Texture Analysis

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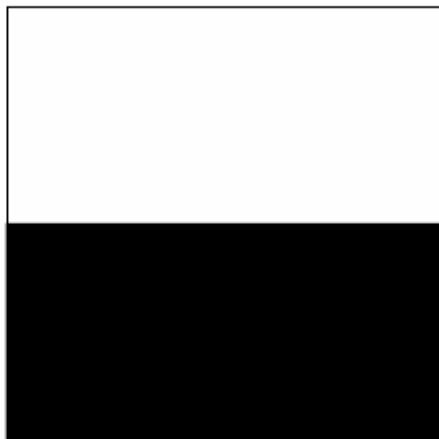
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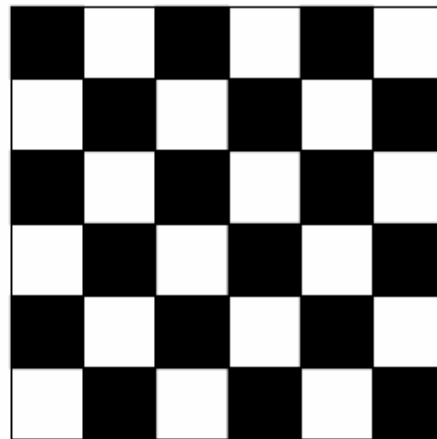
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Texture

- An important approach to image description is to quantify its texture content.

- Texture gives us information about the spatial arrangement of the colors or intensities in an image.

block pattern   **checkerboard**   **striped pattern**

Figure 7.2: Three different textures with the same distribution of black and white.
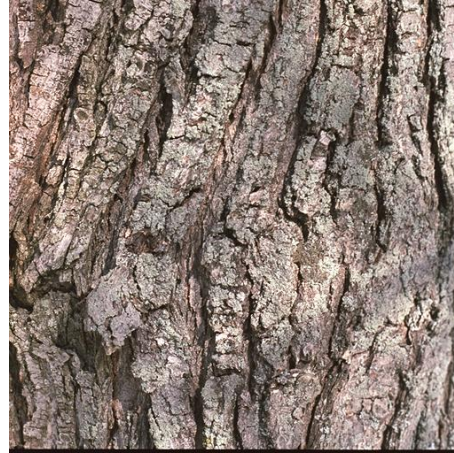
Texture

- Although no formal definition of texture exists, intuitively it can be defined as the uniformity, density, coarseness, roughness, regularity, intensity and directionality of discrete tonal features and their spatial relationships.

- Texture is commonly found in natural scenes, particularly in outdoor scenes containing both natural and man-made objects.
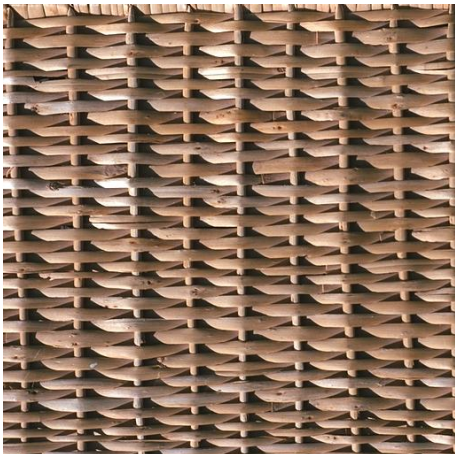
Texture

Bark

Bark

Fabric

Fabric
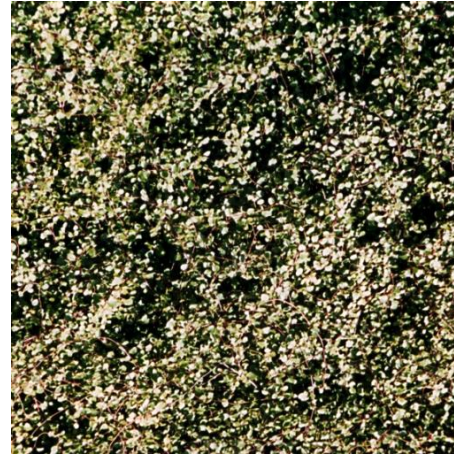
Fabric

Flowers

Flowers

Flowers

Texture
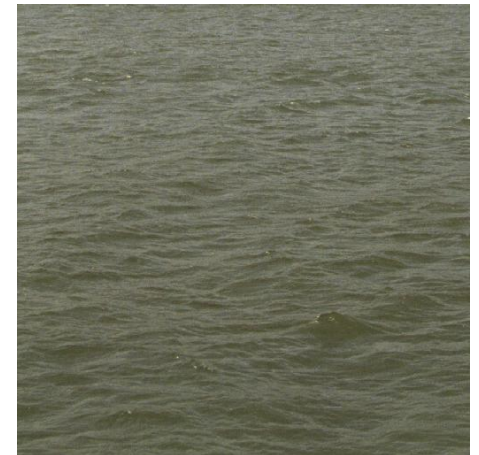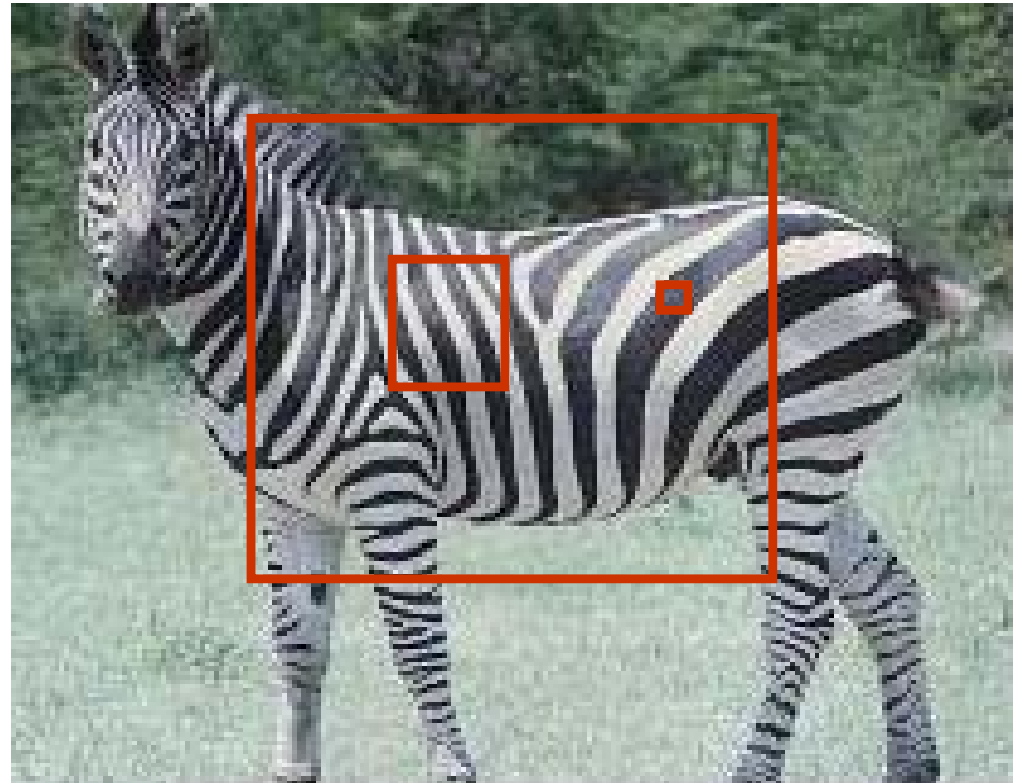

Food

Food

Leaves

Leaves

Leaves

Leaves

Water

Water

Texture

- Whether an effect is a texture or not depends on the scale at which it is viewed.

Texture

- The approaches for characterizing and measuring texture can be grouped as:

  - structural approaches that use the idea that textures are made up of primitives appearing in a near-regular repetitive arrangement,

  - statistical approaches that yield a quantitative measure of the arrangement of intensities.

- While the first approach is appealing and can work well for man-made, regular patterns, the second approach is more general and easier to compute and is used more often in practice.

Structural approaches

- Structural approaches model texture as a set of primitive texels (texture elements) in a particular spatial relationship.

- A structural description of a texture includes a description of the texels and a specification of the spatial relationship.

- Of course, the texels must be identifiable and the relationship must be efficiently computable.

Structural approaches

- Voronoi tessellation of texels by Tuceryan and Jain (PAMI 1990)
    - Texels are extracted by thresholding.
    - Spatial relationships are characterized by their Voronoi tessellation.
    - Shape features of the Voronoi polygons are used to group the polygons into clusters that define uniformly texture regions.
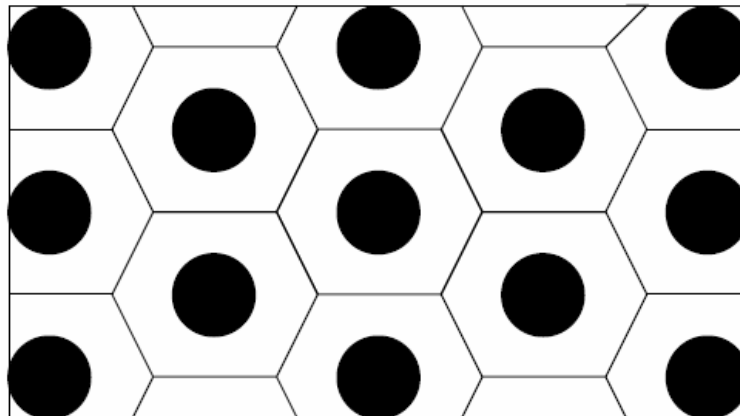

Figure 7.4: The Voronoi tesselation of a set of circular texels.

Structural approaches

- Computational model for periodic pattern perception based on frieze and wallpaper groups by Liu, Collins and Tsin (PAMI 2004)

  - A frieze pattern is a 2D strip in the plane that is periodic along one dimension.

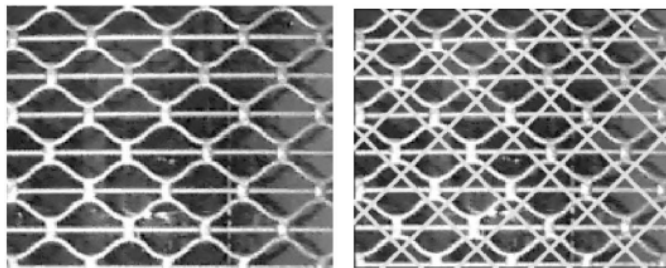 - A periodic pattern extended in two linearly independent directions to cover the 2D plane is known as a wallpaper pattern.
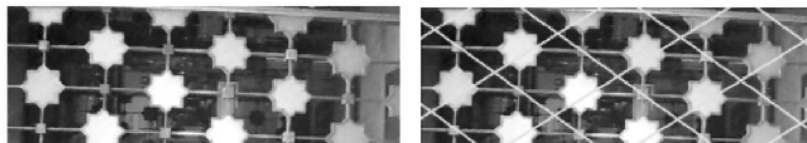
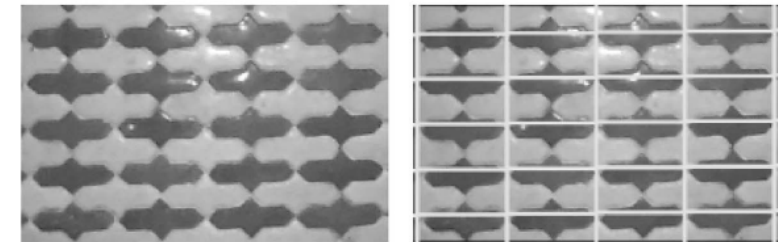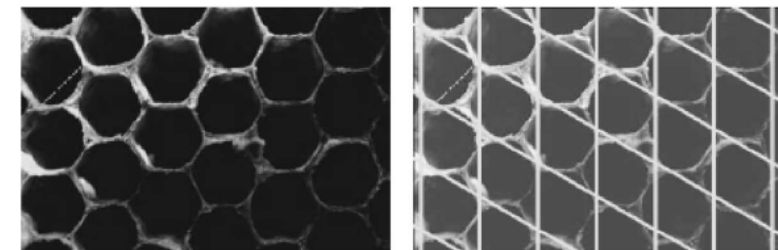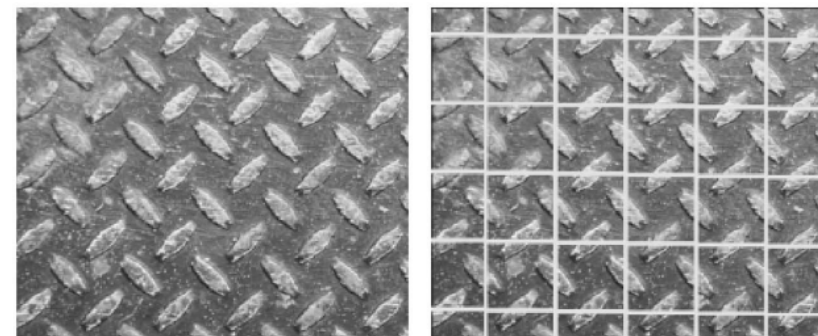# Structural approaches
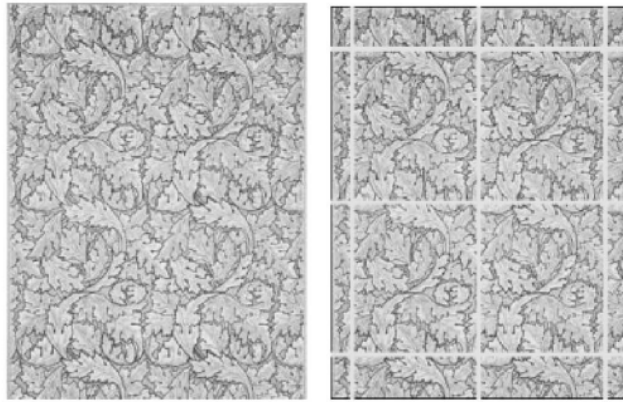
(a)
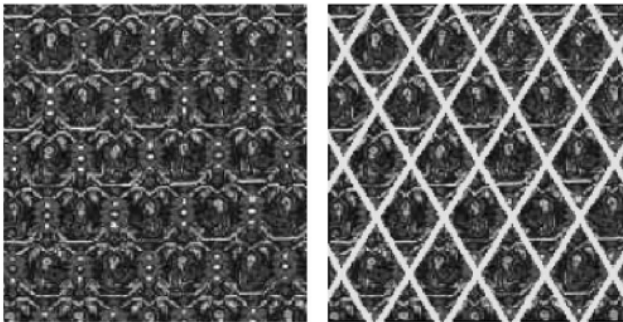
(b)

