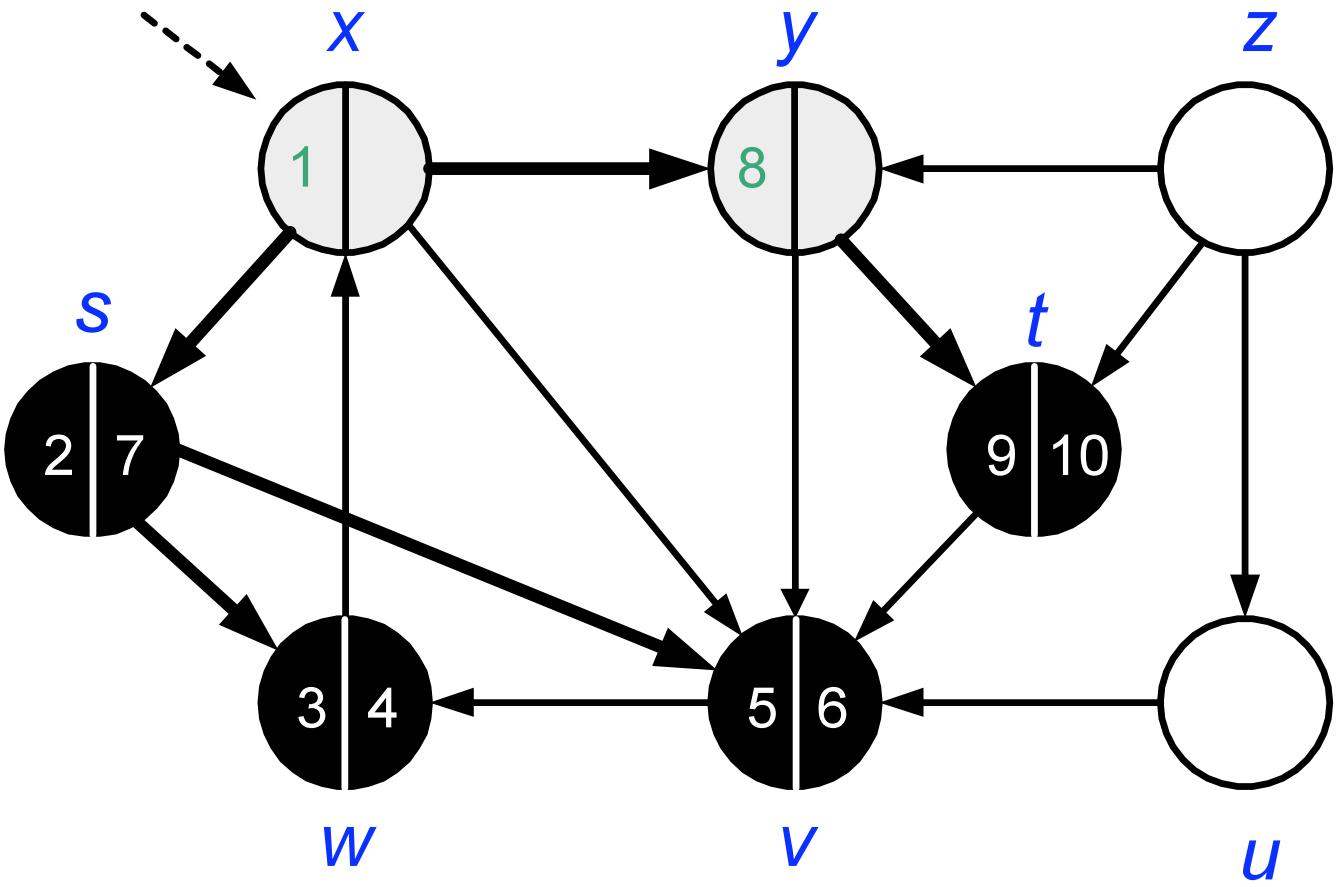

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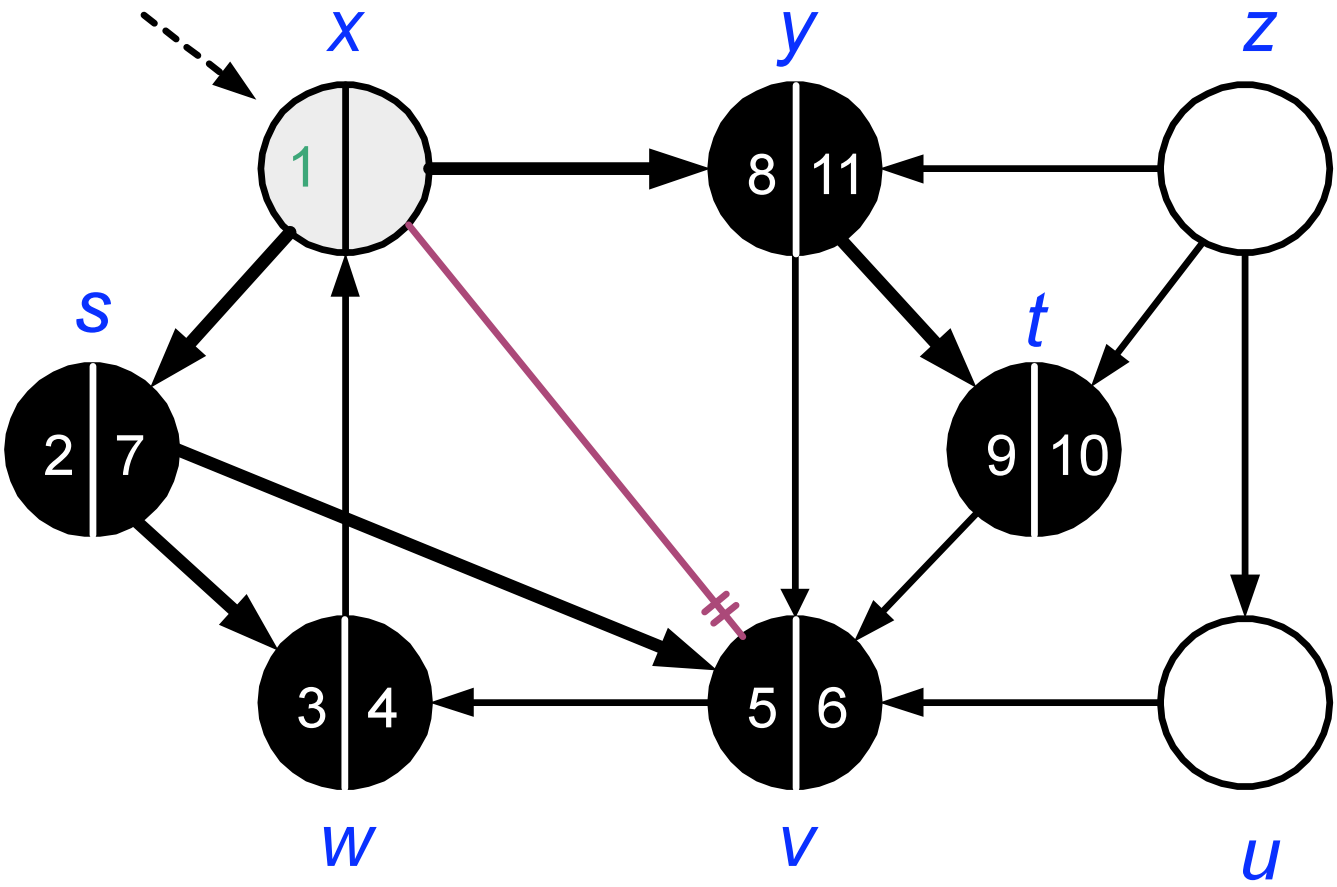

