CS473-Algorithms I

Lecture 15

Graph Searching:

Depth-First Search and Topological Sort

Depth-First Search

- Graph $G=(V,E)$ directed or undirected
- Adjacency list representation
- Goal: Systematically explore every vertex and every edge
- Idea: search deeper whenever possible
  - Using a LIFO queue (Stack; FIFO queue used in BFS)

Depth-First Search

- Maintains several fields for each $v \in V$
- Like BFS, colors the vertices to indicate their states. Each vertex is
  - Initially white,
  - grayed when discovered,
  - blackened when finished
- Like BFS, records discovery of a white v during scanning Adj[u] by $\pi[v] \leftarrow u$

Depth-First Search

- Unlike BFS, predecessor graph G_π produced by DFS forms spanning forest
- $G_\pi=(V,E_\pi)$ where

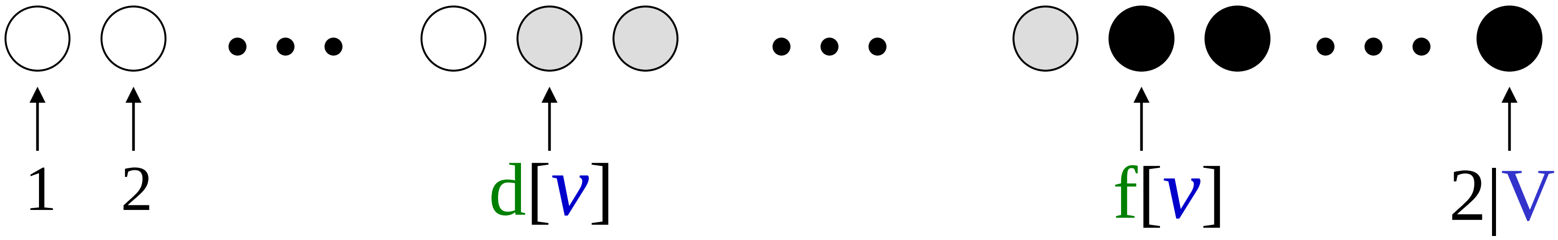
$$E_\pi=\{(\pi[v],v): v \in V \text{ and } \pi[v] \neq \text{NIL}\}$$

- G_π= depth-first forest (DFF) is composed of disjoint depth-first trees (DFTs)

Depth-First Search

- DFS also timestamps each vertex with two timestamps
- $d[v]$: records when v is first discovered and grayed
- $f[v]$: records when v is finished and blackened
- Since there is only one discovery event and finishing event for each vertex we have $1 \le d[v] < f[v] \le 2|V|$

Time axis for the color of a vertex



Depth-First Search

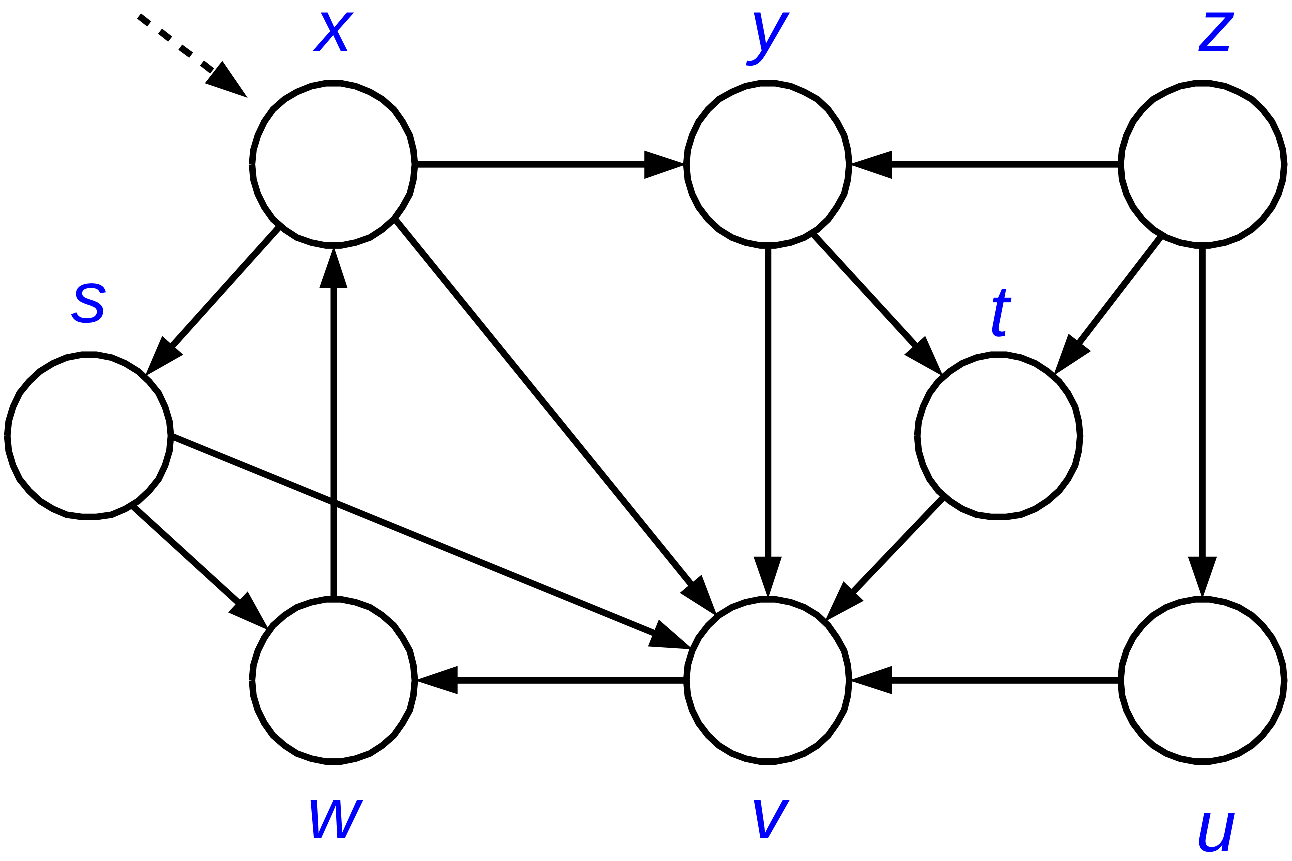
```
DFS(G)
    for each $u \in V$ do
        color[$u$] ← white
        $\pi[u]$ ← NIL
    time ← 0
    for each $u \in V$ do
        if color[$u$] = white then
            DFS-VISIT(G, $u$)
```

```
DFS-VISIT(G, $u$)
    color[$u$] ← gray
    d[$u$] ← time ← time +1
    for each $v \in$ Adj[$u$] do
        if color[$v$] = white then
            $\pi[v]$ ← $u$
            DFS-VISIT(G, $v$)
    color[$u$] ← black
    f[$u$] ← time ← time +1
```

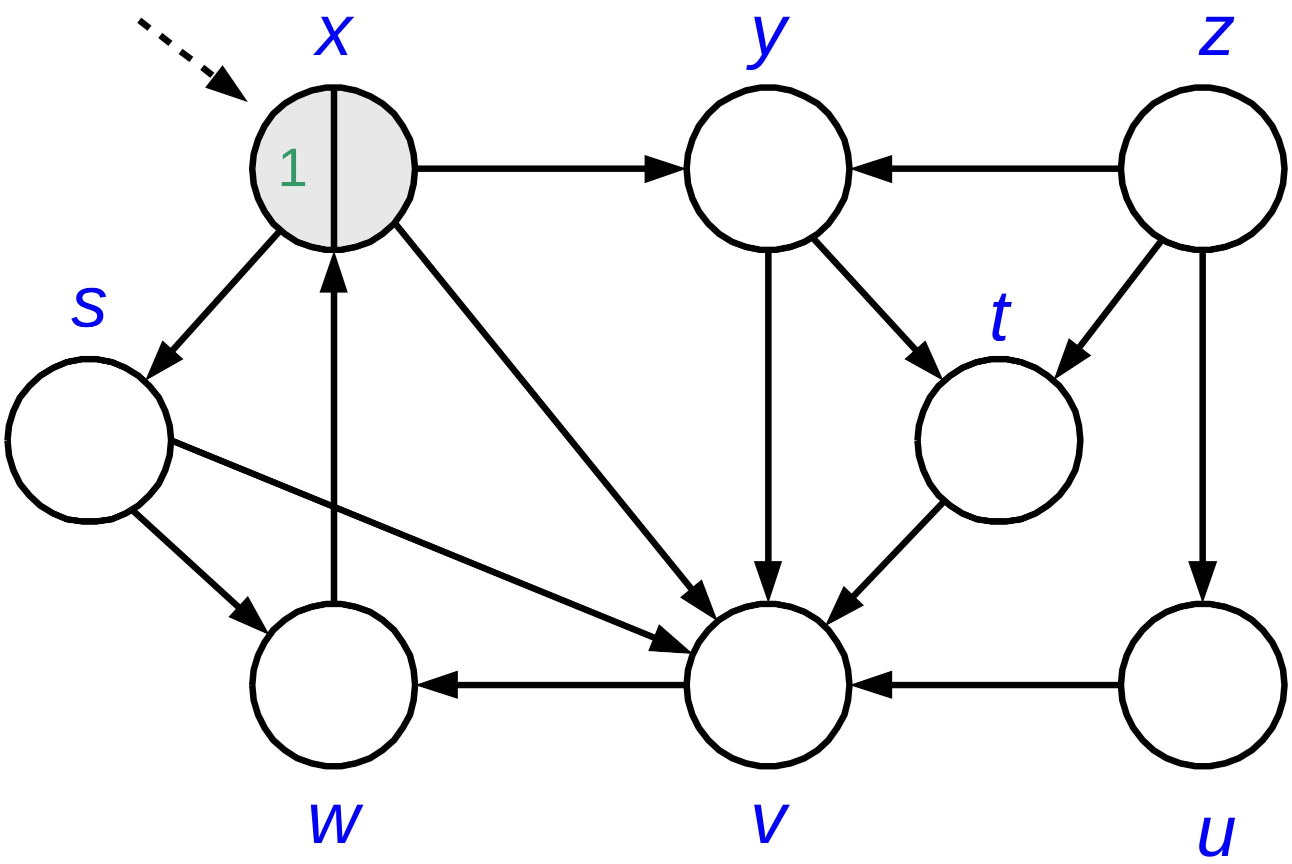
Depth-First Search

- Running time: $\Theta(V+E)$
- Initialization loop in **DFS** : $\Theta(V)$
- Main loop in **DFS**: $\Theta(V)$ exclusive of time to execute calls to **DFS-VISIT**
- **DFS-VISIT** is called exactly once for each $v \in V$ since
  - **DFS-VISIT** is invoked only on white vertices and
  - **DFS-VISIT(G, *u*)** immediately colors u as gray
- For loop of **DFS-VISIT(G, *u*)** is executed $|\text{Adj}[u]|$ time
- Since $\Sigma\, |\text{Adj}[u]| = E$, total cost of executing loop of **DFS-VISIT** is $\Theta(E)$