(c)

(d)

(a)

(b)

(c)

(d)

Statistical approaches

- Usually, segmenting out the texels is difficult or even impossible in real images.

- Instead, numeric quantities or statistics that describe a texture can be computed from the gray tones or colors themselves.

- This approach is less intuitive, but is computationally efficient and often works well.

Statistical approaches

- Some statistical approaches for texture:
    - Edge density and direction
    - Co-occurrence matrices
    - Local binary patterns
    - Statistical moments
    - Autocorrelation
    - Markov random fields
    - Autoregressive models
    - Mathematical morphology
    - Interest points
    - Fourier power spectrum
    - Gabor filters

Edge density and direction

- Use an edge detector as the first step in texture analysis.

- The number of edge pixels in a fixed-size region tells us how busy that region is.

- The directions of the edges also help characterize the texture.

Edge density and direction

- **Edge-based texture measures:**
  - Edgeness per unit area

    $$F_{edgeness} = | \{ p \mid gradient\_magnitude(p) \geq threshold \} | / N$$

    where N is the size of the unit area.
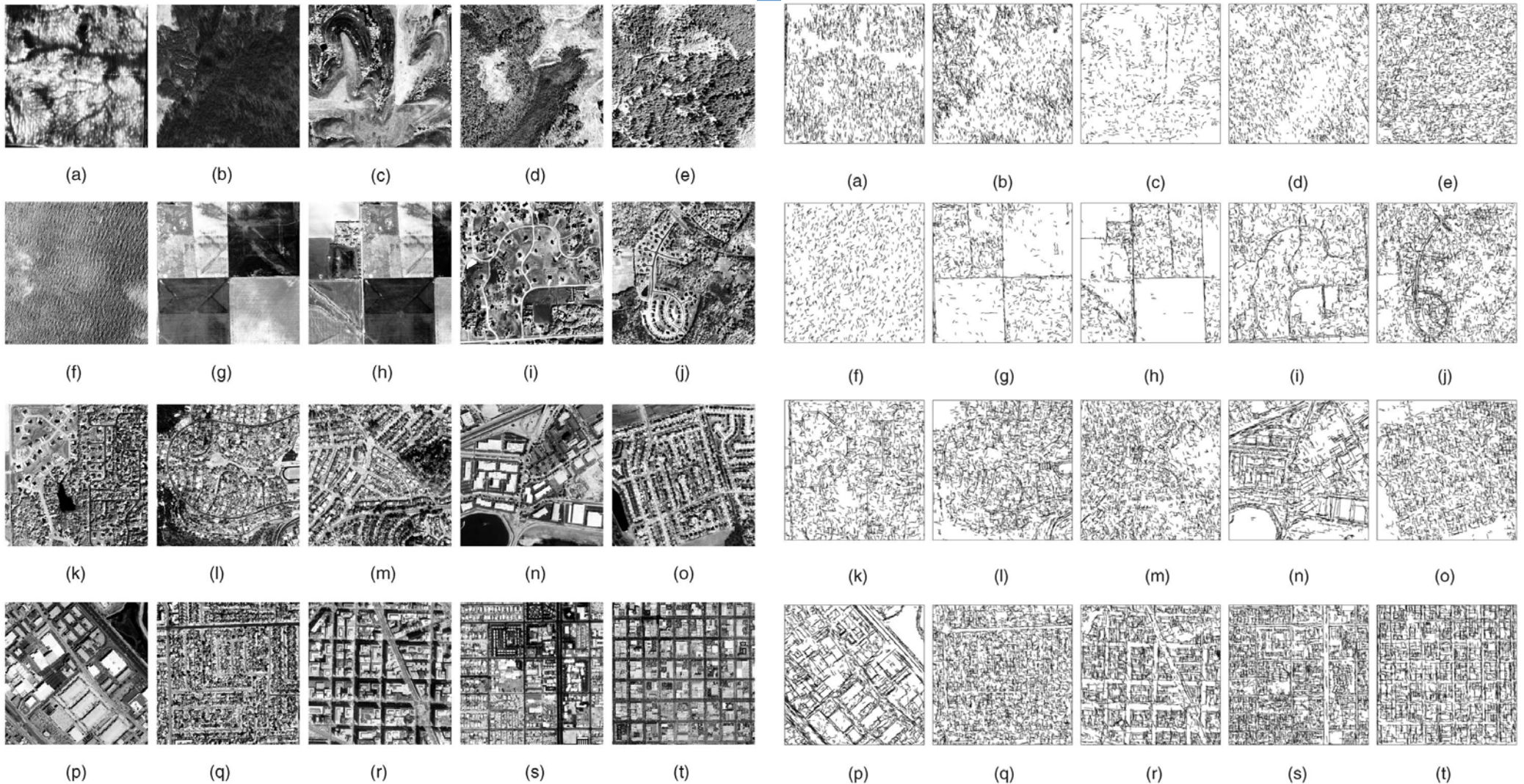
  - Edge magnitude and direction histograms

    $$F_{magdir} = (H_{magnitude}, H_{direction})$$

    where these are the normalized histograms of gradient magnitudes and gradient directions, respectively.

- Two histograms can be compared by computing their L_1 distance.

Edge texture

Satellite images sorted according to the amount of land development (left). Properties of the arrangements of line segments can be used to model the organization in an area (right).

Co-occurrence matrices

- Co-occurrence, in general form, can be specified in a matrix of relative frequencies $P(i, j; d, \theta)$ with which two texture elements separated by distance d at orientation θ occur in the image, one with property i and the other with property j.

- In gray level co-occurrence, as a special case, texture elements are pixels and properties are gray levels.

Co-occurrence matrices

- The spatial relationship can also be specified as a displacement vector (dr, dc) where dr is a displacement in rows and dc is a displacement in columns.

- For a particular displacement, the resulting square matrix can be normalized by dividing each entry by the number of elements used to compute that matrix.

Co-occurrence matrices

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

(a)

4x4 image
with gray levels
0-3.

		Gray Level			
		0	1	2	3
	0	#(0,0)	#(0,1)	#(0,2)	#(0,3)
Gray	1	#(1,0)	#(1,1)	#(1,2)	#(1,3)
Level	2	#(2,0)	#(2,1)	#(2,2)	#(2,3)
	3	#(3,0)	#(3,1)	#(3,2)	#(3,3)

(b)   General form   of   co-occurrence
matrices $P(i,j;d,\theta)$ for   gray   levels
0-3   where $\#(i,j)$ stands   for   number
of   times   gray   levels i and j have   been
neighbors.

$$P(i,j;1,0^\circ) = \begin{pmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 0 \\ 1 & 0 & 6 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

(c) $(d,\theta) = (1,0^\circ)$

$$P(i,j;1,45^\circ) = \begin{pmatrix} 2 & 1 & 3 & 0 \\ 1 & 2 & 1 & 0 \\ 3 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

(d) $(d,\theta) = (1,45^\circ)$

$$P(i,j;1,90^\circ) = \begin{pmatrix} 6 & 0 & 2 & 0 \\ 0 & 4 & 2 & 0 \\ 2 & 2 & 2 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

(e) $(d,\theta) = (1,90^\circ)$

$$P(i,j;1,135^\circ) = \begin{pmatrix} 4 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

(f) $(d,\theta) = (1,135^\circ)$

Co-occurrence matrices

	1	1	0	0
	1	1	0	0
	0	0	2	2
	0	0	2	2

Image I

$$
\begin{array}{c c}
 & j \\
i & \begin{array}{c c c}
 & 0 \quad 1 \quad 2 \\
0 & 4 \quad 0 \quad 2 \\
1 & 2 \quad 2 \quad 0 \\
2 & 0 \quad 0 \quad 2
\end{array}
\end{array}
$$

$C_{(0,1)}$

i	j

$$
\begin{array}{c c c}
 & 0 \quad 1 \quad 2 \\
0 & 4 \quad 0 \quad 2 \\
1 & 2 \quad 2 \quad 0 \\
2 & 0 \quad 0 \quad 2
\end{array}
$$

$C_{(1,0)}$

i
j

$$
\begin{array}{c c c}
 & 0 \quad 1 \quad 2 \\
0 & 2 \quad 0 \quad 2 \\
1 & 2 \quad 1 \quad 1 \\
2 & 0 \quad 0 \quad 1
\end{array}
$$

$C_{(1,1)}$

Co-occurrence matrices

- If a texture is coarse and the distance d used to compute the co-occurrence matrix is small compared to the sizes of the texture elements, pairs of pixels at separation d should usually have similar gray levels.

- This means that high values in the matrix $P(i, j; d, \theta)$ should be concentrated on or near its main diagonal.

- Conversely, for a fine texture, if d is comparable to the texture element size, then the gray levels of points separated by d should often be quite different, so that values in $P(i, j; d, \theta)$ should be spread out relatively uniformly.

Co-occurrence matrices

- Similarly, if a texture is directional, i.e., coarser in one direction than another, the degree of spread of the values about the main diagonal in $P(i, j; d, \theta)$ should vary with the orientation θ.

- Thus texture directionality can be analyzed by comparing spread measures of $P(i, j; d, \theta)$ for various orientations.

Co-occurrence matrices

Grayscale image

(a,d) = (0,1)   (a,d) = (0,2)

(a,d) = (0,3)   (a,d) = (0,4)
Co-occurrence matrices
(a,d) = (orientation,distance)

Grayscale image

(a,d) = (0,1)   (a,d) = (0,2)

(a,d) = (0,3)   (a,d) = (0,4)
Co-occurrence matrices
(a,d) = (orientation,distance)
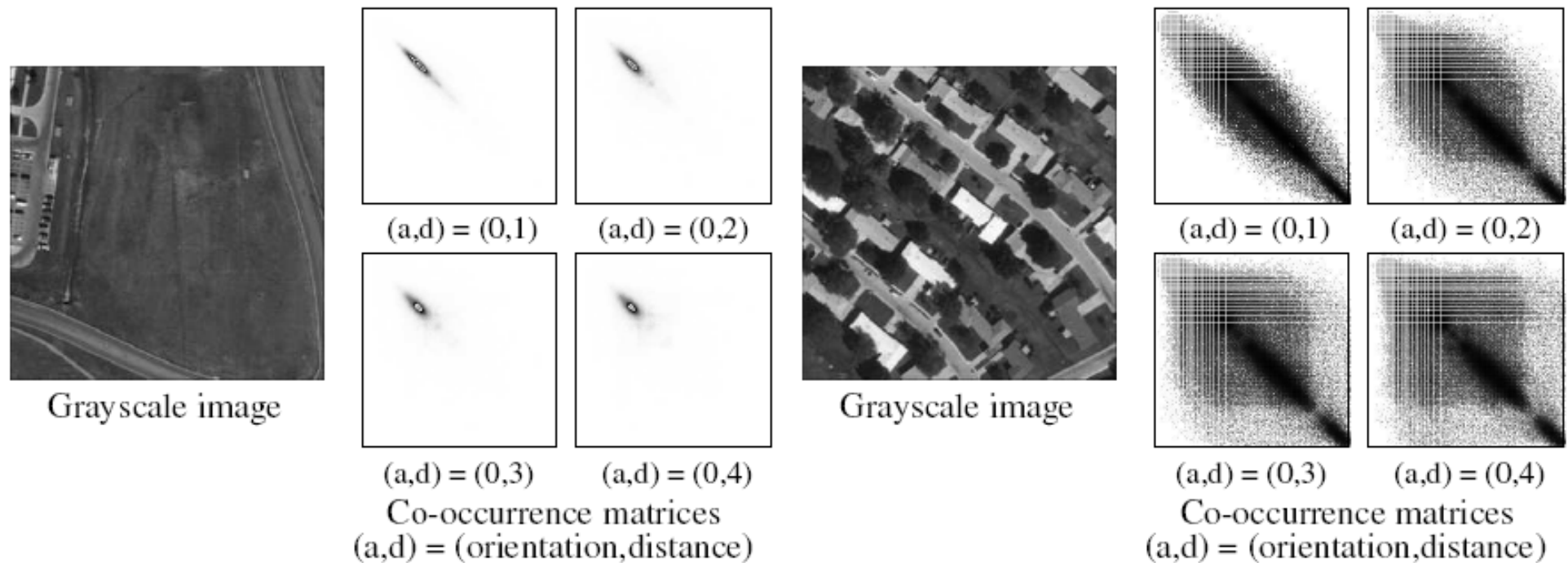
(a) Co-occurrence matrices for an image with a small amount of local spatial variations.