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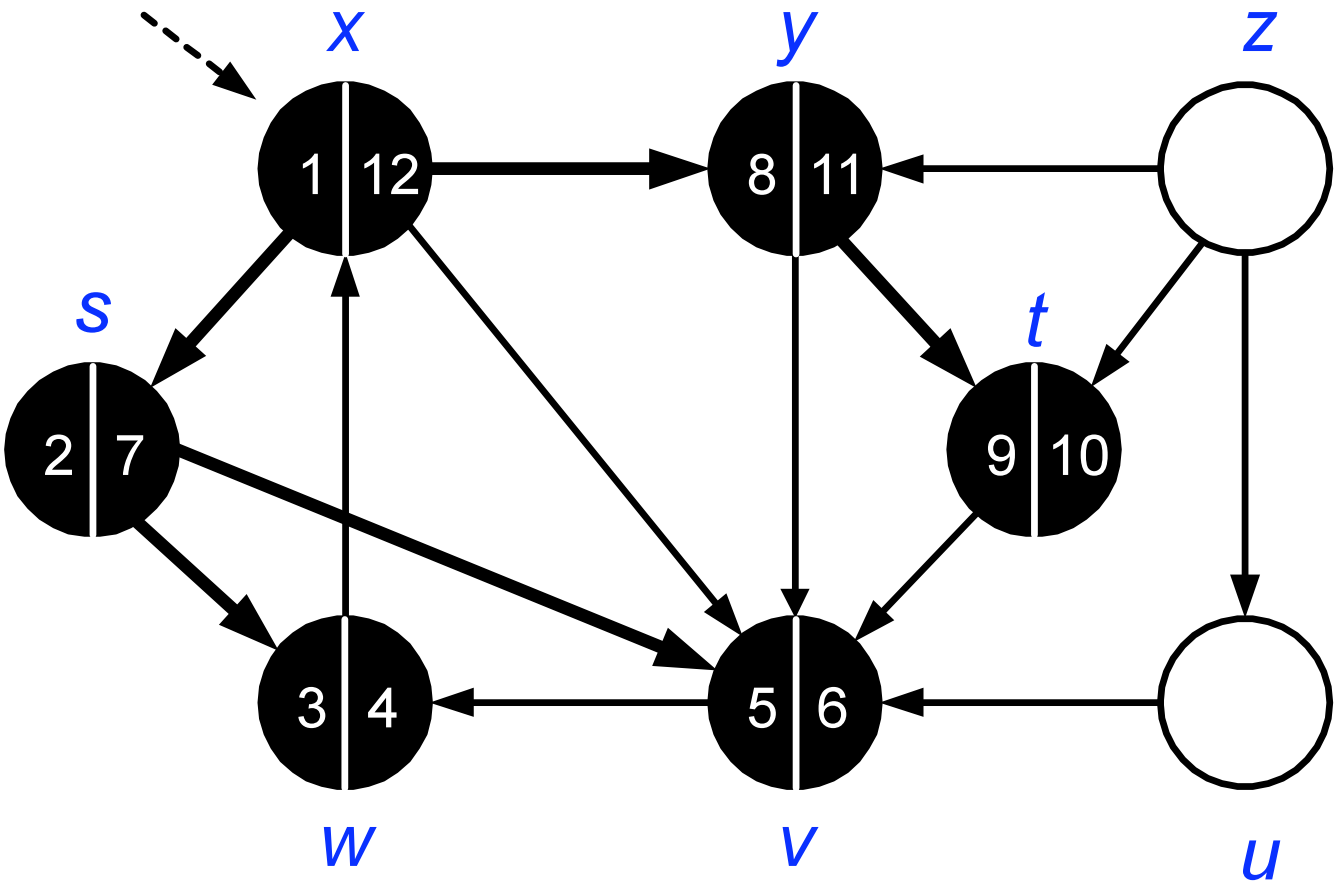