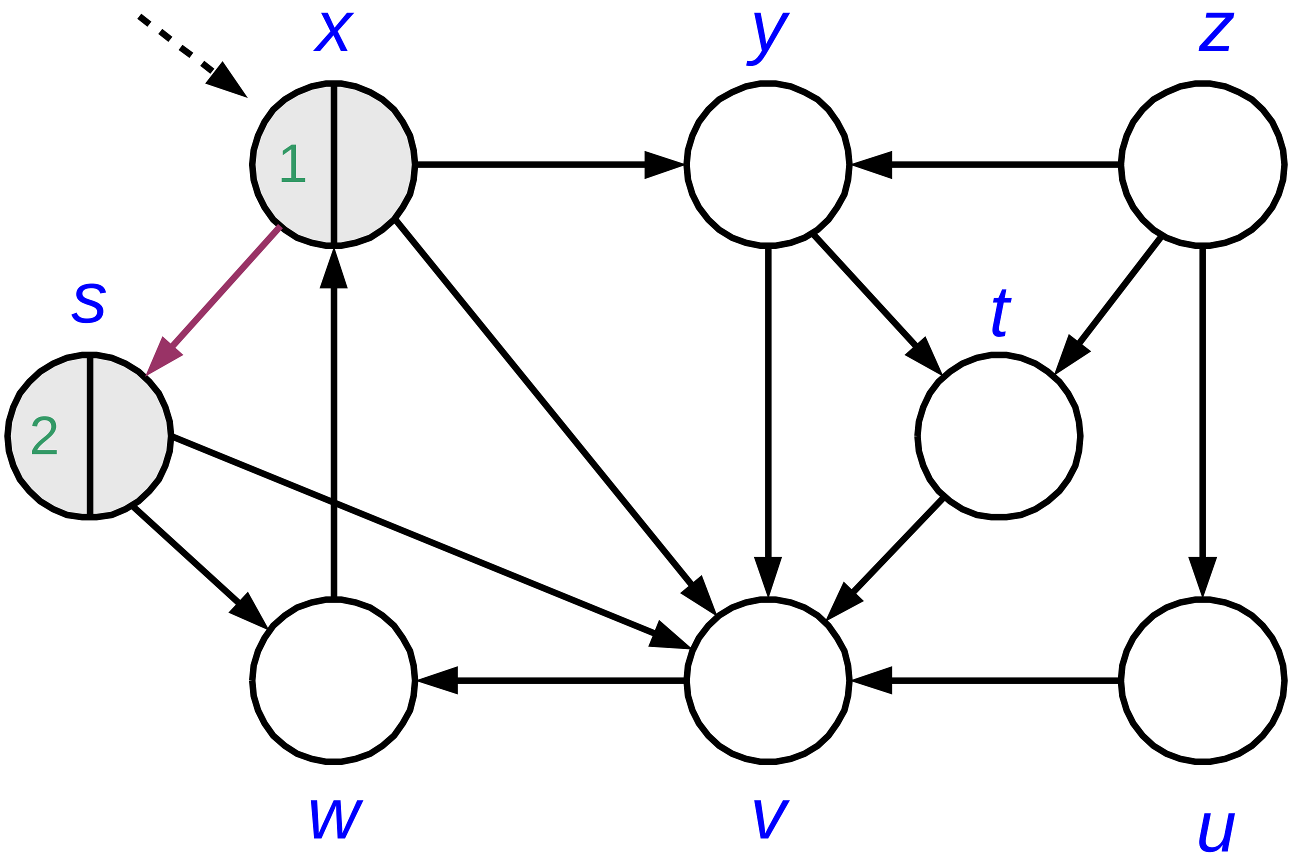
Depth-First Search: Example



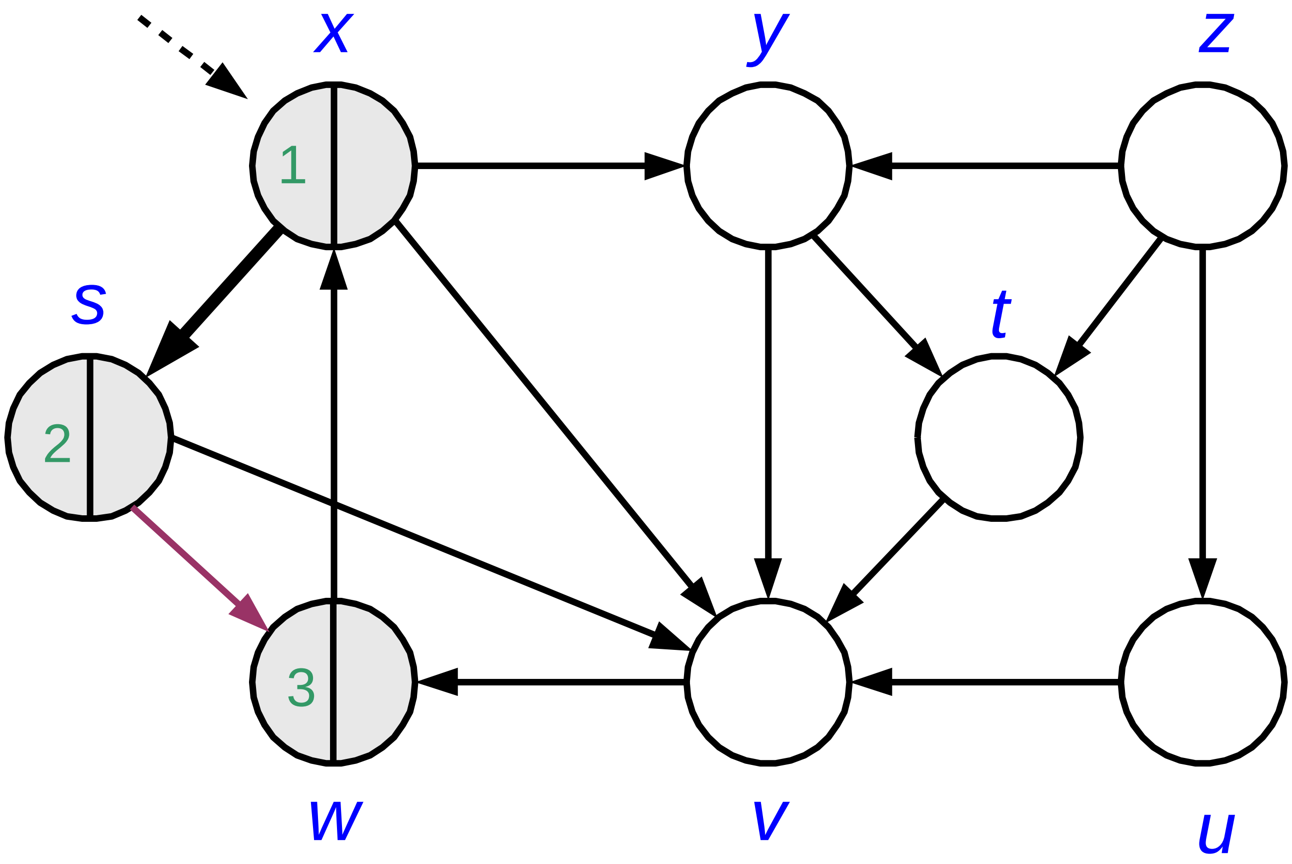
Depth-First Search: Example



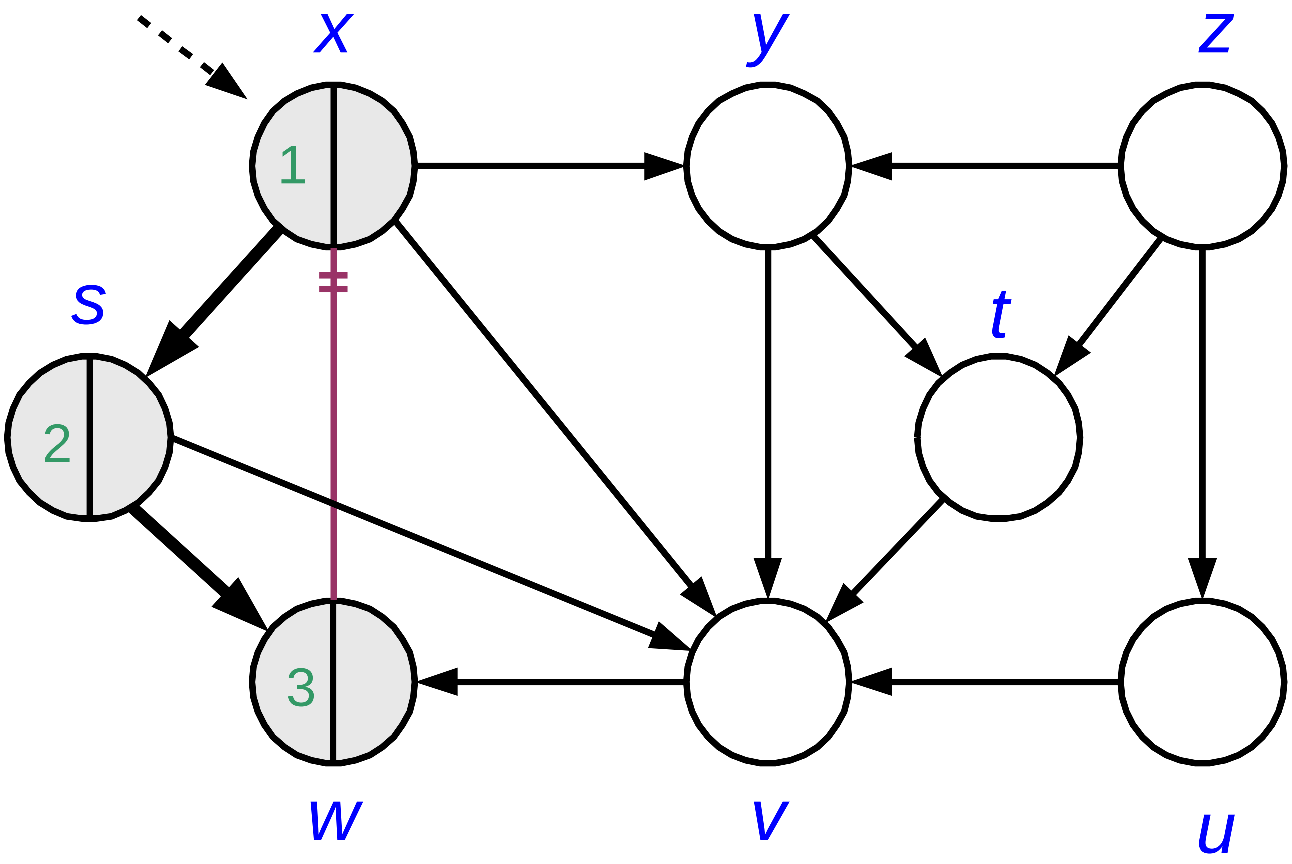
Depth-First Search: Example



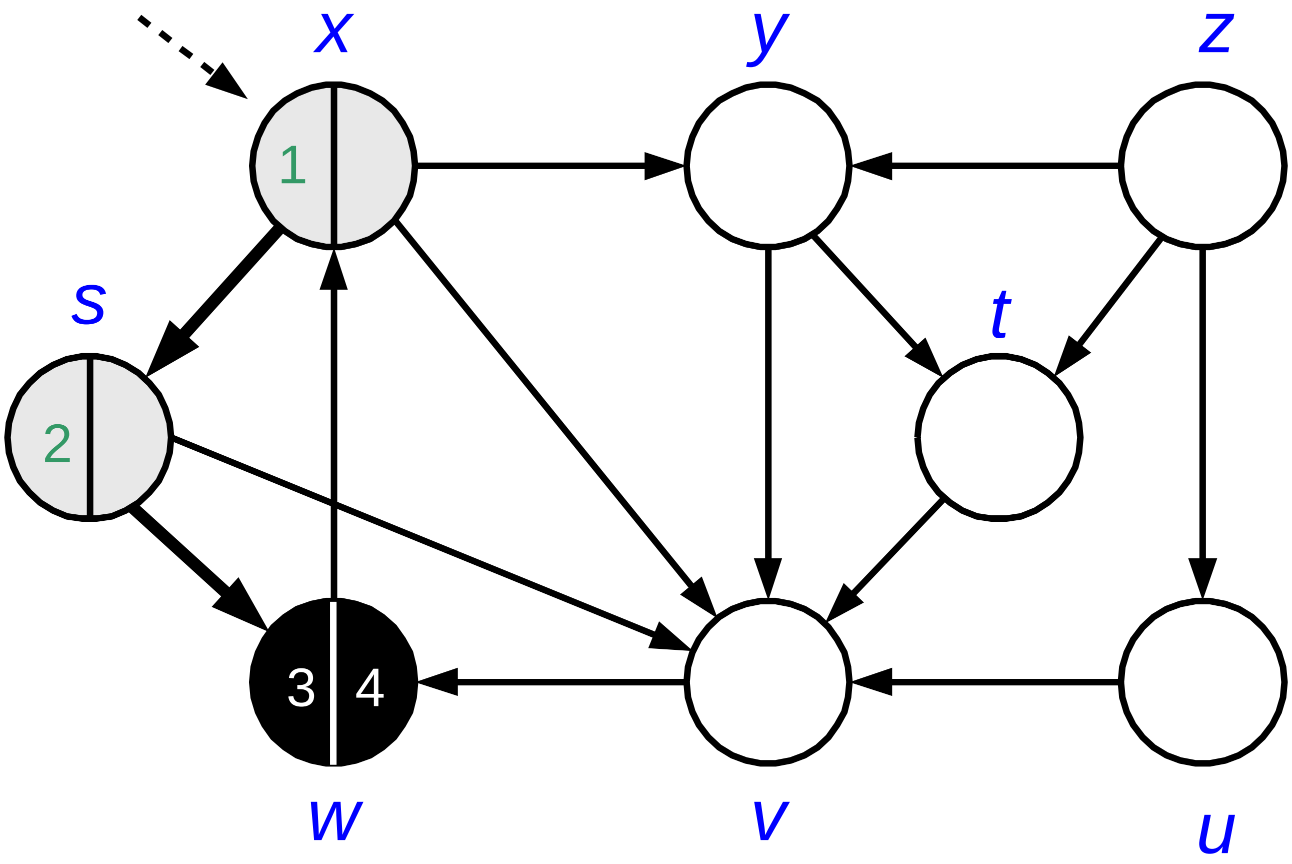
Depth-First Search: Example