(b) Co-occurrence matrices for an image with a large amount of local spatial variations.

Figure 4. Example co-occurrence matrices.

Co-occurrence matrices

- In order to use the information contained in co-occurrence matrices, Haralick et al. (SMC 1973) defined 14 statistical features that capture textural characteristics such as homogeneity, contrast, organized structure, and complexity.

- Zucker and Terzopoulos (CGIP 1980) suggested using a chi-square statistical test to select the values of d that have the most structure for a given class of images.

$$\chi^2(d) = (\sum_i \sum_j \frac{N_d^2(i,j)}{N_d(i)N_d(j)} - 1)$$

Co-occurrence matrices

$$Energy \;\; = \;\; \sum_i \sum_j N_d^2(i,j)$$

$$Entropy \;\; = \;\; -\sum_i \sum_j N_d(i,j) log_2 N_d(i,j)$$

$$Contrast \;\; = \;\; \sum_i \sum_j (i-j)^2 N_d(i,j)$$

$$Homogeneity \;\; = \;\; \sum_i \sum_j \frac{N_d(i,j)}{1+ \mid i-j \mid}$$

$$Correlation \;\; = \;\; \frac{\sum_i \sum_j (i-\mu_i)(j-\mu_j)N_d(i,j)}{\sigma_i \sigma_j}$$

where μ_i, μ_j are the means and σ_i, σ_j are the standard deviations of the row and column sums $N_d(i)$ and $N_d(j)$ defined by

$$N_d(i) \;\; = \;\; \sum_j N_d(i,j)$$

$$N_d(j) \;\; = \;\; \sum_i N_d(i,j)$$

Co-occurrence matrices
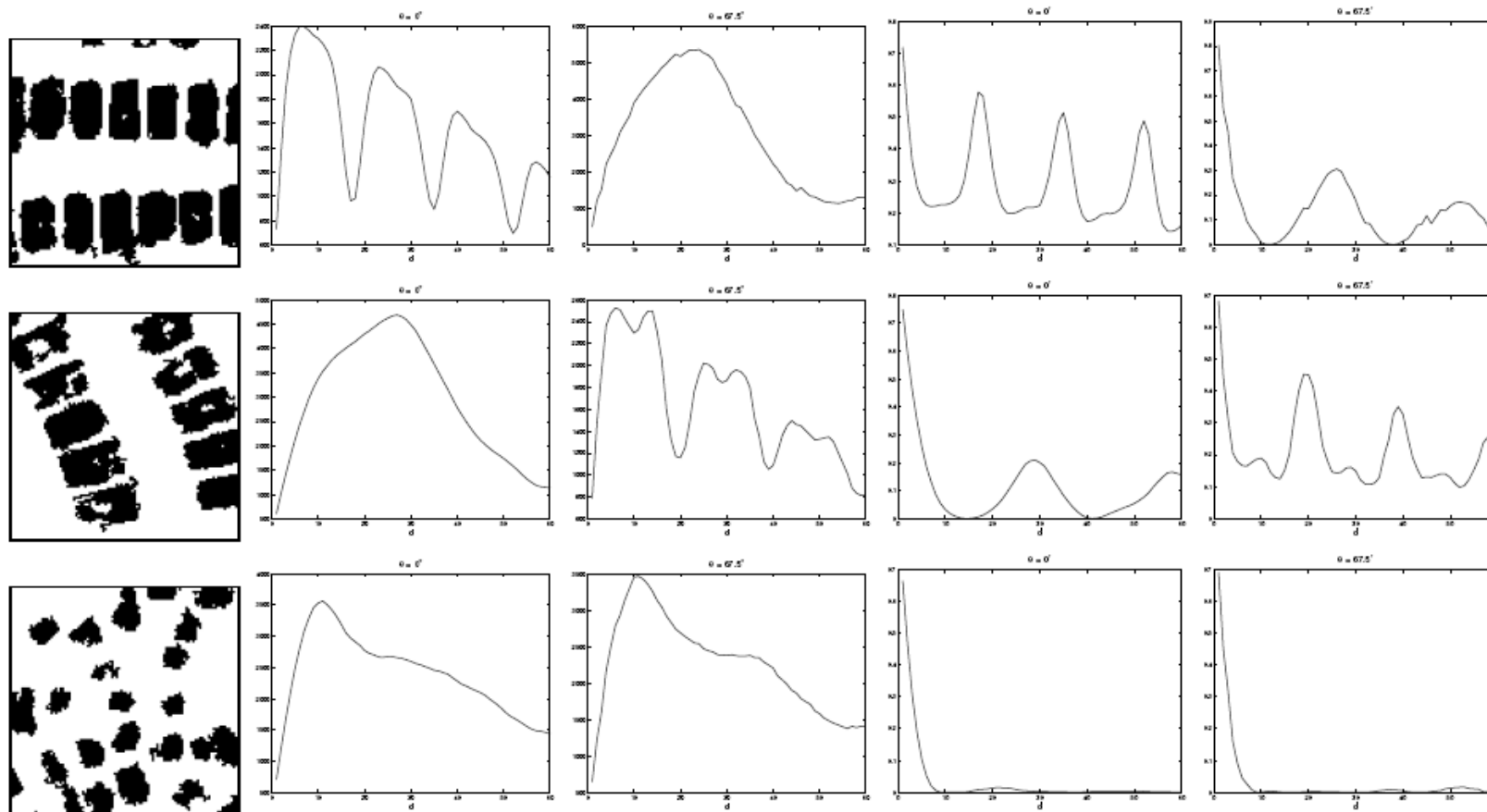
Example building groups (first column), the contrast features for 0 and 67.5 degree orientations (second and third columns), and the chi-square features for 0 and 67.5 degree orientations (fourth and fifth columns). X-axes represent inter-pixel distances of 1 to 60. The features at a particular orientation exhibit a periodic structure as a function of distance if the neighborhood contains a regular arrangement of buildings along that direction. On the other hand, features are very similar for different orientations if there is no particular arrangement in the neighborhood.

Local binary patterns

- For each pixel p, create an 8-bit number $b_1 \ b_2 \ b_3 \ b_4 \ b_5 \ b_6 \ b_7 \ b_8$, where $b_i = 0$ if neighbor i has value less than or equal to p's value and 1 otherwise.

- Represent the texture in the image (or a region) by the histogram of these numbers.

	1	2	3	
	100	101	103	
8	40	50	80	4
	50	60	90	
	7	6	5	

→ | 1 1 1 1 1 1 0 0 |
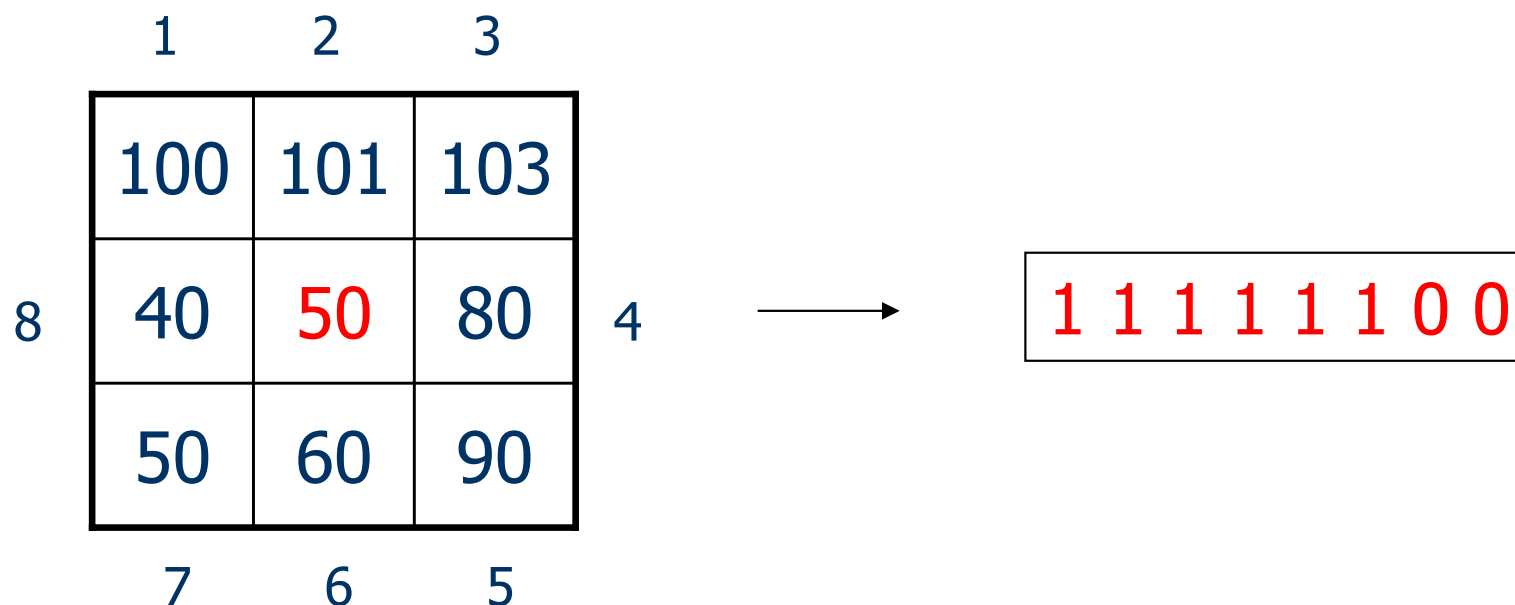
Local binary patterns

- The fixed neighborhoods were later extended to multi-scale circularly symmetric neighbor sets.
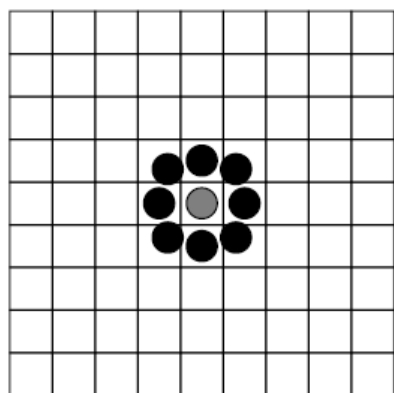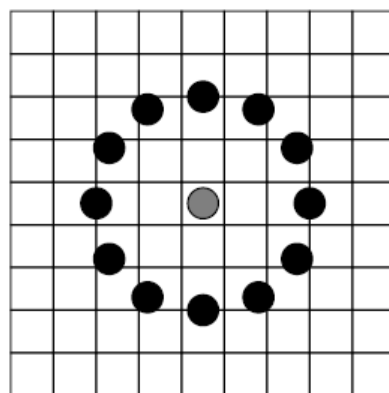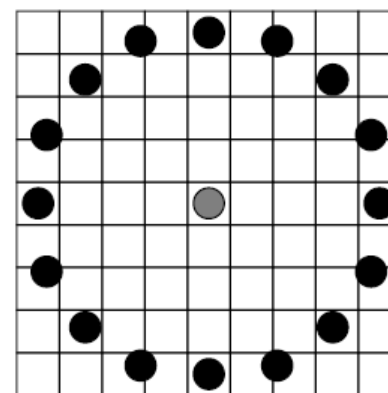
P=8, R=1.0   P=12, R=2.5   P=16, R=4.0

- Texture primitives detected by the LBP:
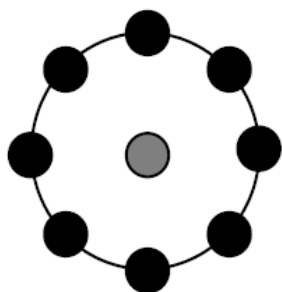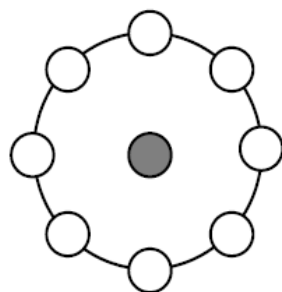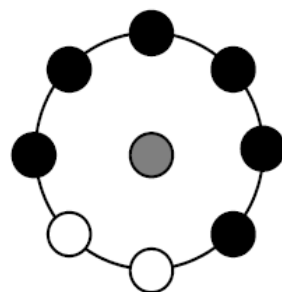
Spot   Spot/flat   Line end   Edge   Corner
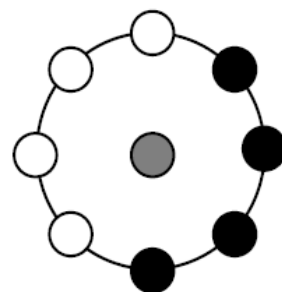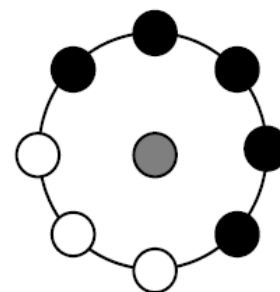
Autocorrelation

- The autocorrelation function of an image can be used to
  - detect repetitive patterns of texture elements, and
  - describe the fineness/coarseness of the texture.