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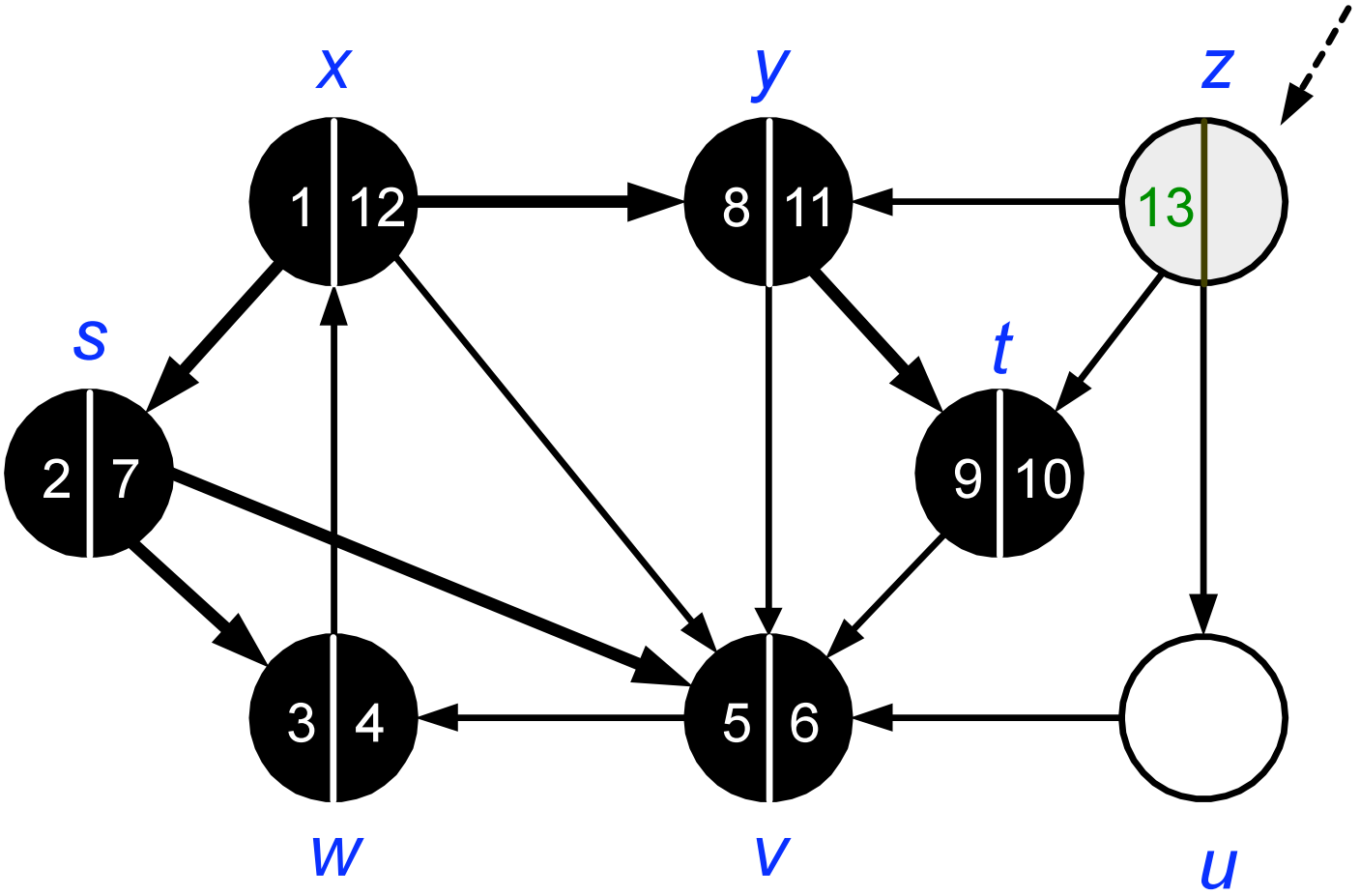

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