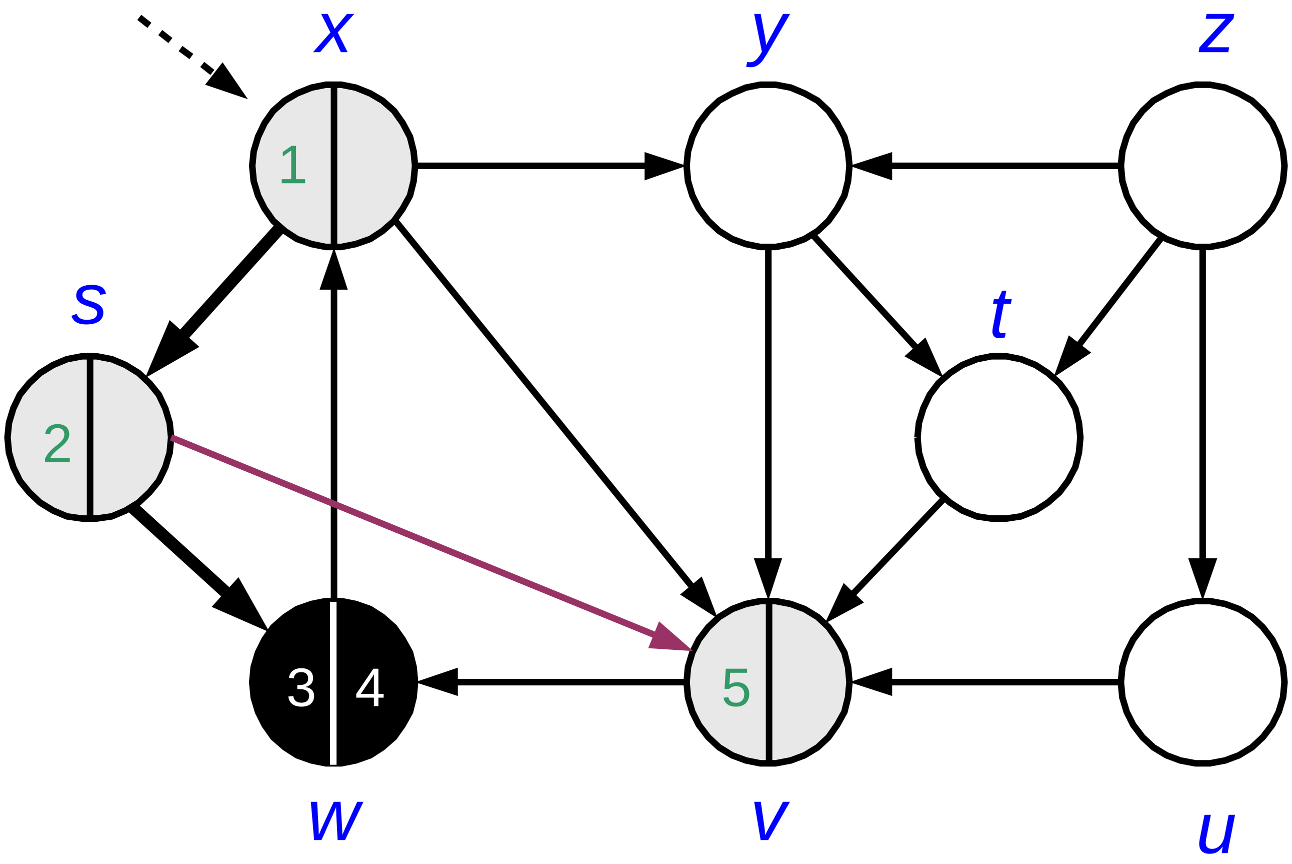
Depth-First Search: Example



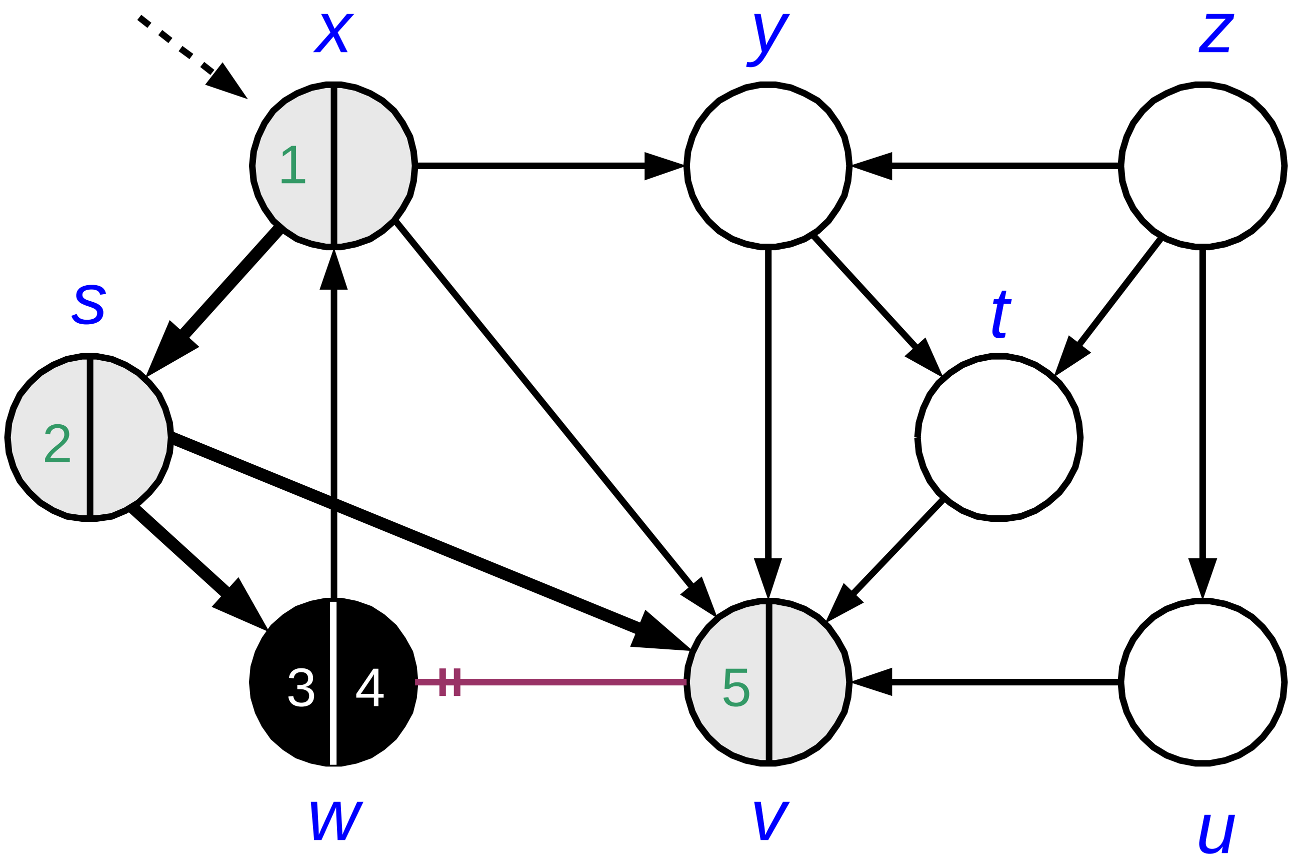
Depth-First Search: Example



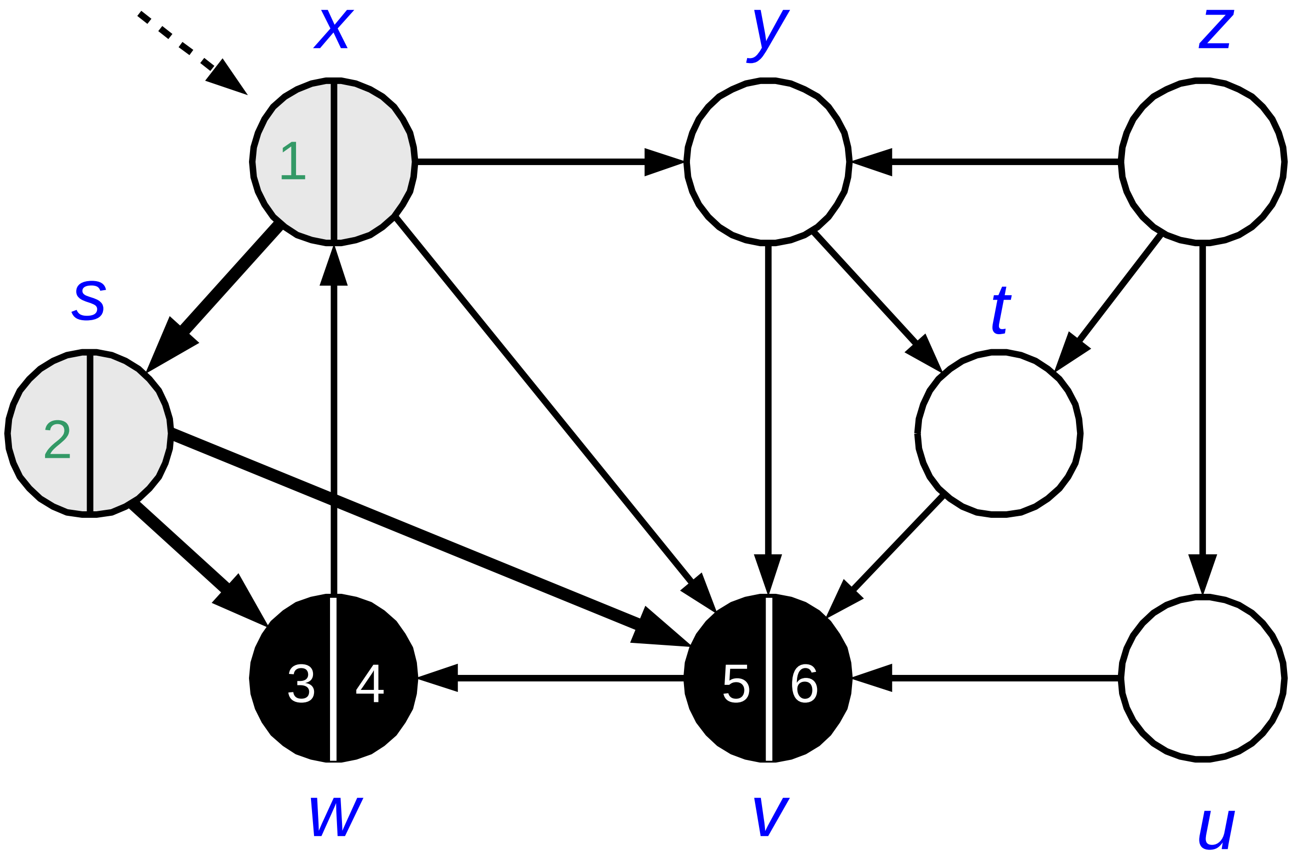
Depth-First Search: Example



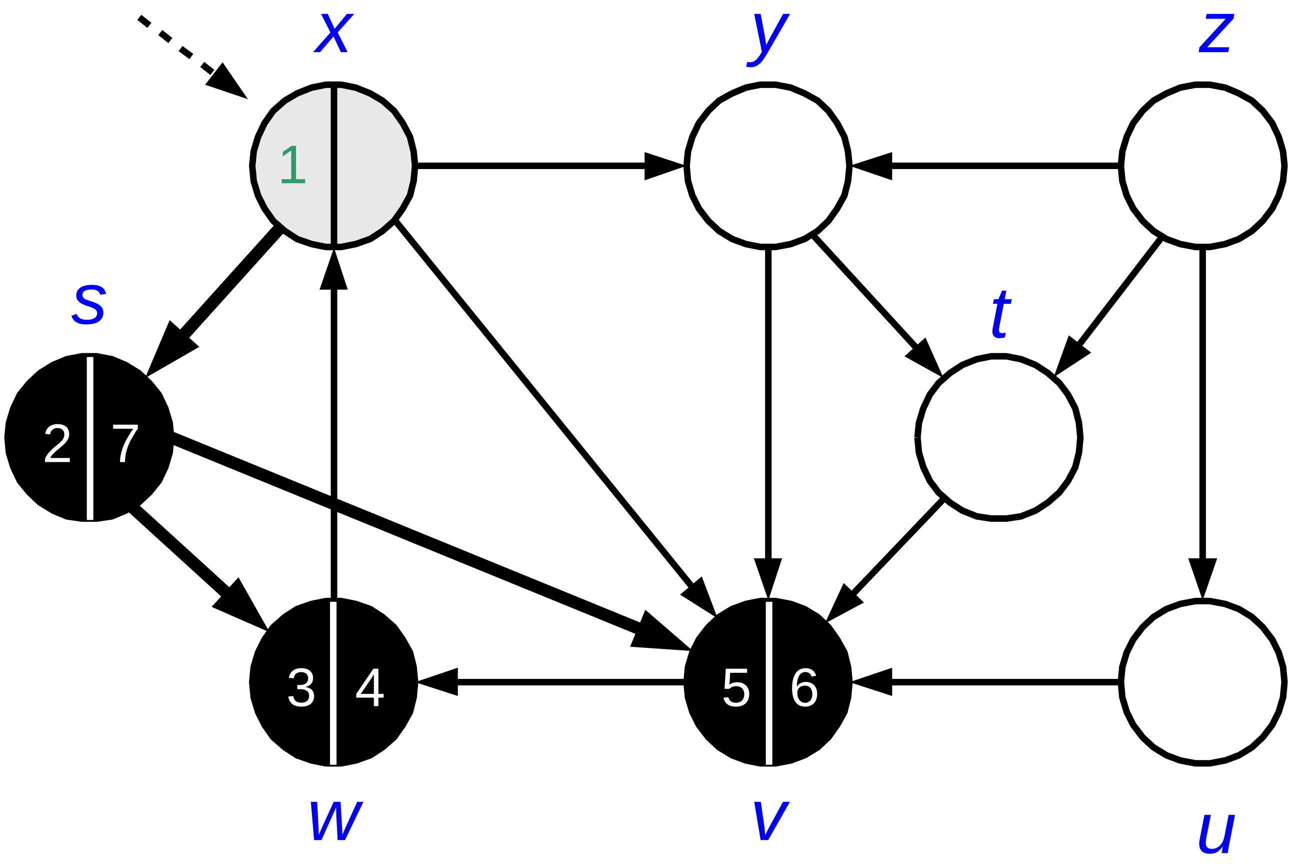