- The autocorrelation function $\rho(dr,dc)$ for displacement d=(dr,dc) is given by

$$\rho(dr, dc) = \frac{\sum_{r=0}^{N} \sum_{c=0}^{N} I[r,c]I[r+dr,c+dc]}{\sum_{r=0}^{N} \sum_{c=0}^{N} I^2[r,c]}$$

$$= \frac{I[r,c] \circ I_d[r,c]}{I[r,c] \circ I[r,c]}$$

Autocorrelation

- Interpreting autocorrelation:
  - Coarse texture → function drops off slowly
  - Fine texture → function drops off rapidly
  - Can drop differently for r and c
  - Regular textures → function will have peaks and valleys; peaks can repeat far away from [0,0]
  - Random textures → only peak at [0,0]; breadth of peak gives the size of the texture

Fourier power spectrum

- The autocorrelation function is related to the power spectrum of the Fourier transform.

- The power spectrum contains texture information because

  - prominent peaks in the spectrum give the principal direction of the texture patterns,

  - location of the peaks gives the fundamental spatial period of the patterns.

Fourier power spectrum

- The power spectrum, represented in polar coordinates, can be integrated over regions bounded by circular rings (for frequency content) and wedges (for orientation content).
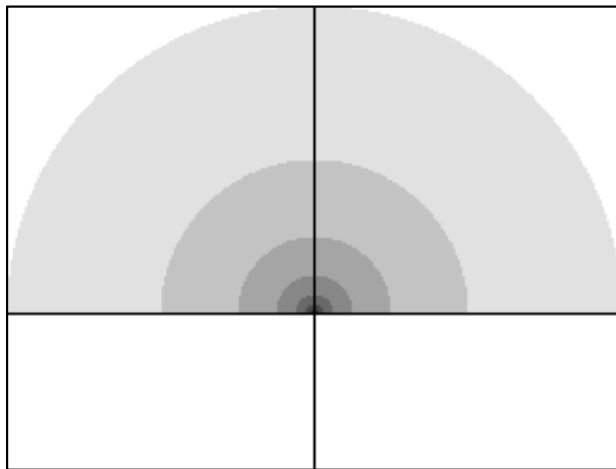
$$x_i = \sum_{r=r_i}^{r_{i+1}} \sum_{\theta=0}^{\pi} S(r, \theta)$$
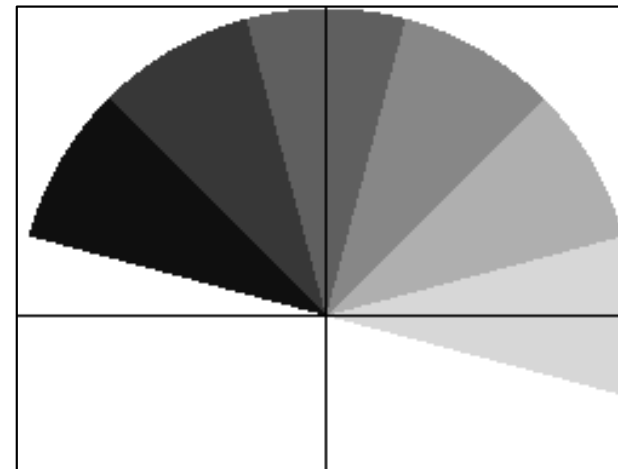
$$y_i = \sum_{\theta=\theta_i}^{\theta_{i+1}} \sum_{r=1}^{r_{max}} S(r, \theta)$$

Fourier power spectrum

FIGURE 11.24 (a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$. (Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)

a b
c d
e f
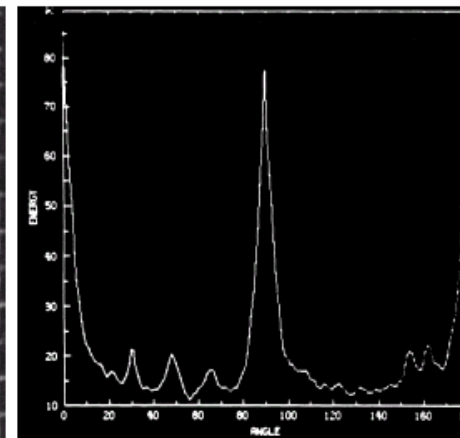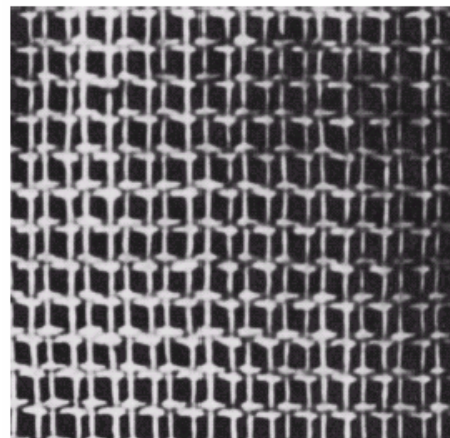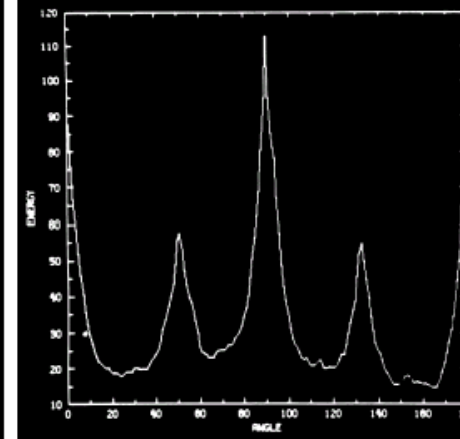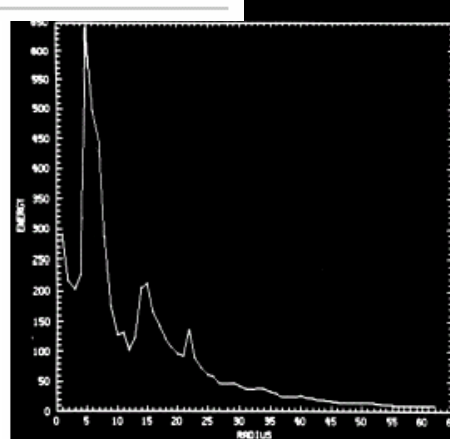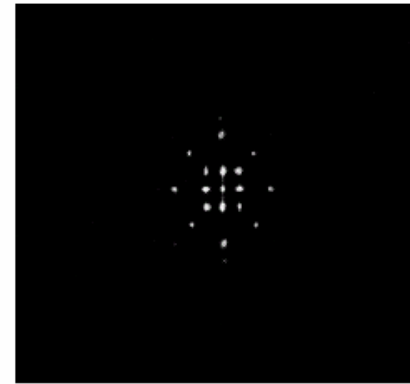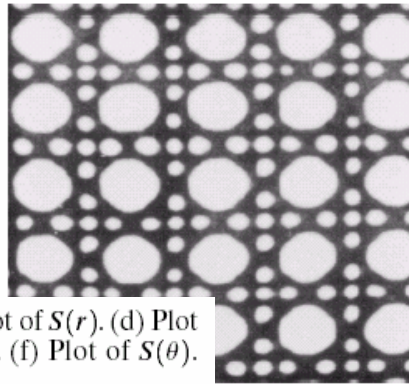
Fourier power spectrum
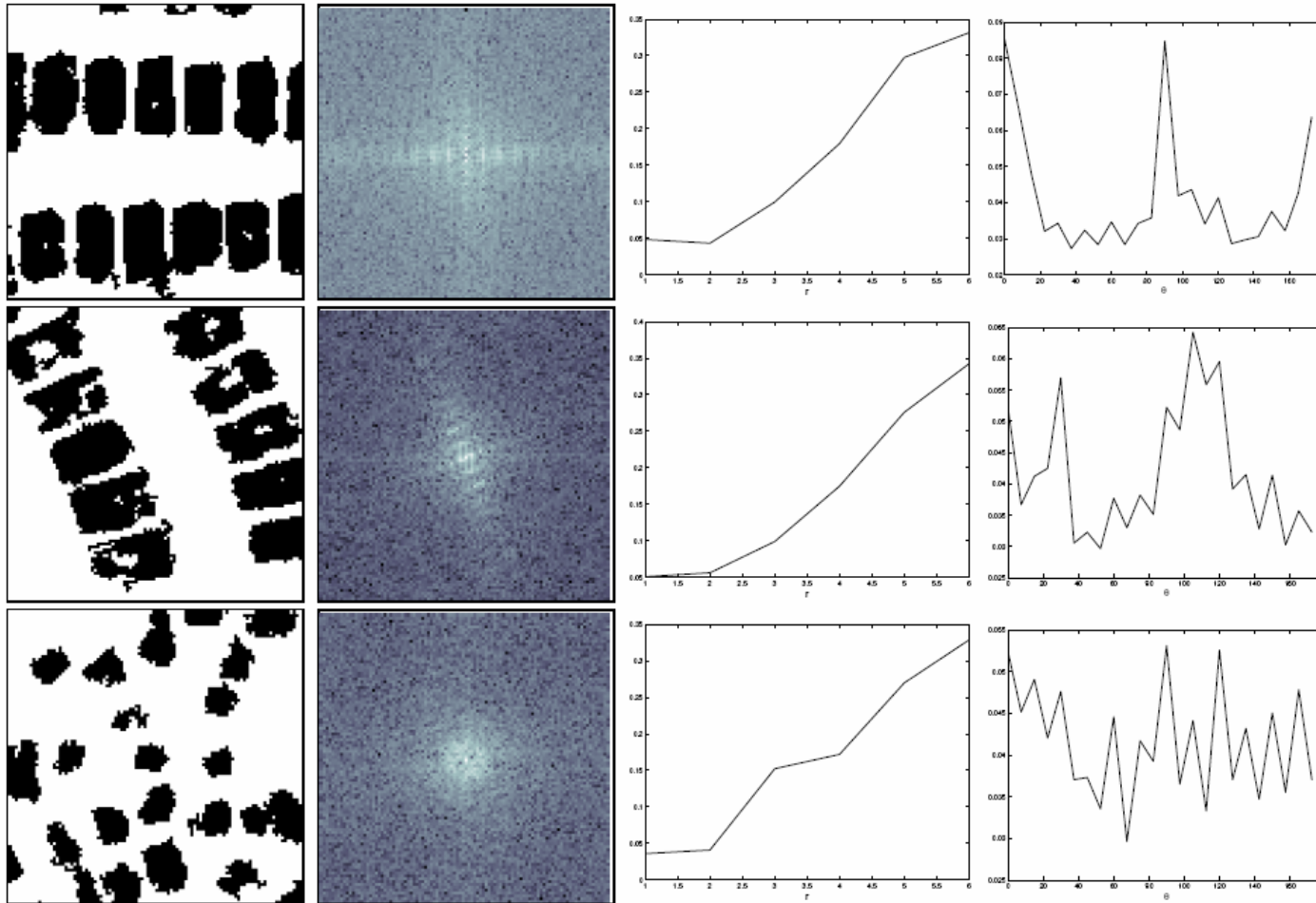
Example building groups (first column), Fourier spectrum of these images (second column), and the corresponding ring- and wedge-based features (third and fourth columns). X-axes represent the rings in the third column and the wedges in the fourth column plots. The peaks in the features correspond to the periodicity and directionality of the buildings, whereas no dominant peaks can be found when there is no regular building pattern.