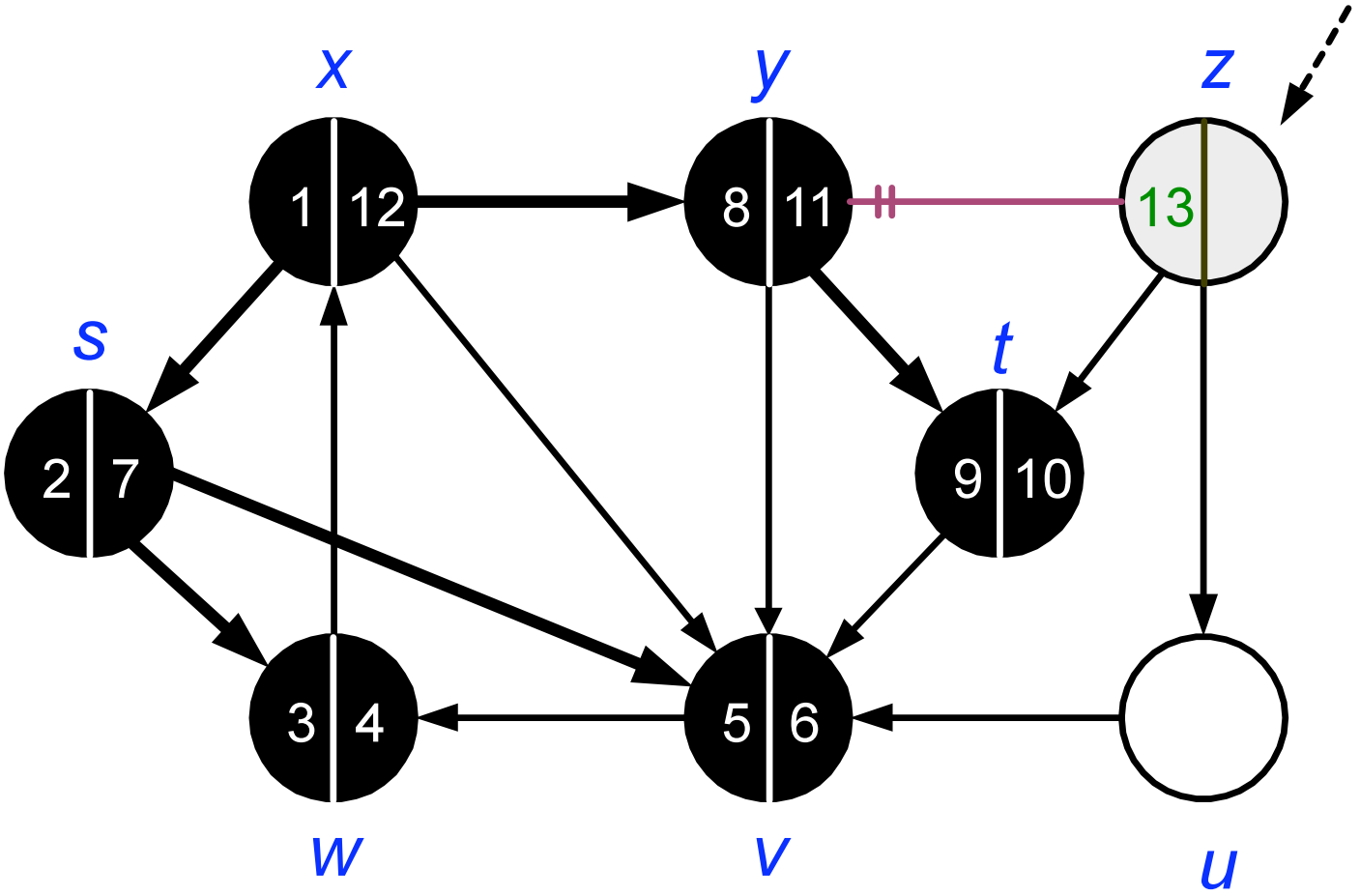

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