Depth-First Search: Example



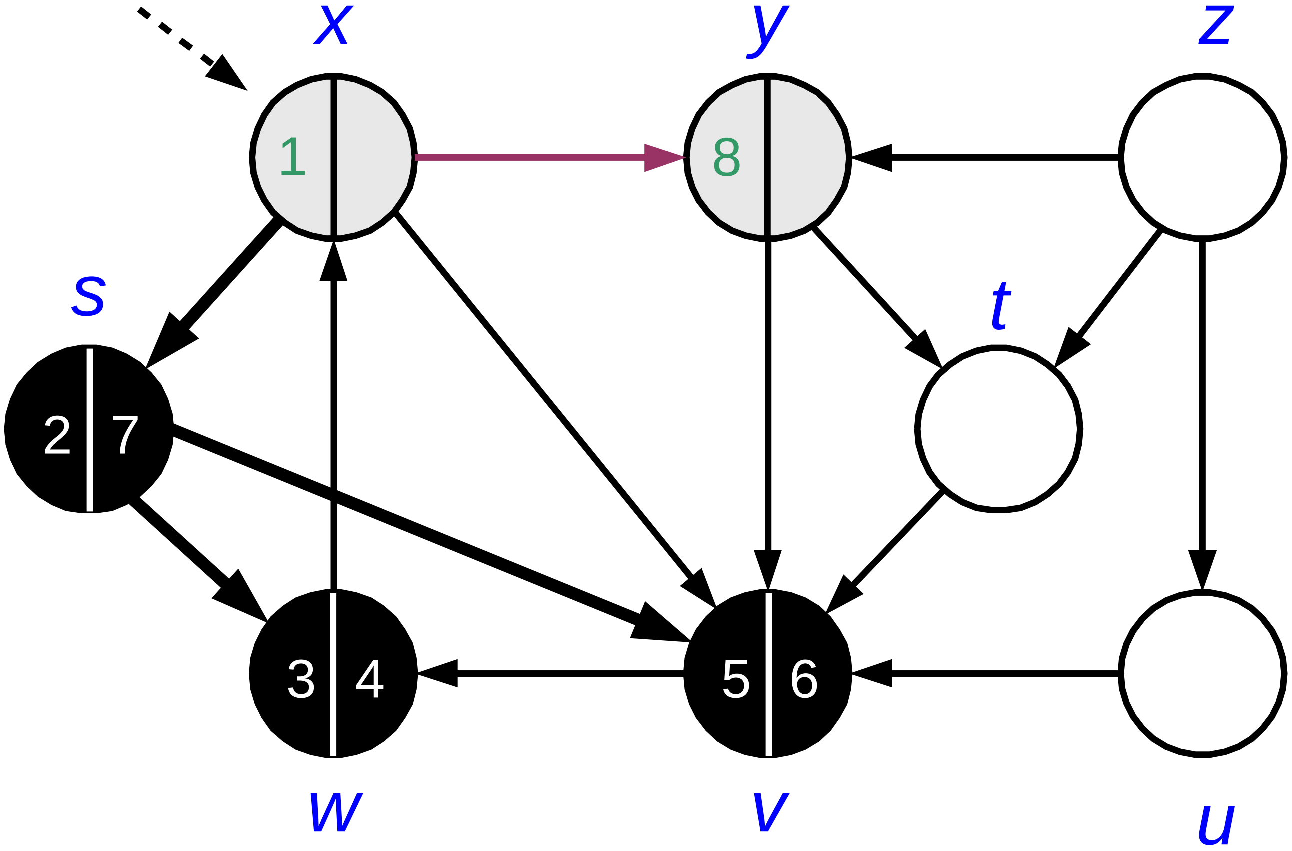
Depth-First Search: Example



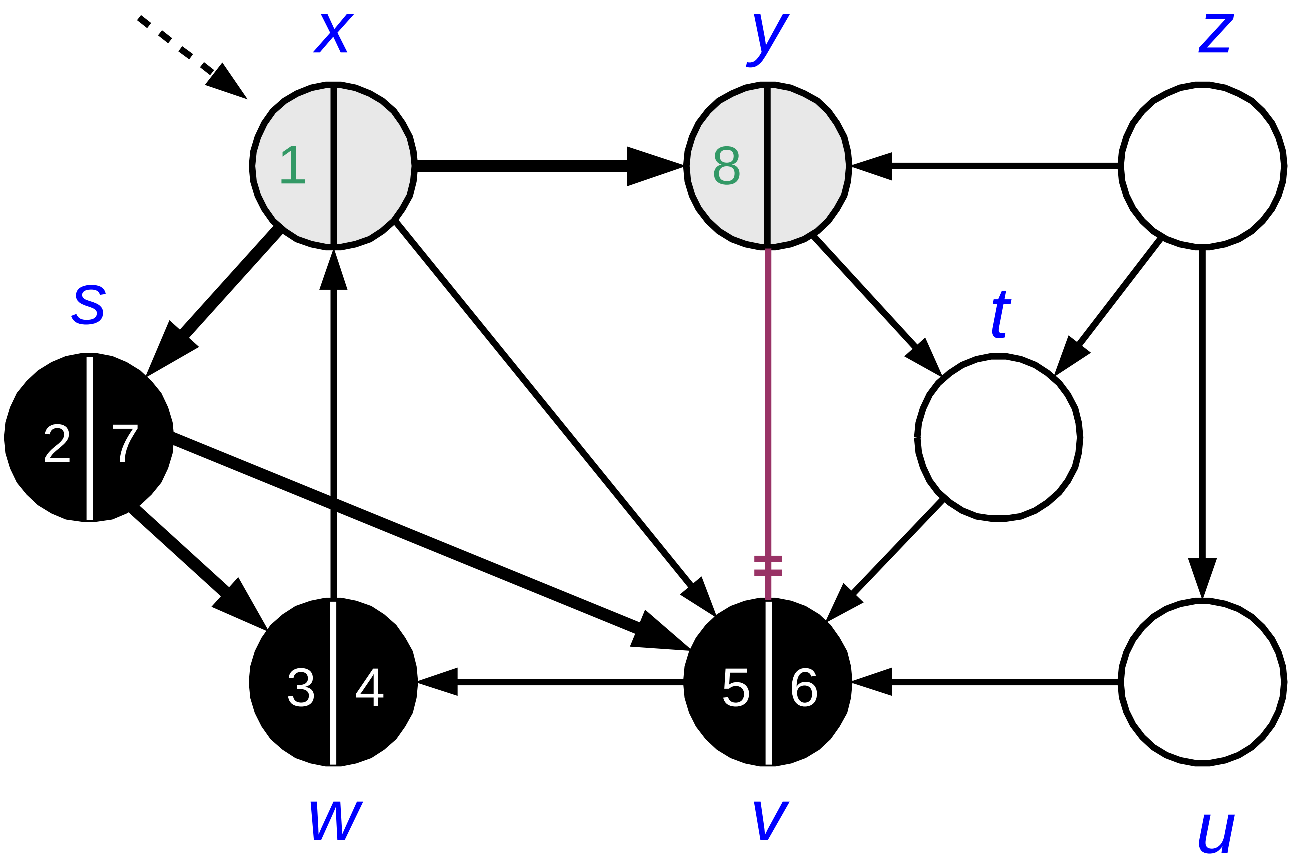
Depth-First Search: Example



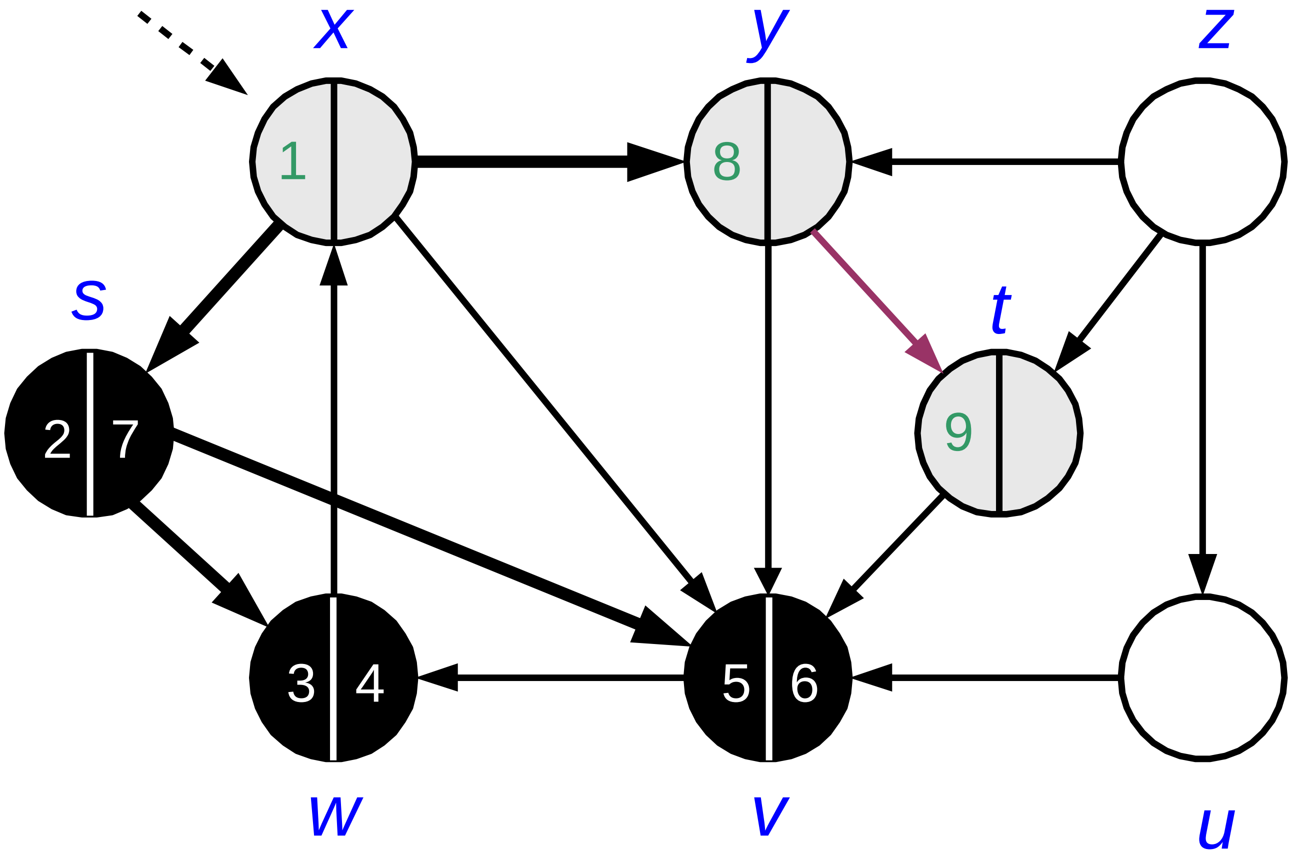
Depth-First Search: Example



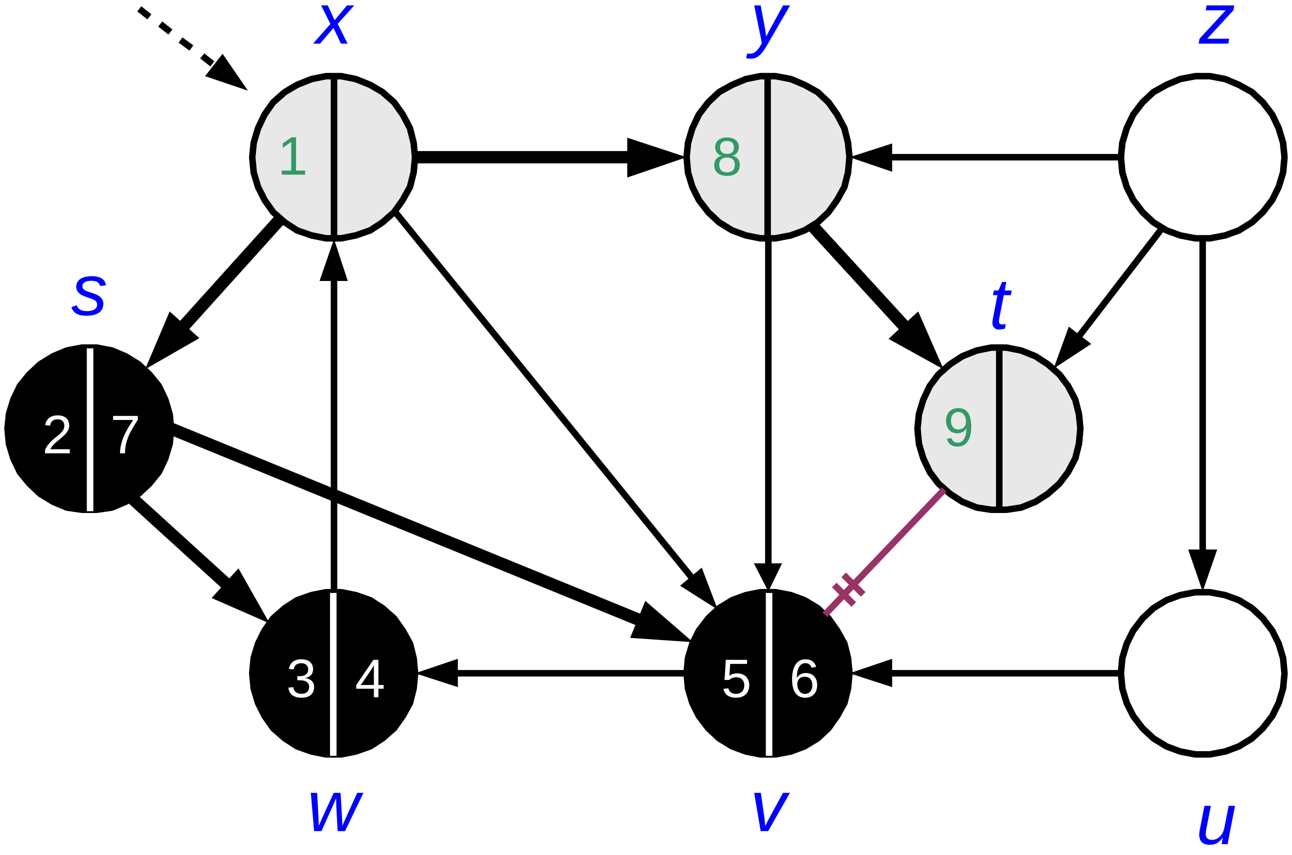