Gabor filters

- The Gabor representation has been shown to be optimal in the sense of minimizing the joint two-dimensional uncertainty in space and frequency.

- These filters can be considered as orientation and scale tunable edge and line detectors.

- Fourier transform achieves localization in either spatial or frequency domain but the Gabor transform achieves simultaneous localization in both spatial and frequency domains.

Gabor filters

- A 2D Gabor function g(x,y) and its Fourier transform G(u,v) can be written as

$$g(x,y) = \left(\frac{1}{2\pi\sigma_x\sigma_y}\right)\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right) + 2\pi j W x\right]$$

$$G(u,v) = \exp\left\{-\frac{1}{2}\left[\frac{(u-W)^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2}\right]\right\}$$

where $\sigma_u = 1/2\pi\sigma_x$ and $\sigma_v = 1/2\pi\sigma_y$.

Gabor filters

- Let U_l and U_h denote the lower and upper center frequencies of interest, K be the number of orientations, and S be the number of scales, the filter parameters can be selected as

$$a = \left(U_h / U_l\right)^{\frac{1}{S-1}}, \quad \sigma_u = \frac{(a-1)U_h}{(a+1)\sqrt{2\ln 2}},$$

$$\sigma_v = \tan\left(\frac{\pi}{2k}\right)\left[U_h - 2\ln\left(\frac{2\sigma_u^2}{U_h}\right)\right]\left[2\ln 2 - \frac{(2\ln 2)^2 \sigma_u^2}{U_h^2}\right]^{-\frac{1}{2}}$$

where $W = U_h$ and $m = 0, 1, \ldots, S-1$.

Gabor filters
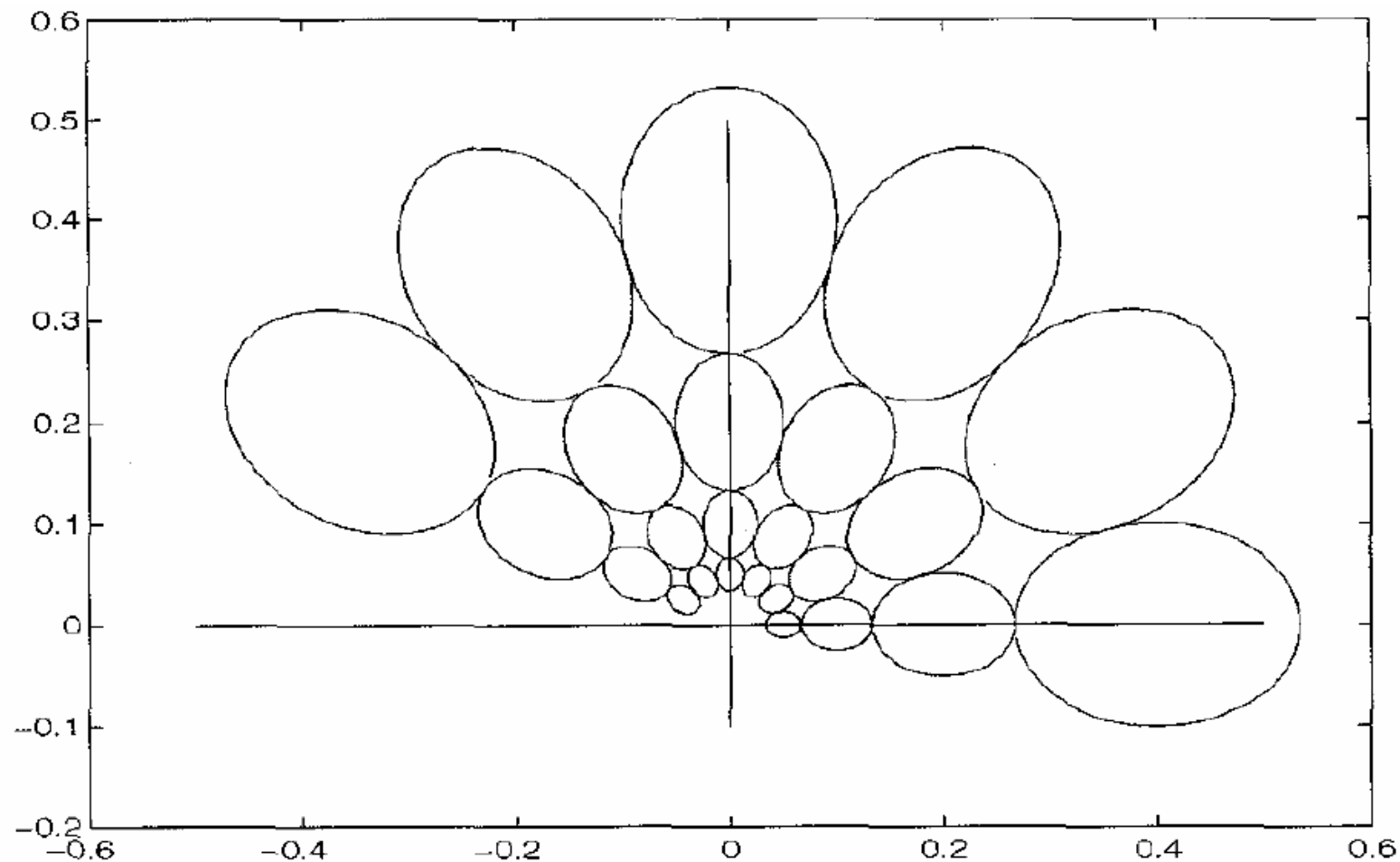
Fig. 1. The contours indicate the half-peak magnitude of the filter responses in the Gabor filter dictionary. The filter parameters used are $U_h = 0.4$, $U_l = 0.05$, $K = 6$, and $S \approx 4$.

Gabor filters

Filters at multiple scales and orientations.

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Gabor filters

$$e^{-\frac{x^2+y^2}{2\sigma^2}} \cos\left(2\pi\left(k_x x + k_y y\right)\right)$$
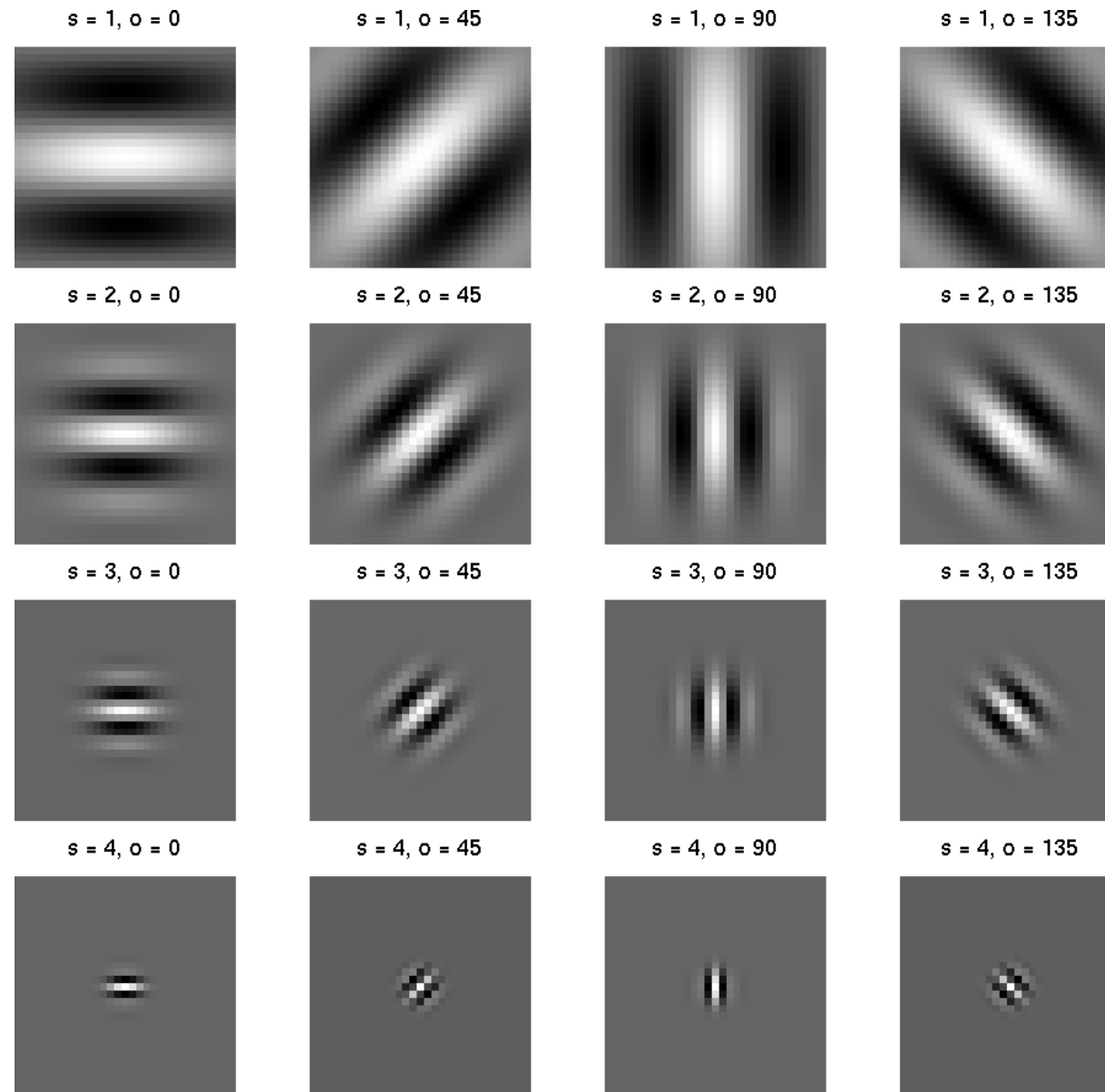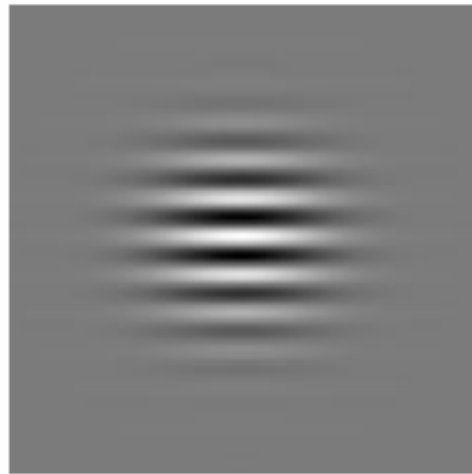
High frequency along axis
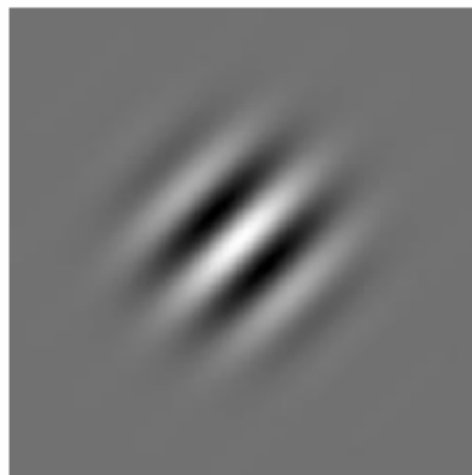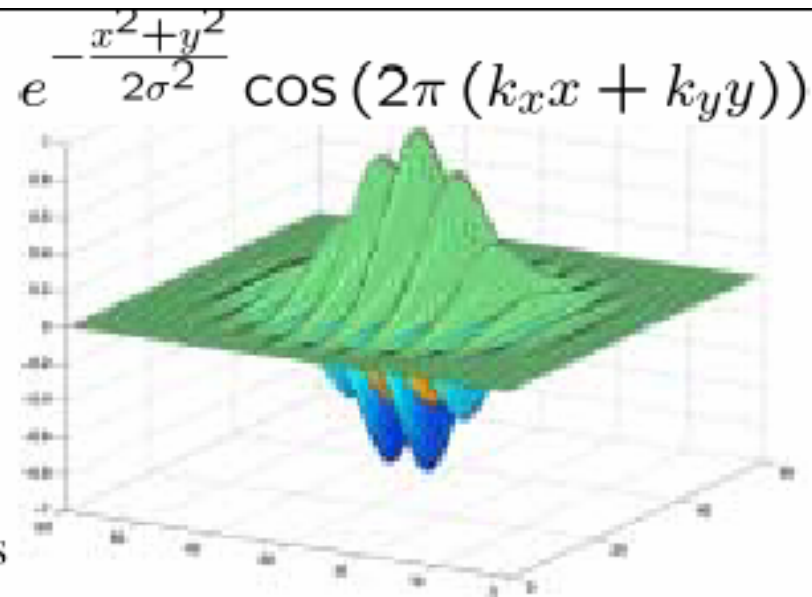
Lower frequency
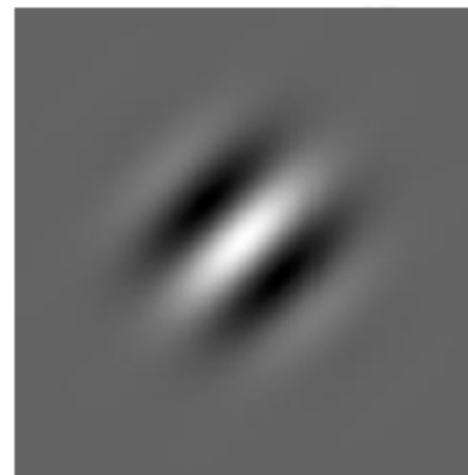
Even lower frequency