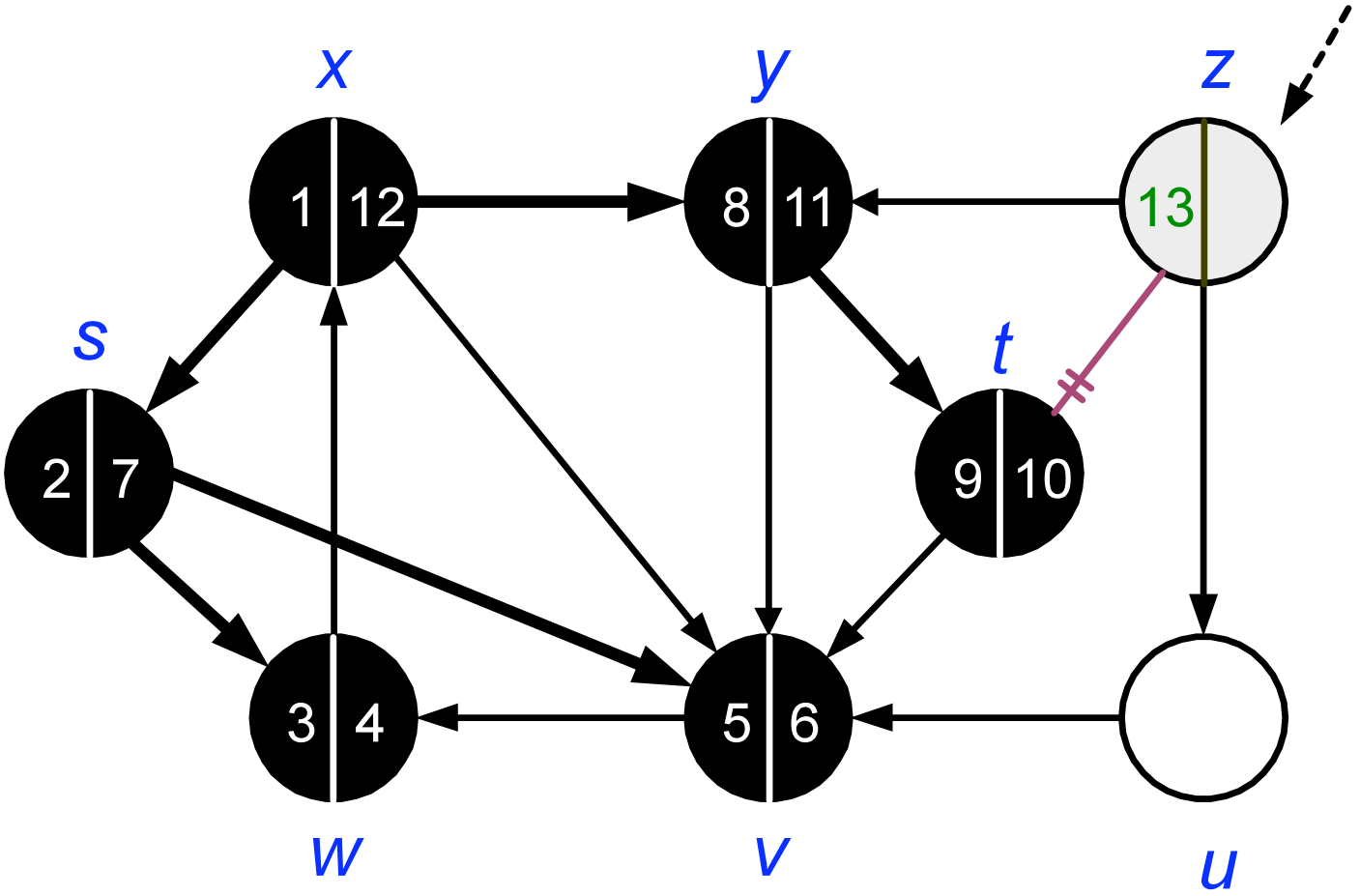

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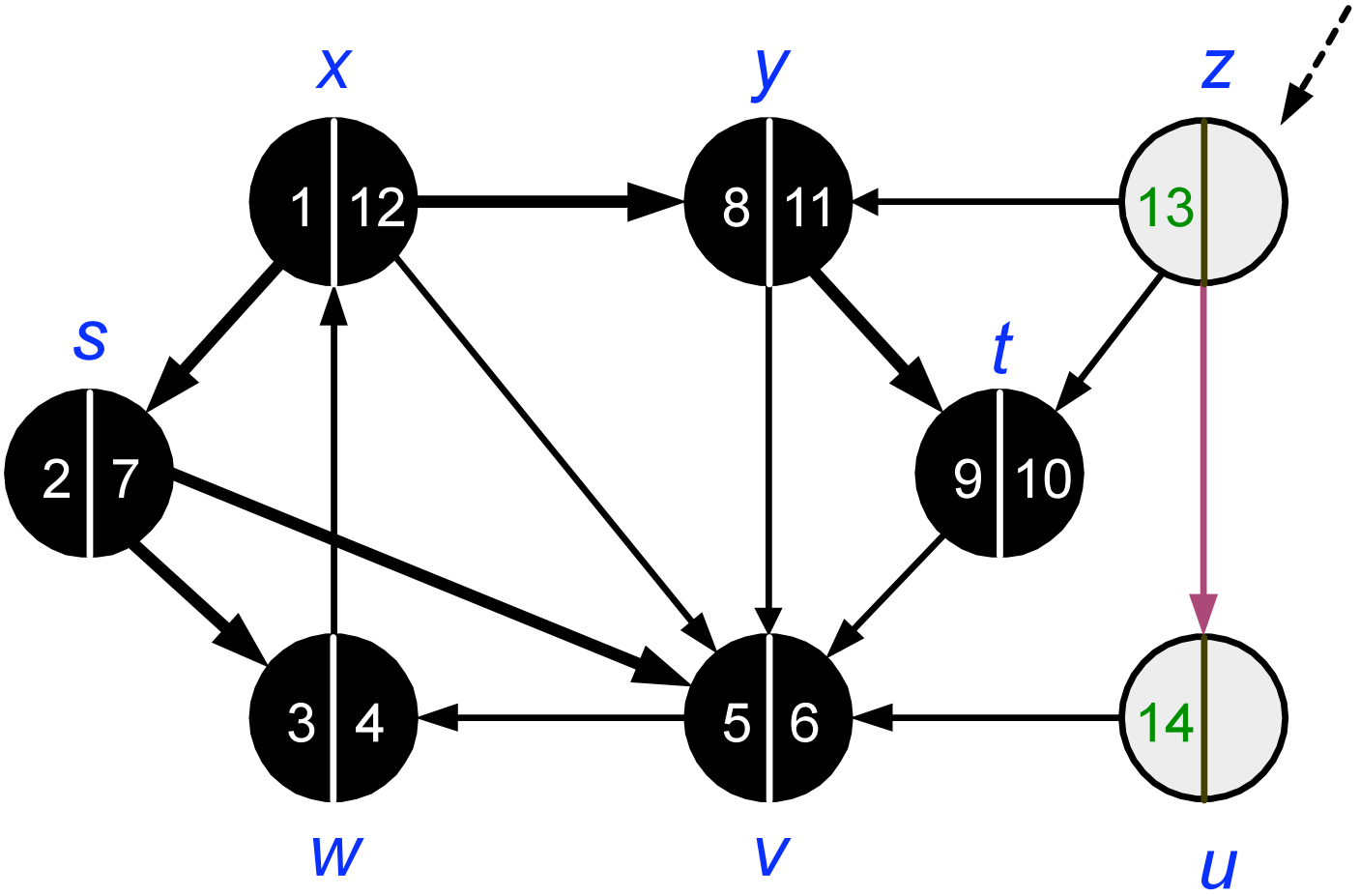