Depth-First Search: Example



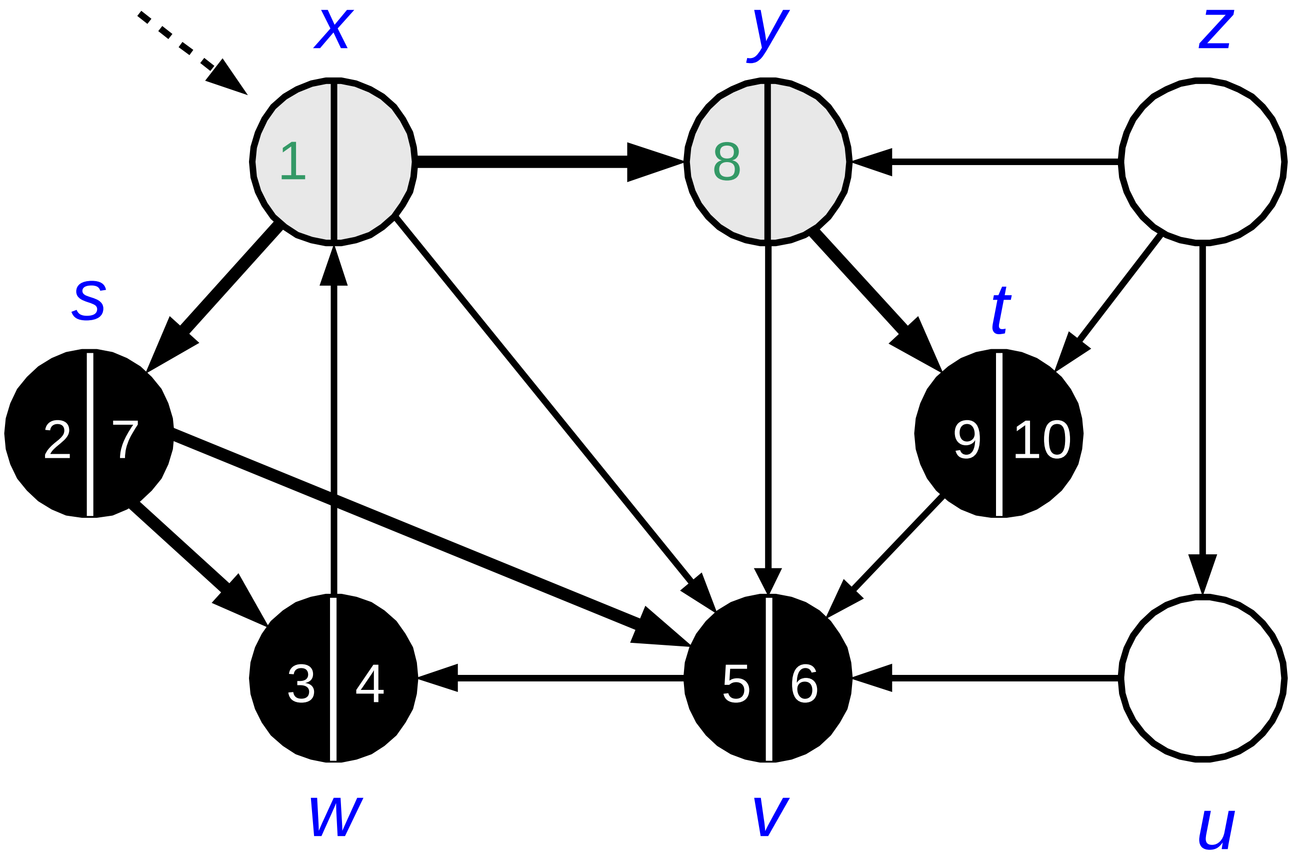
Depth-First Search: Example



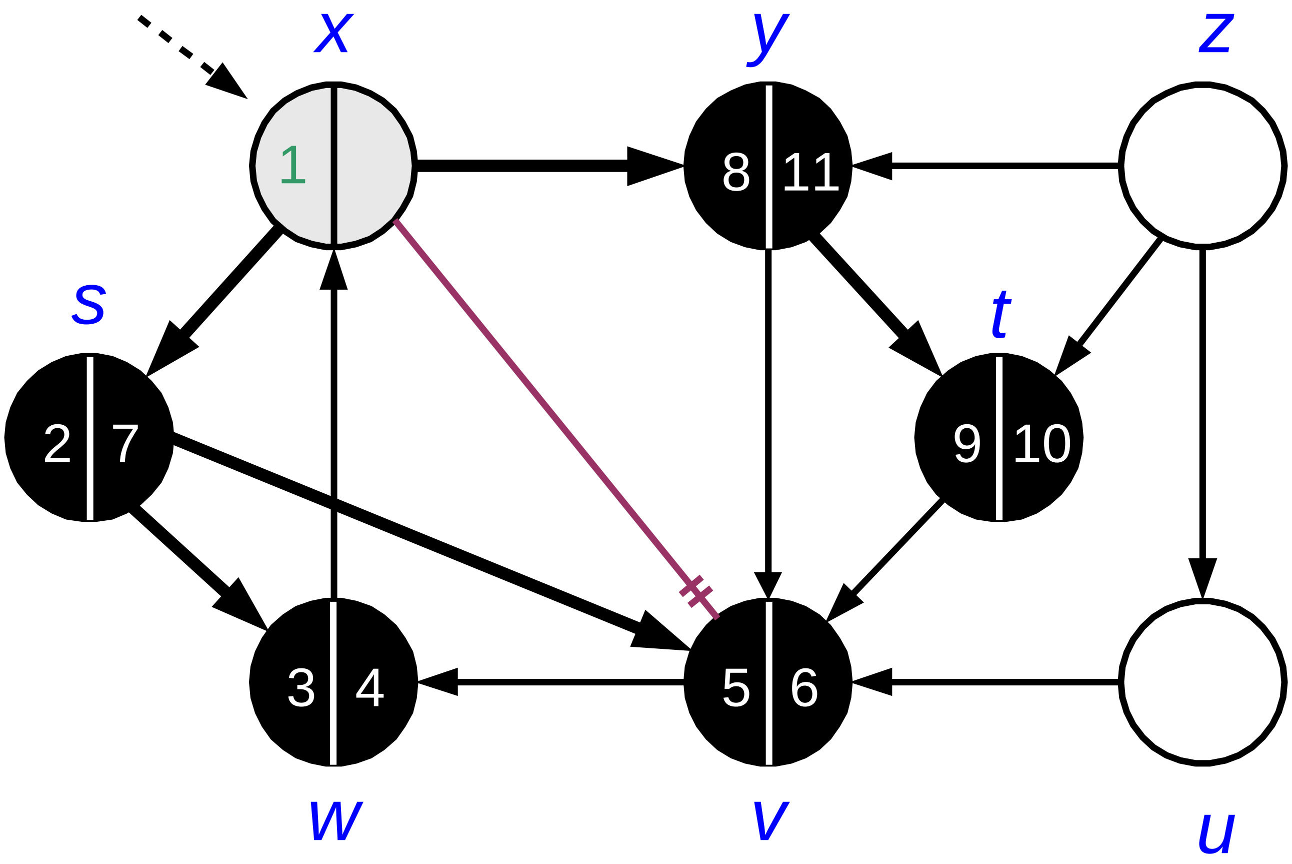
Depth-First Search: Example



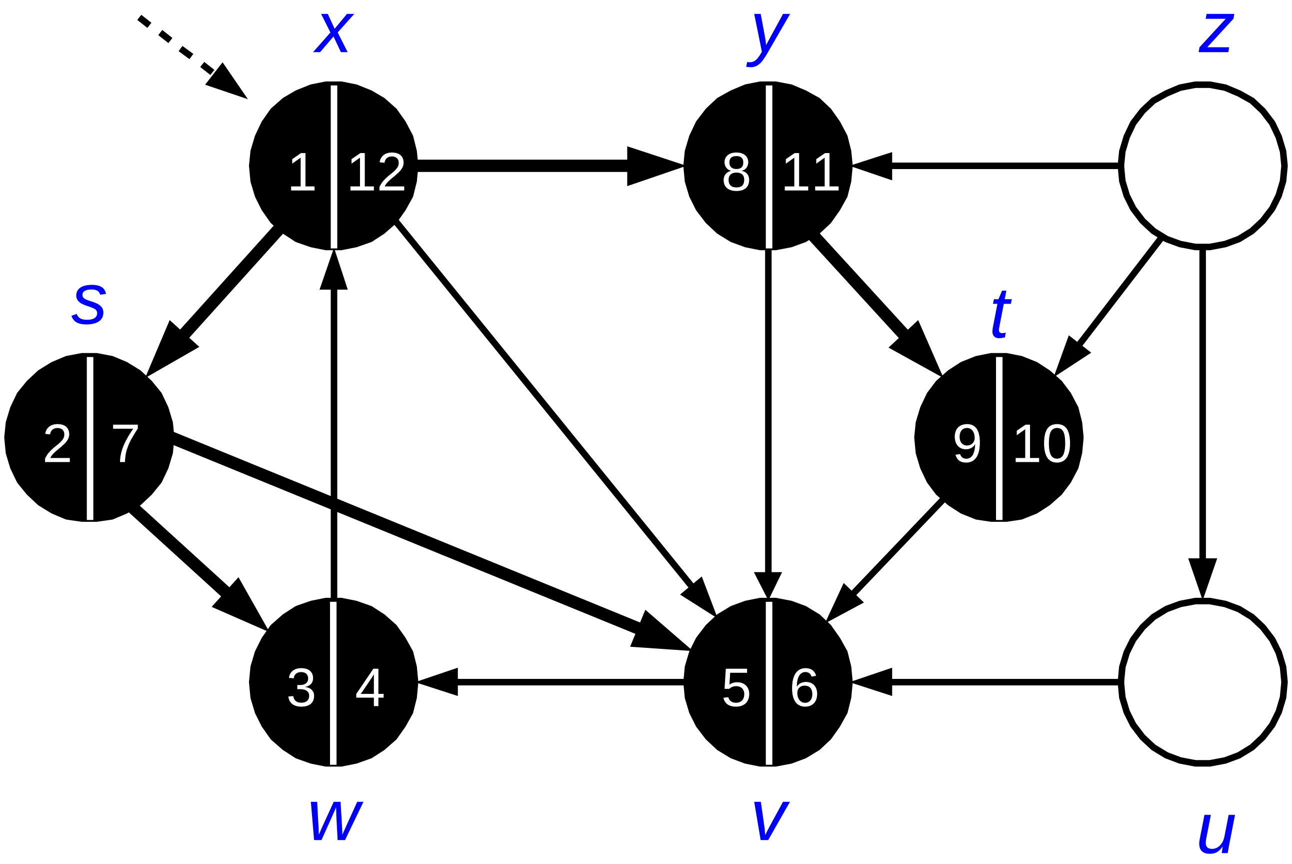
Depth-First Search: Example