Gabor filters
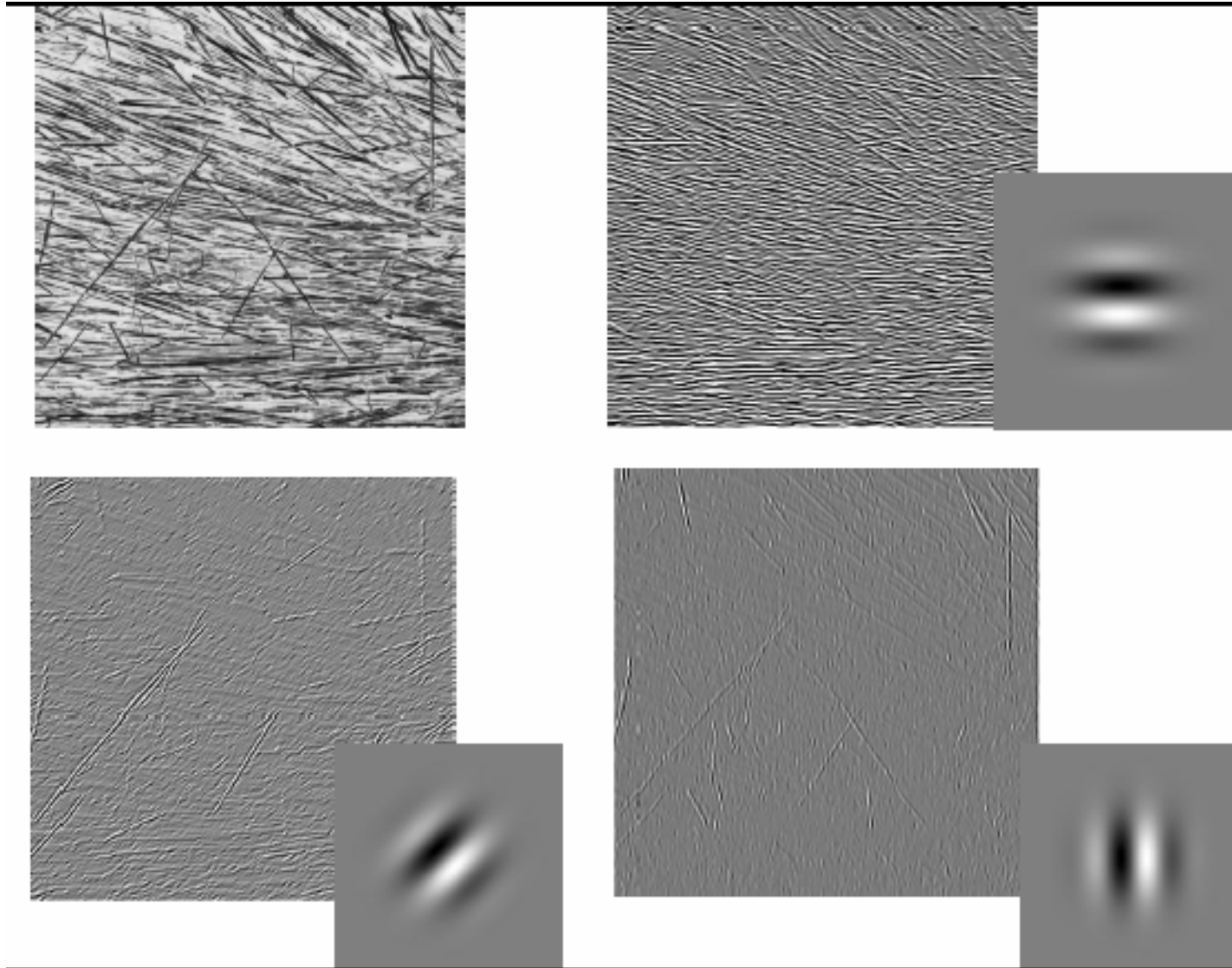
Gabor filter responses for an example image.

Gabor filters

Gabor filter responses for a satellite image.

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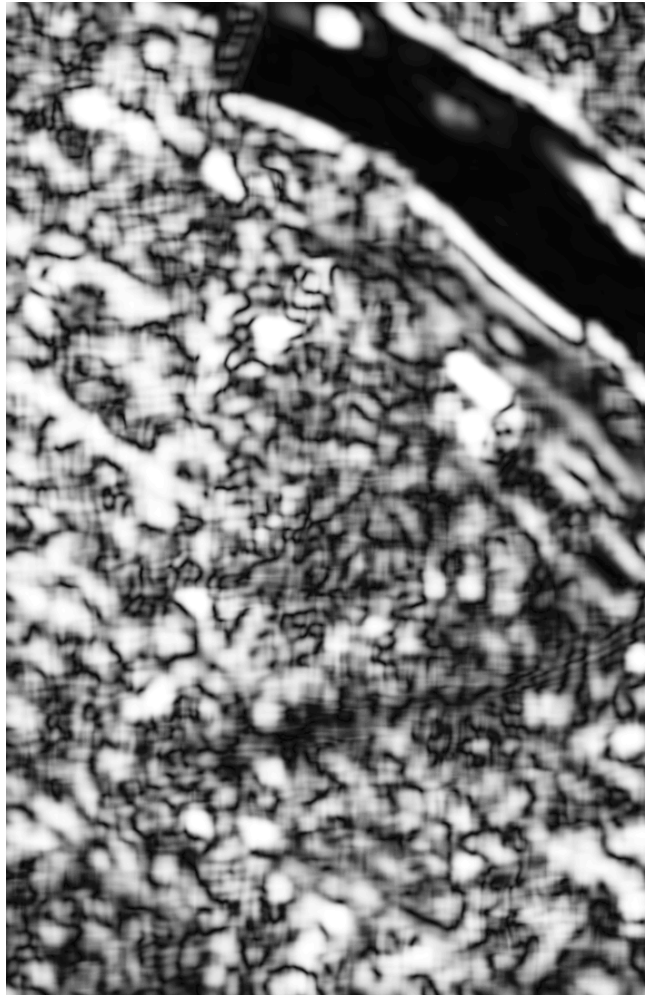
Gabor filters

Gabor filter responses for a satellite image.
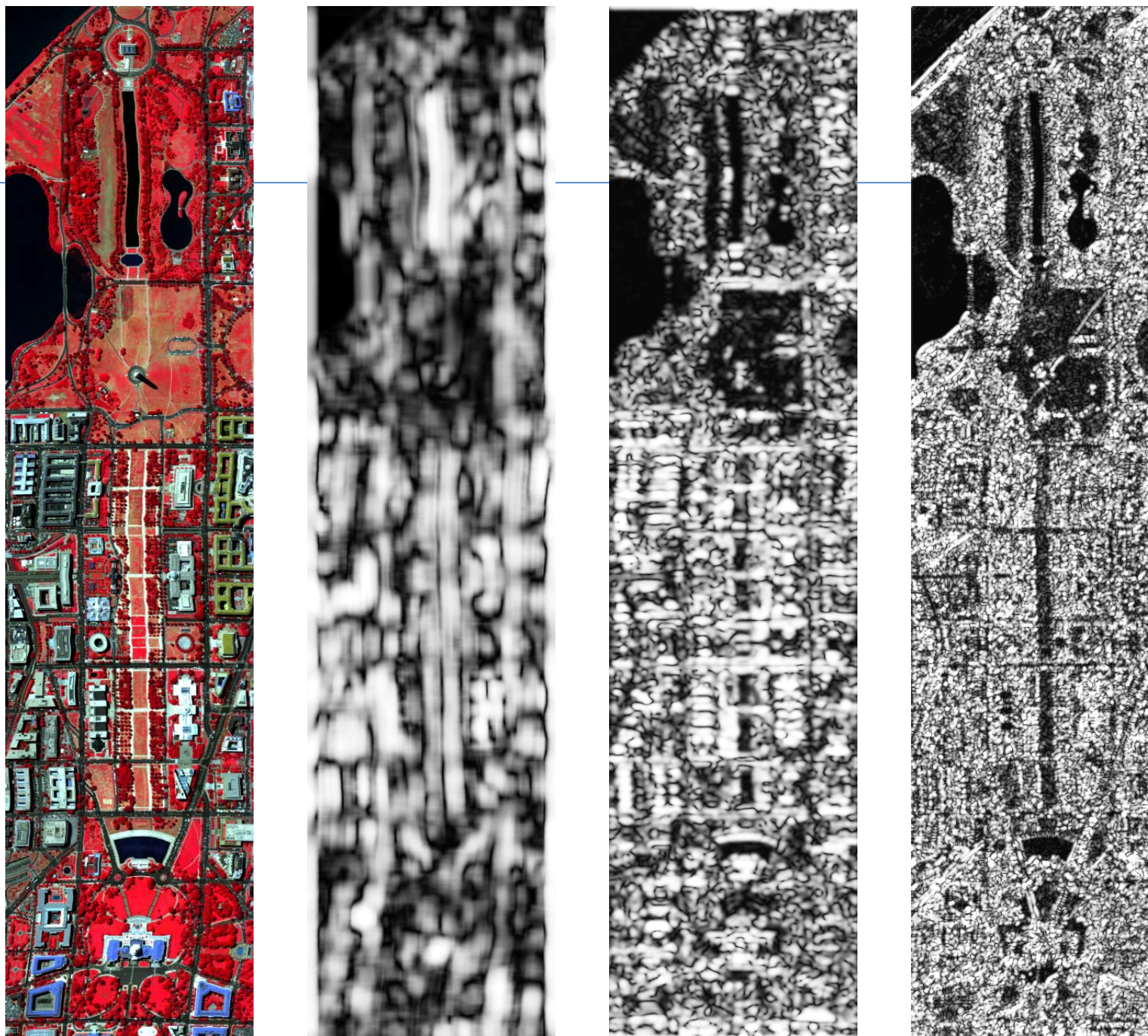
Gabor filter responses for a satellite image.
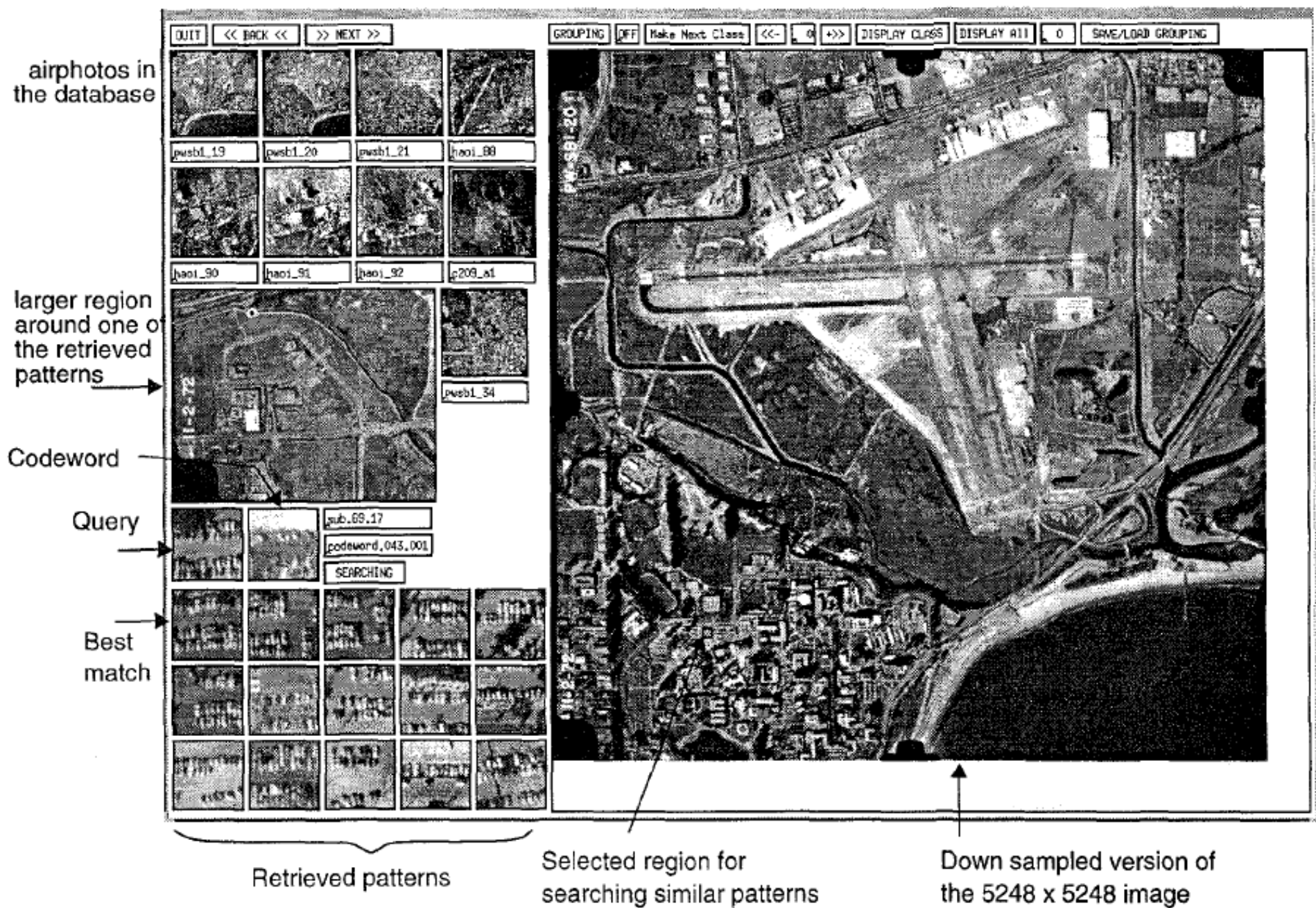
Figure 4: Snapshot of an aerial photograph browsing demonstration. The example shown indicates a query pattern containing a parking lot. Next to the query is the image codeword used to index the database. The browser can retrieve almost 99% of all the parking lots in the aerial photo database.