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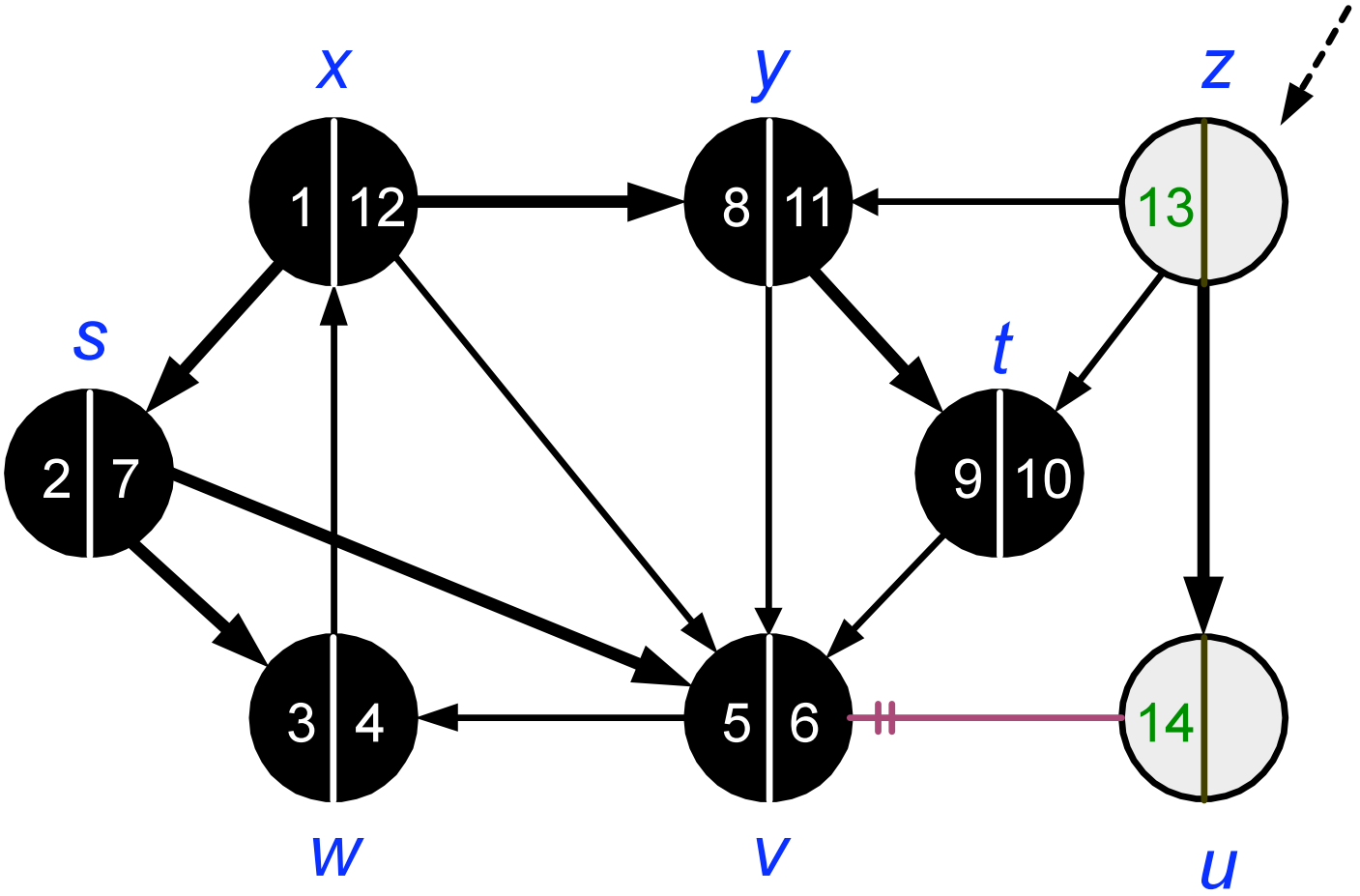