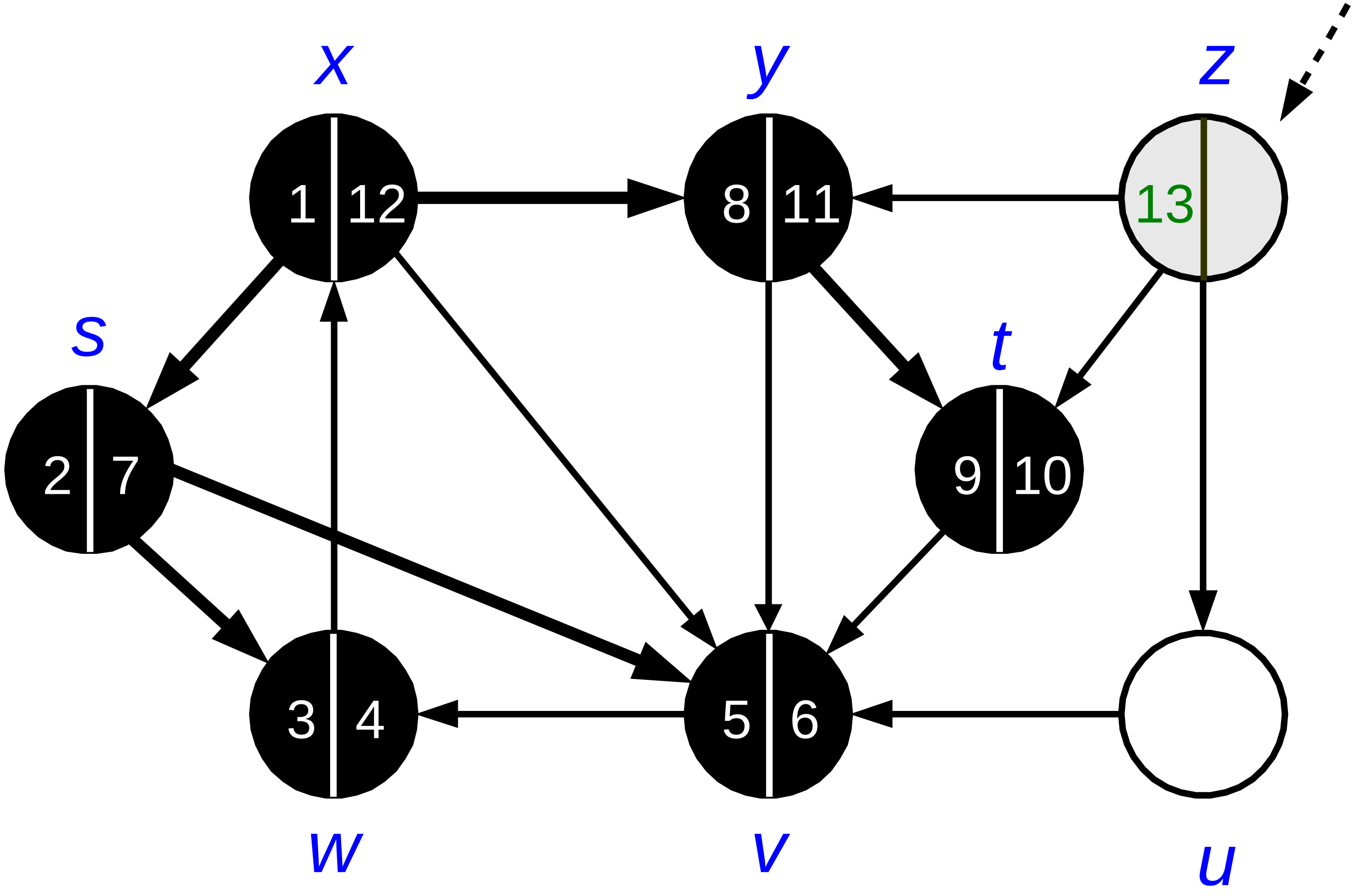
Depth-First Search: Example



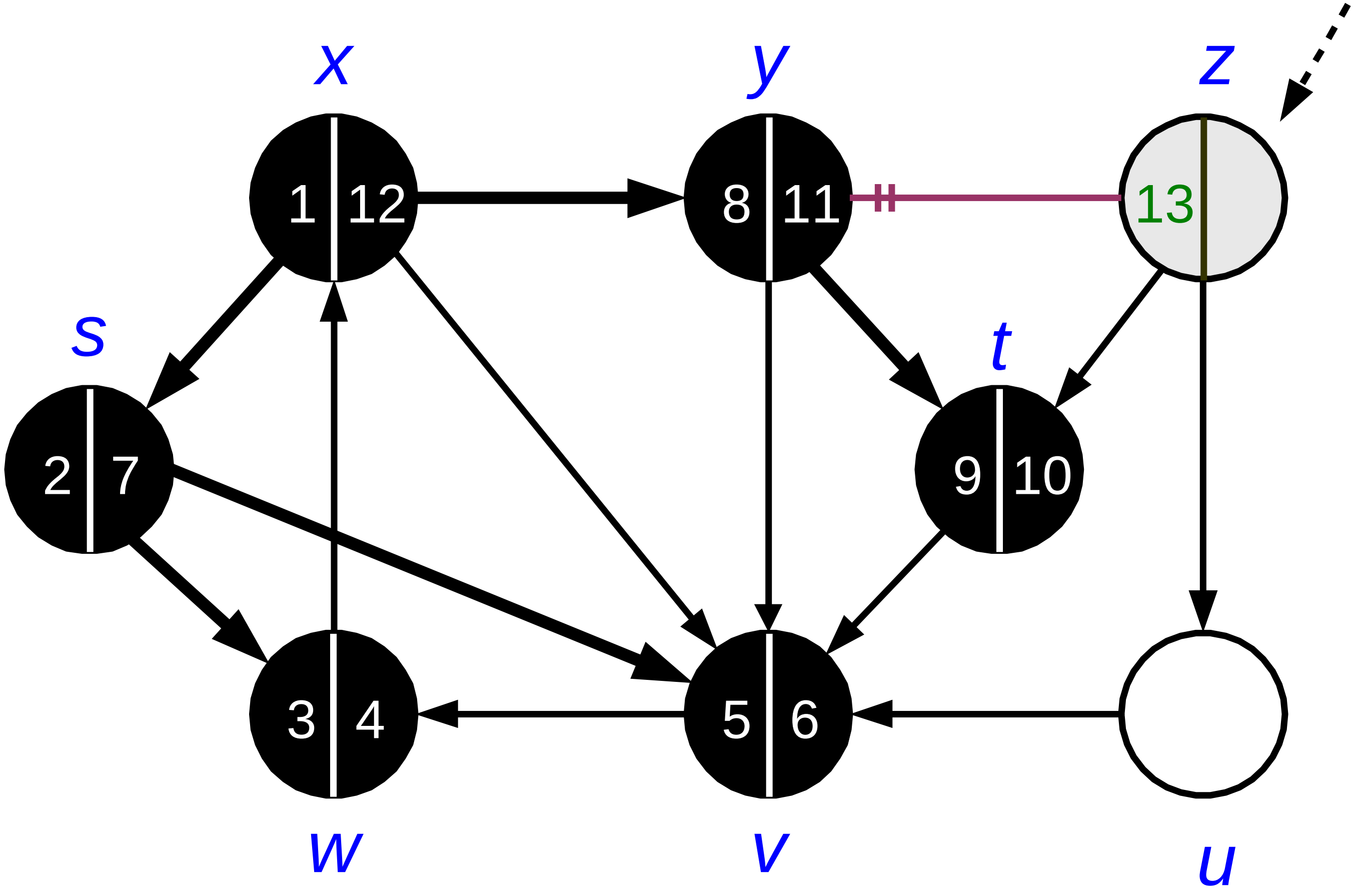
Depth-First Search: Example



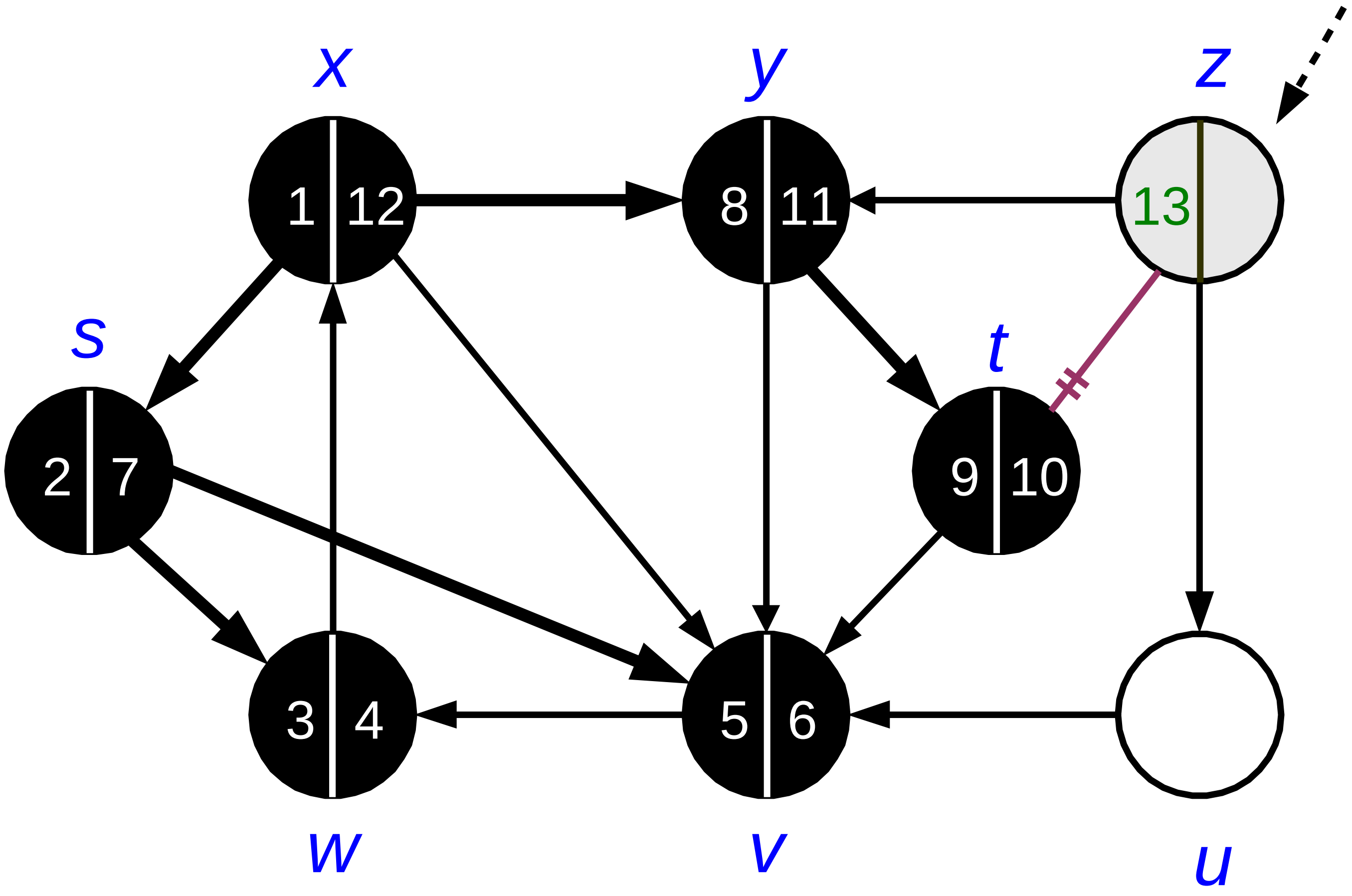
Depth-First Search: Example



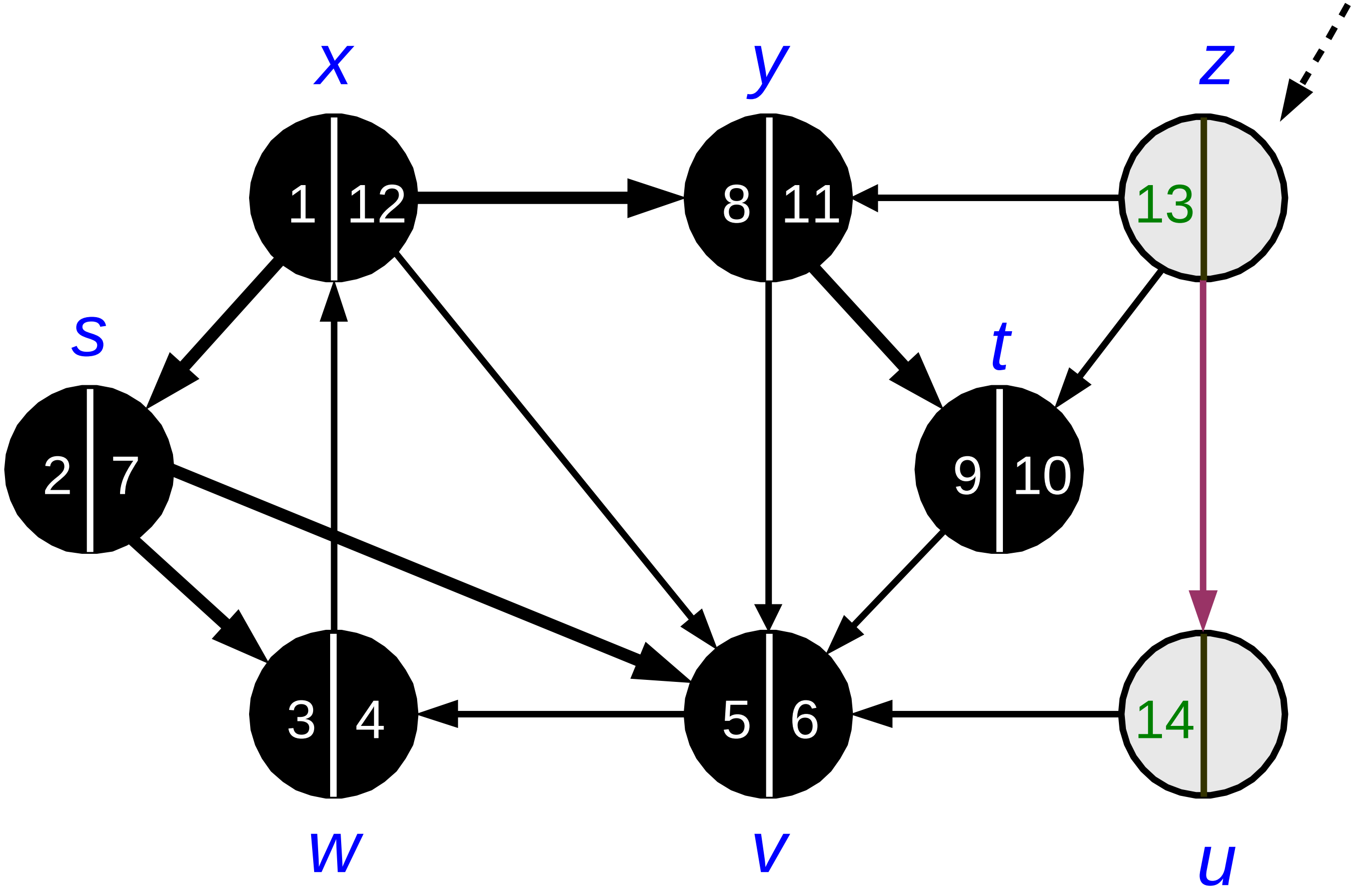
Depth-First Search: Example