Query pattern

Codeword

Retrieval

Selected image tile

#1   #2   #3

#4   #5   #9
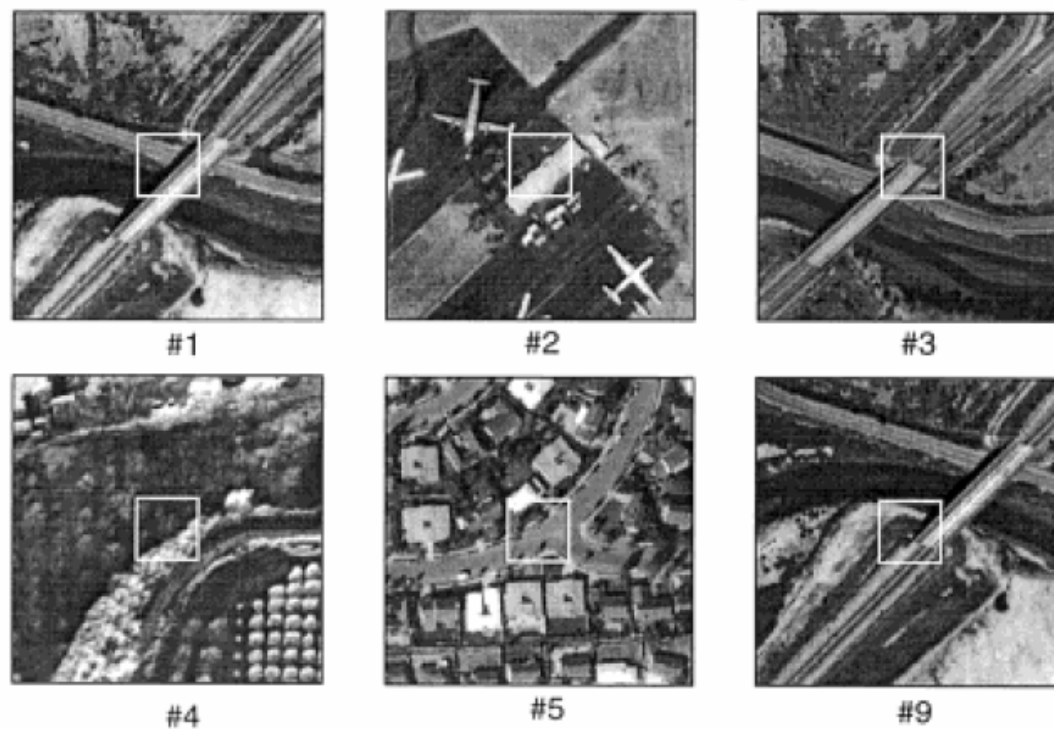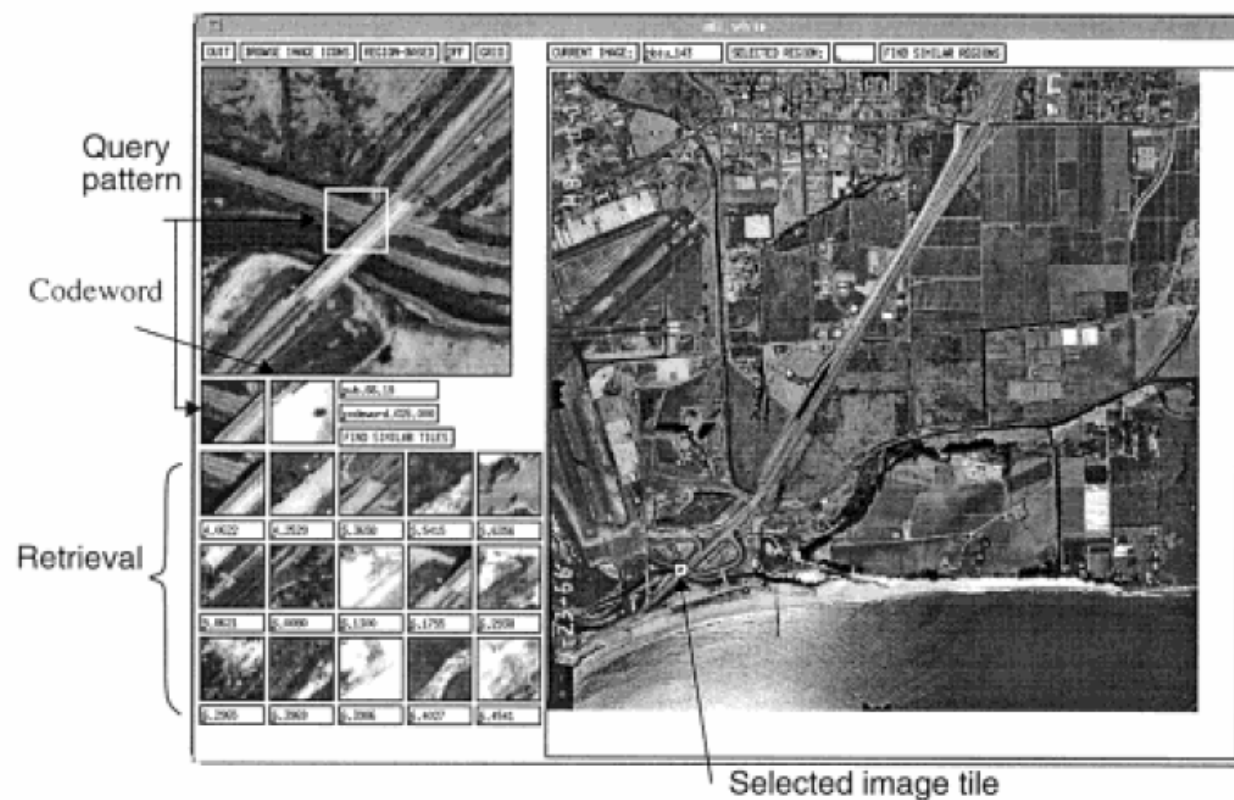
Full resolution of some retrieved tiles

Gabor filters
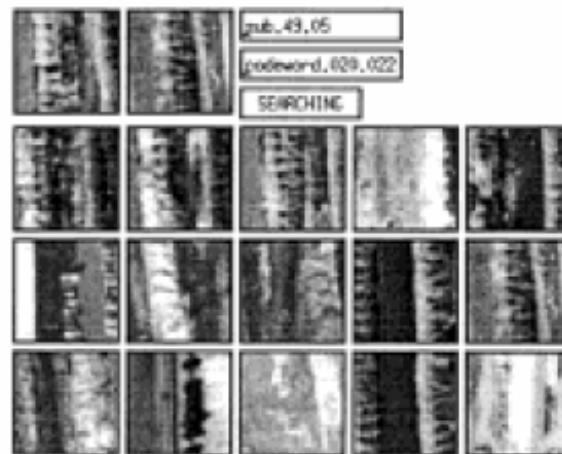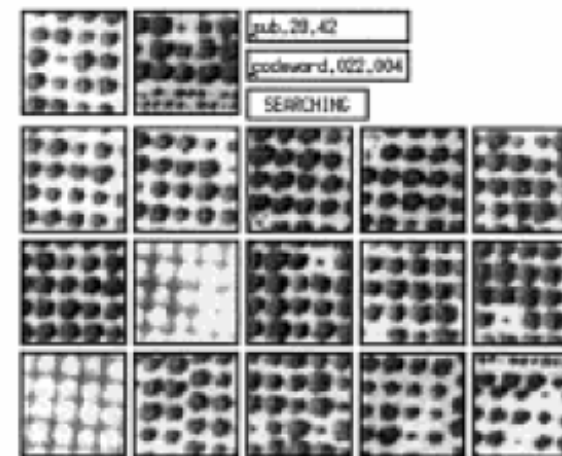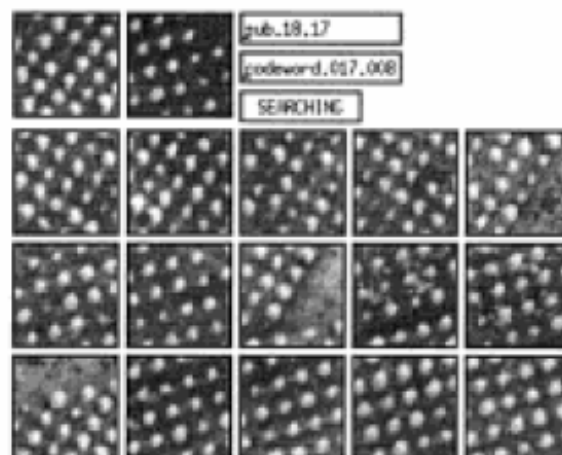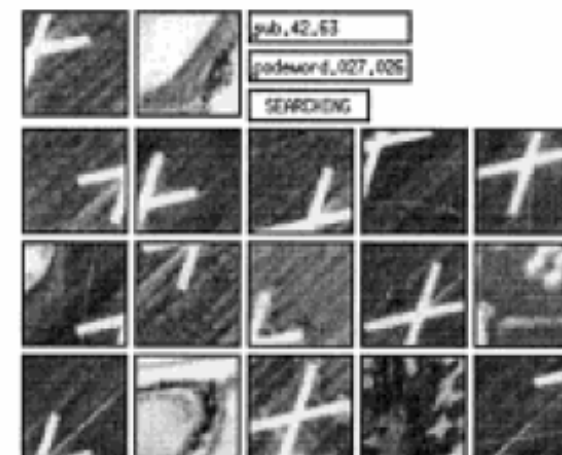
Texture synthesis

- Goal of *texture analysis*: compare textures and decide if they are similar.

- Goal of *texture synthesis*: construct large regions of texture from small example images.

- It is an important problem for rendering in computer graphics.

- Strategy: to think of a texture as a sample from some probability distribution and then to try and obtain other samples from that same distribution.