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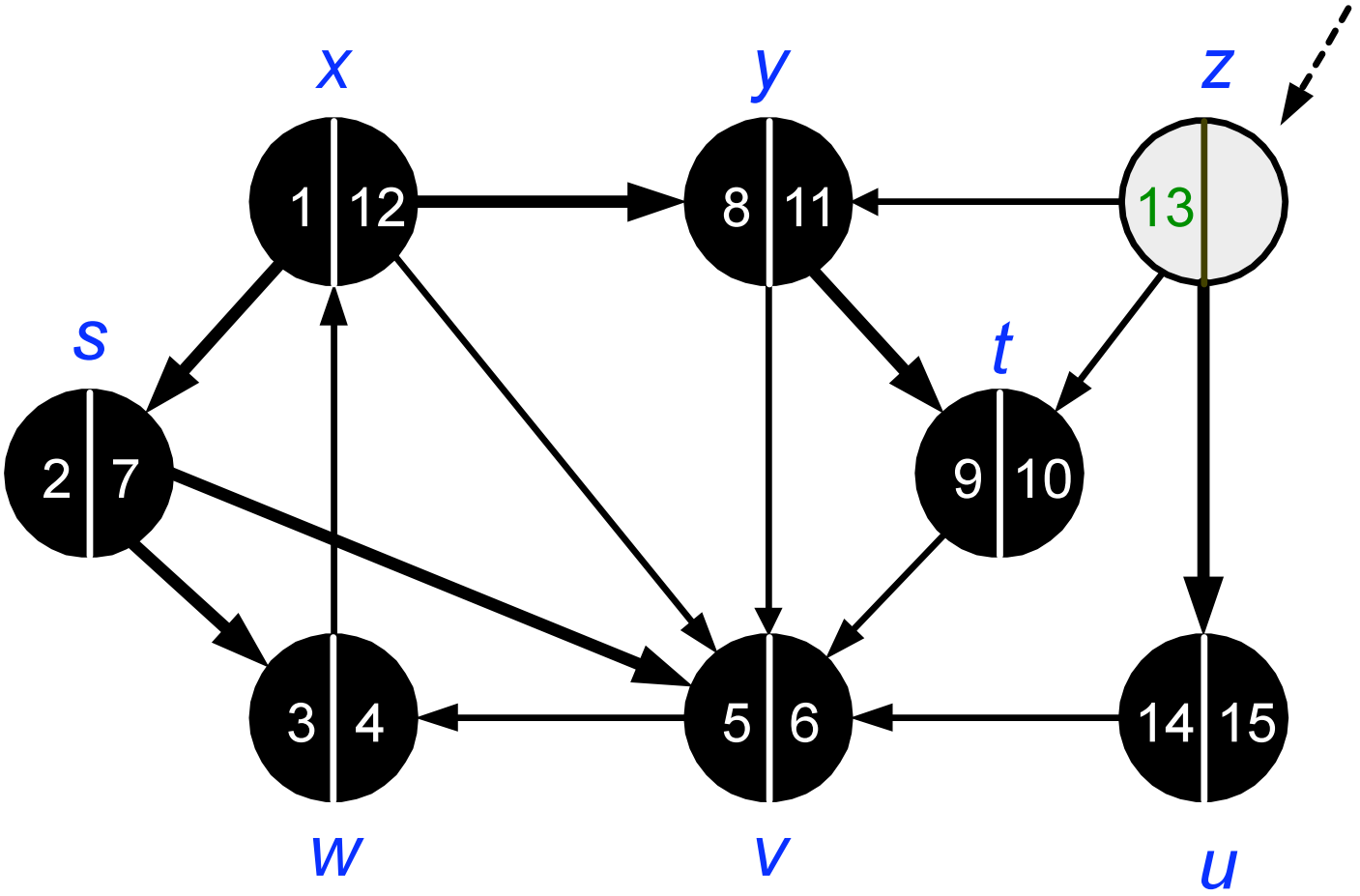

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