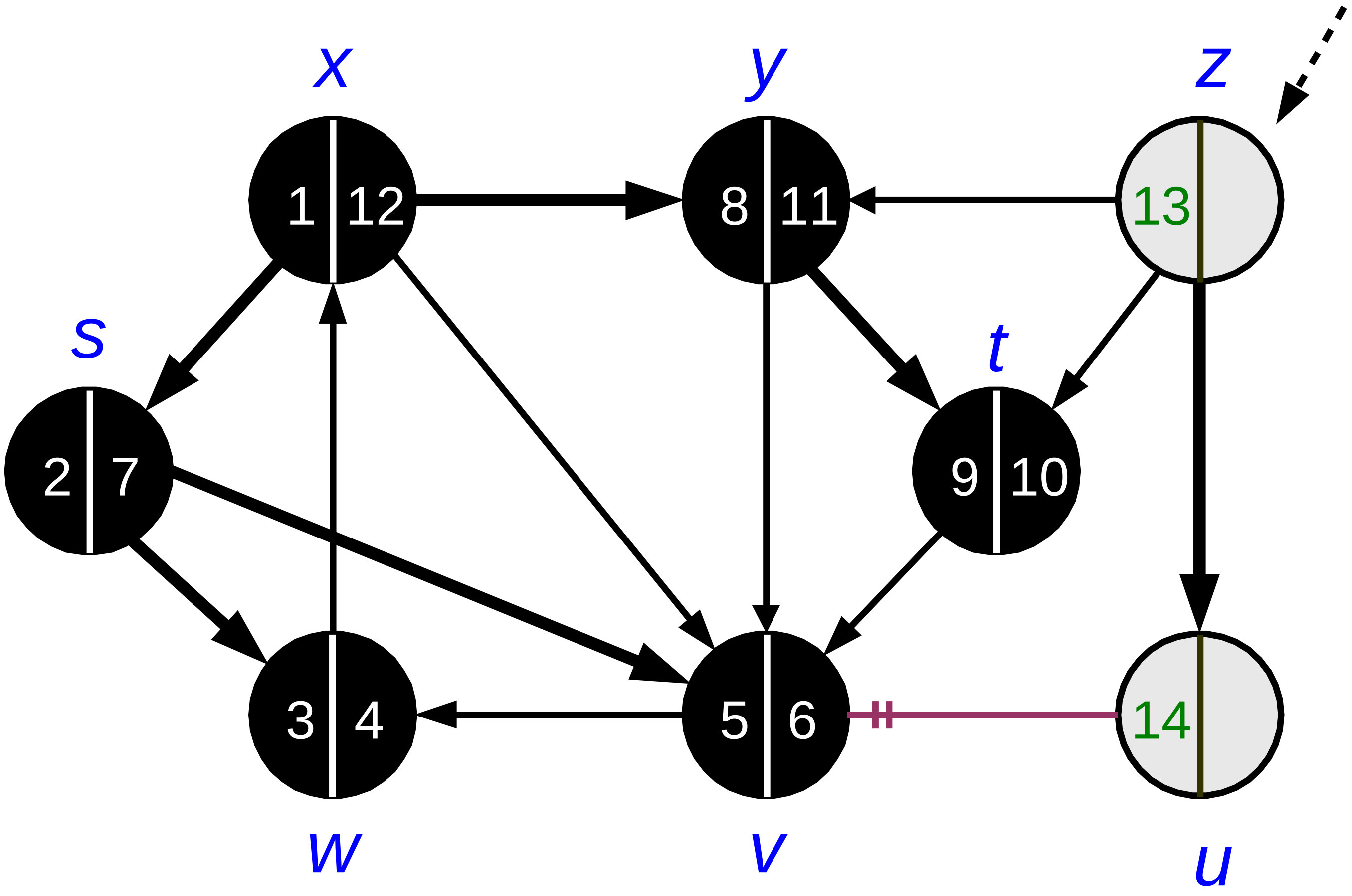
Depth-First Search: Example



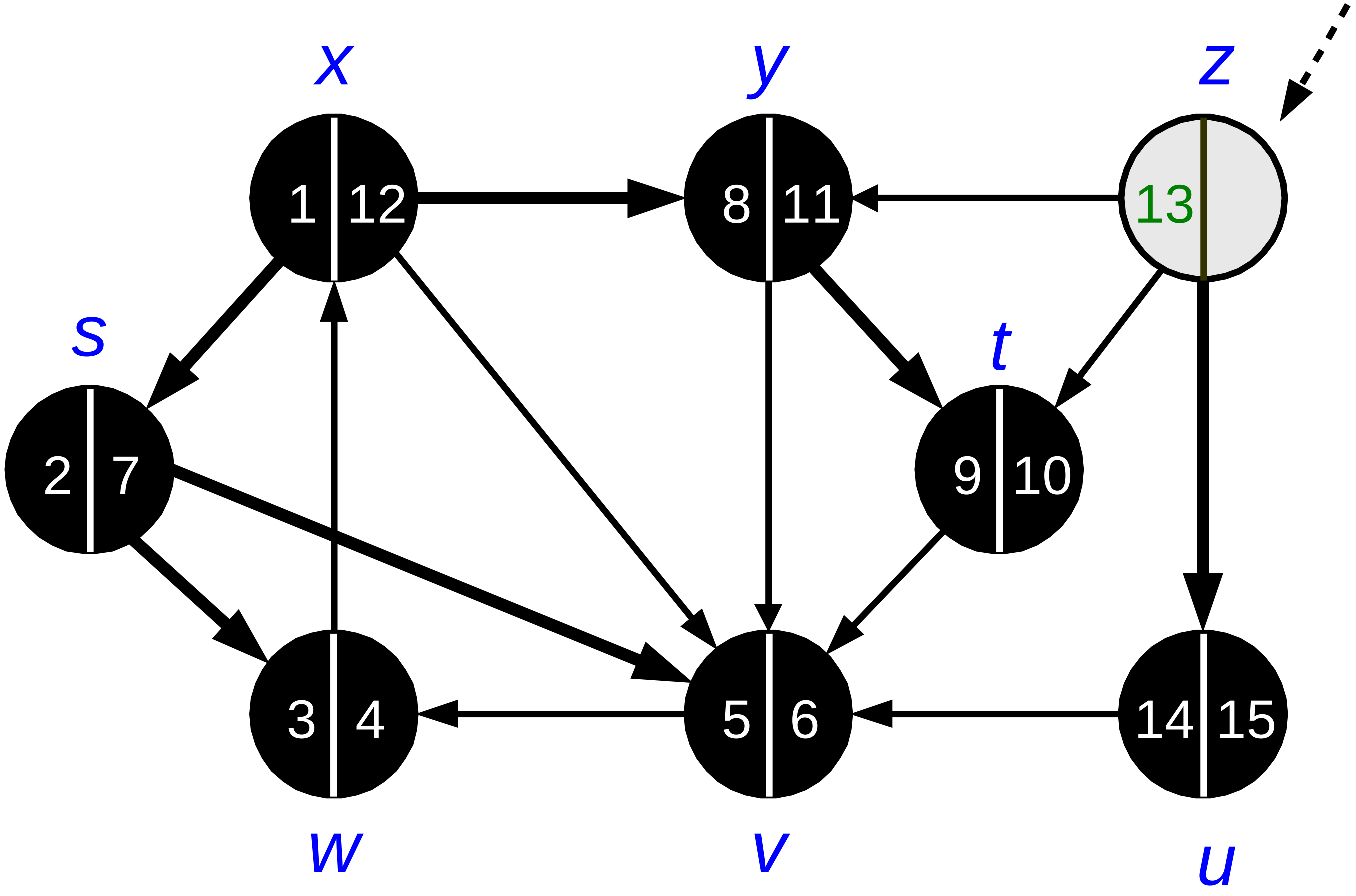
Depth-First Search: Example



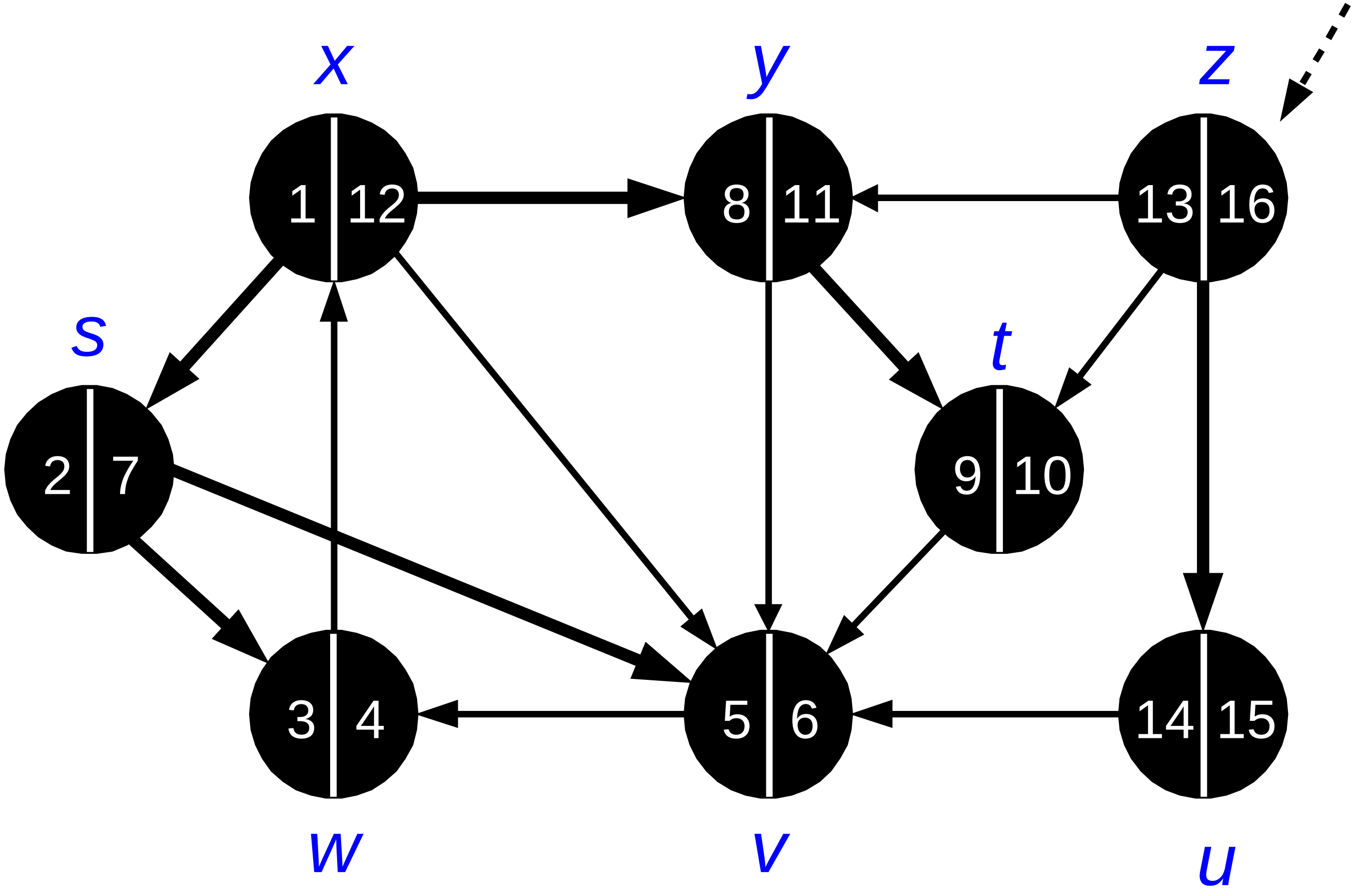
Depth-First Search: Example



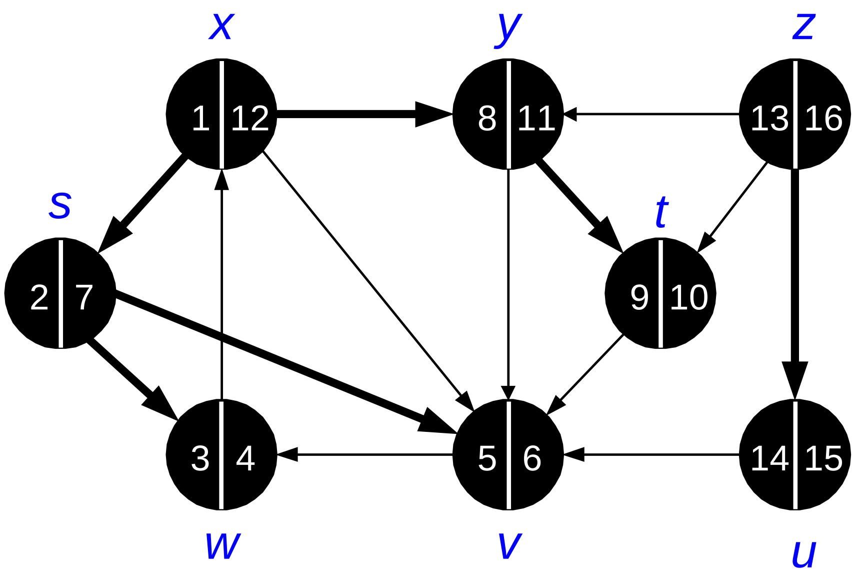