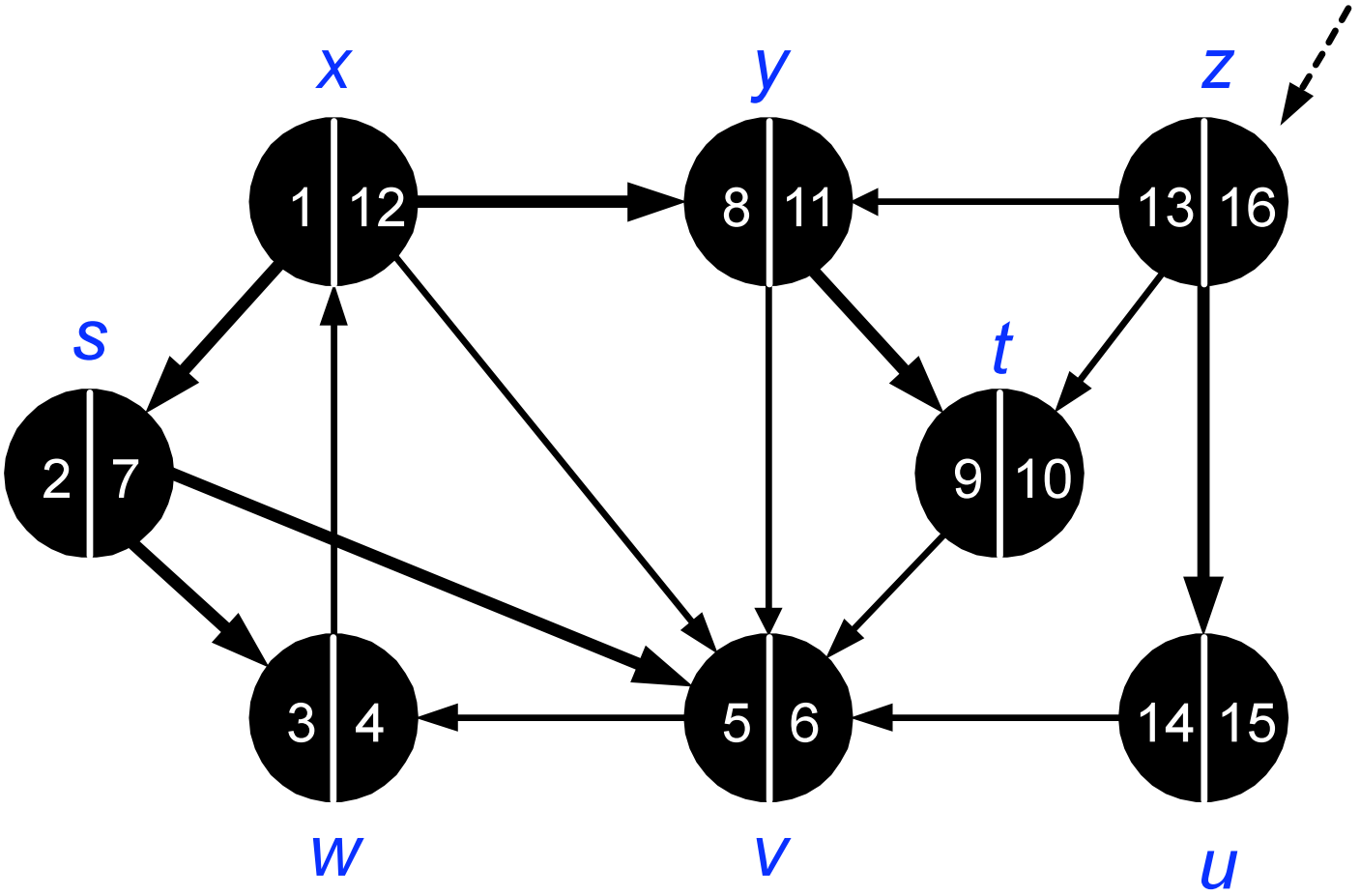

Depth-First Search: Example

DFS(G) terminated	Depth-first forest (DFF)