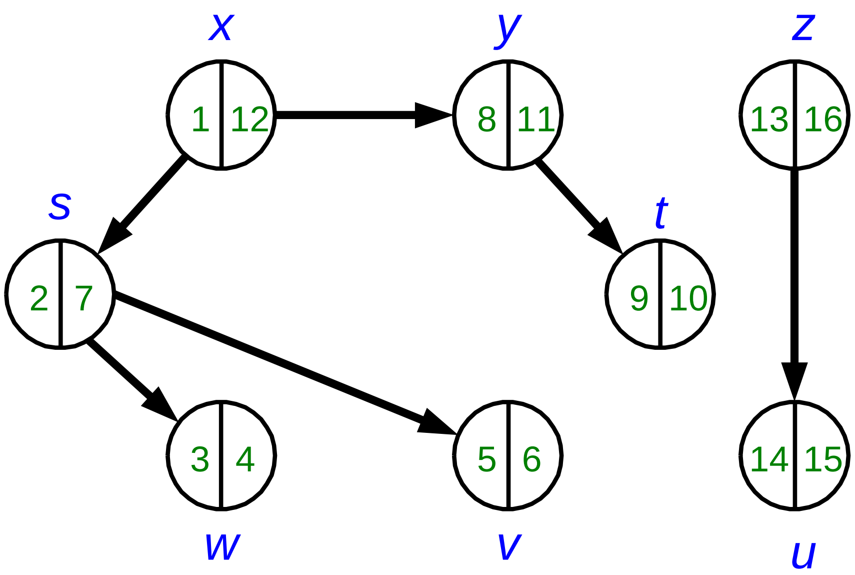
Depth-First Search: Example



Depth-First Search: Example



Depth-First Search: Example

DFS(G) terminated	Depth-first forest (DFF)
 x 1 12 y 8 11 z 13 16 s 2 7 t 9 10 3 4 w 5 6 v 14 15 u	 x 1 12 y 8 11 z 13 16 s 2 7 t 9 10 3 4 w 5 6 v 14 15 u