Neighborhood window
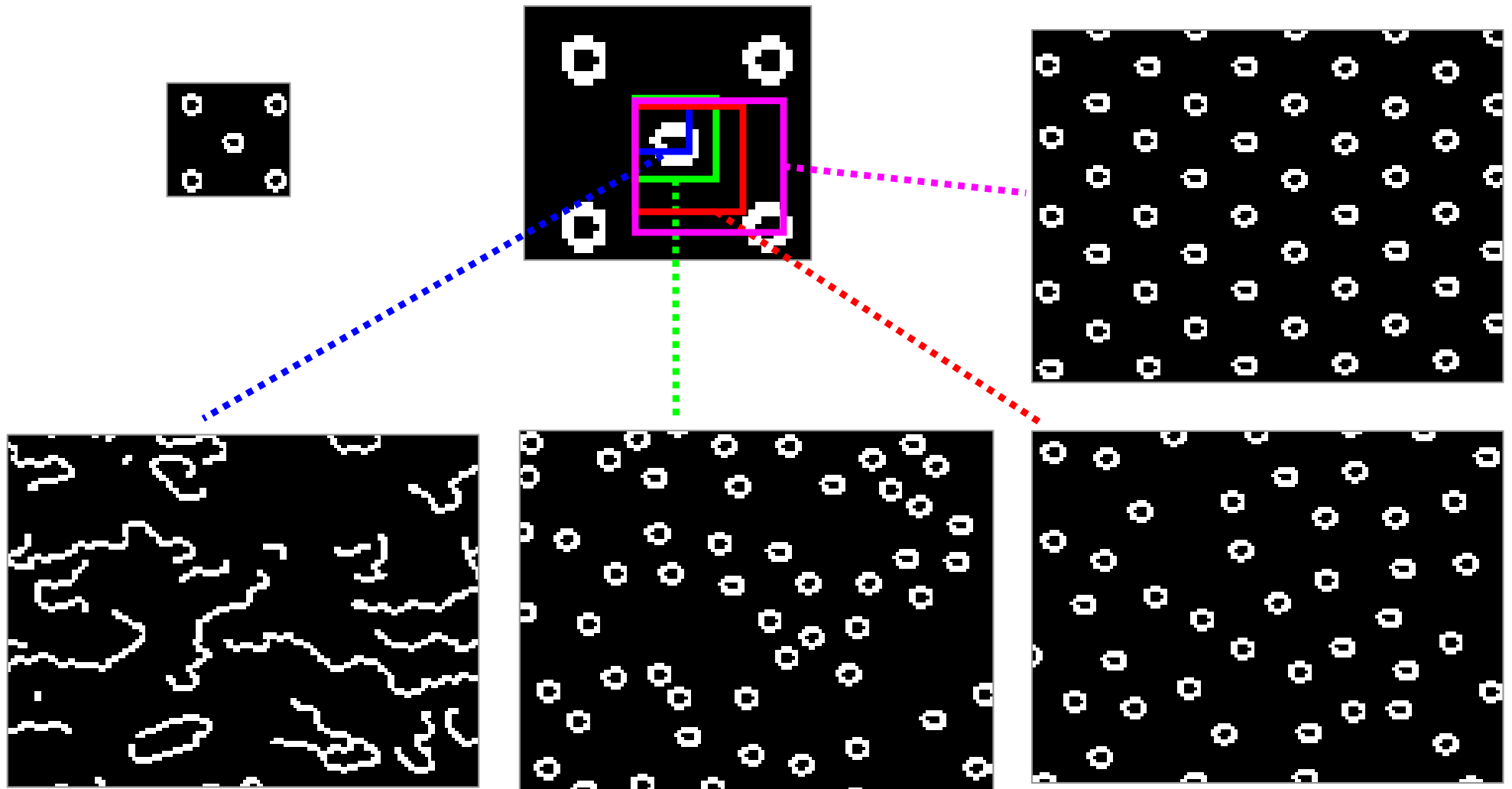
Varying window size
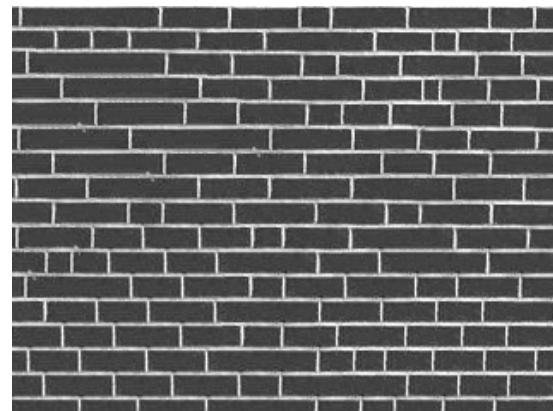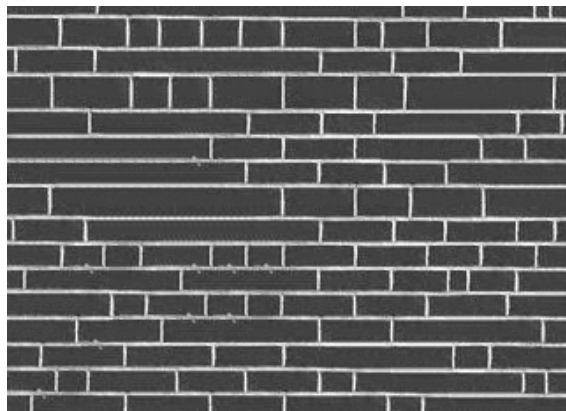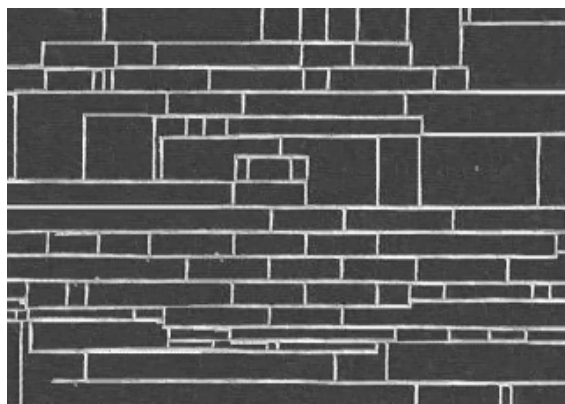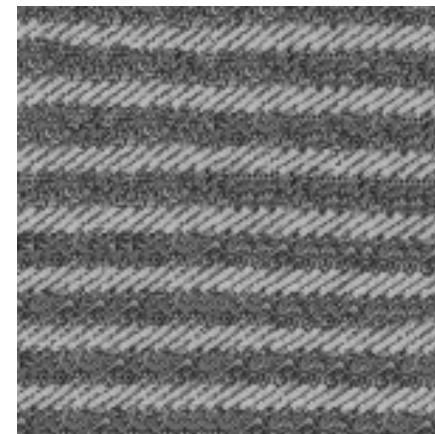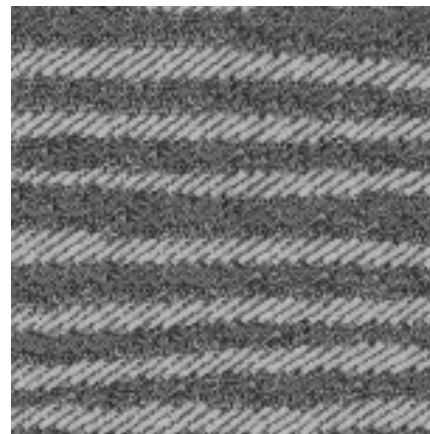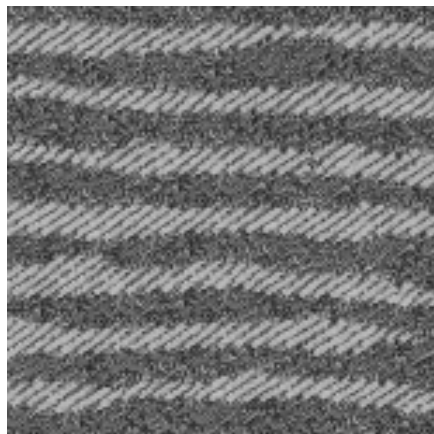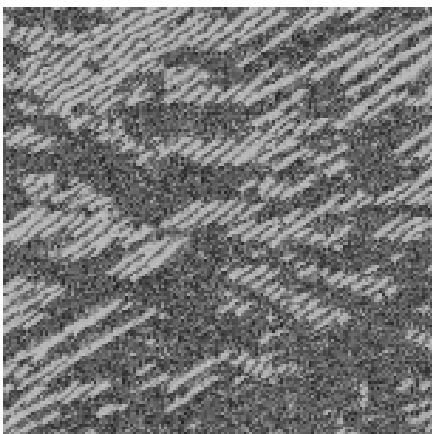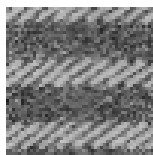
Increasing window size →

Examples
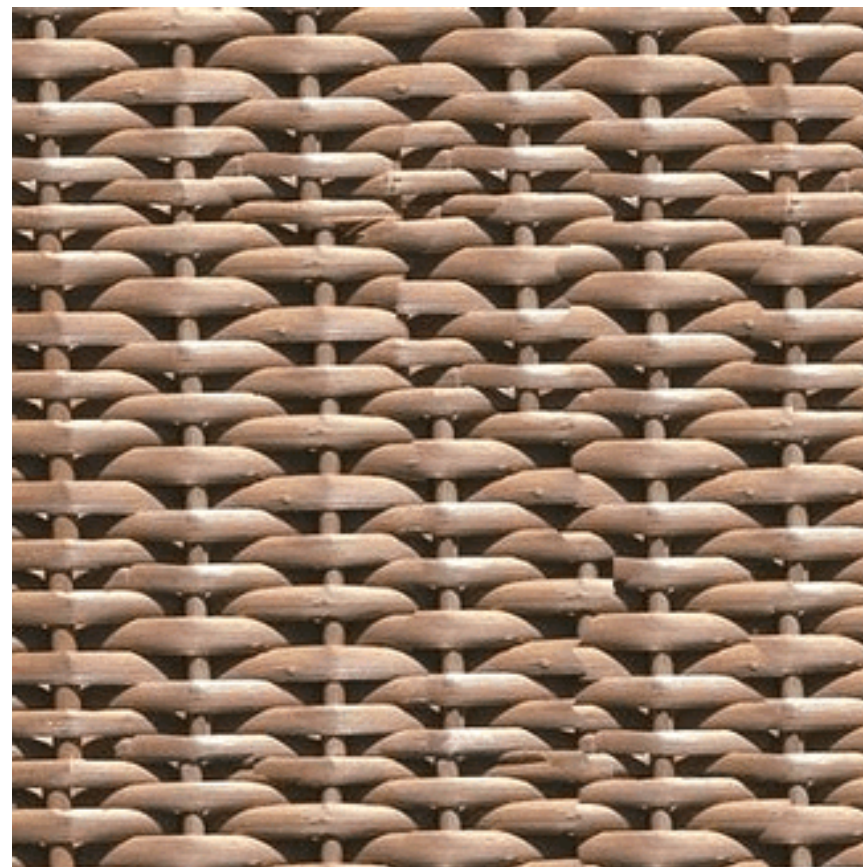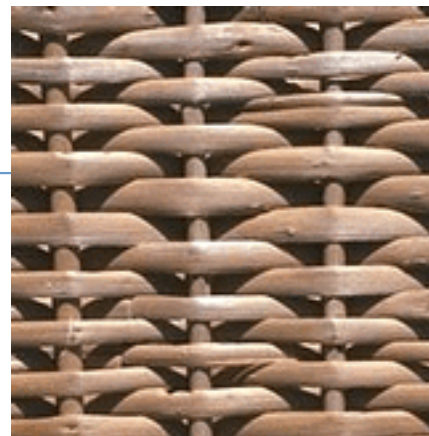
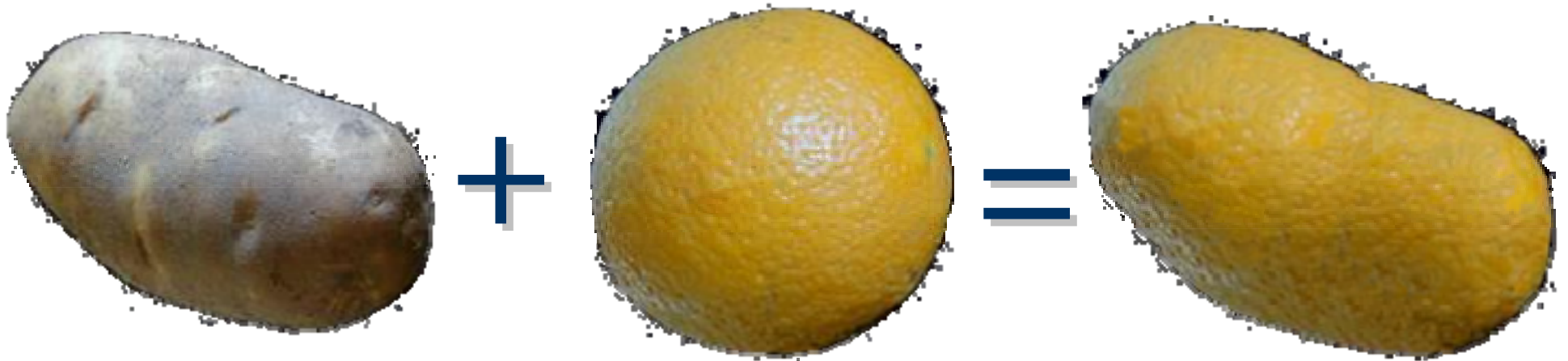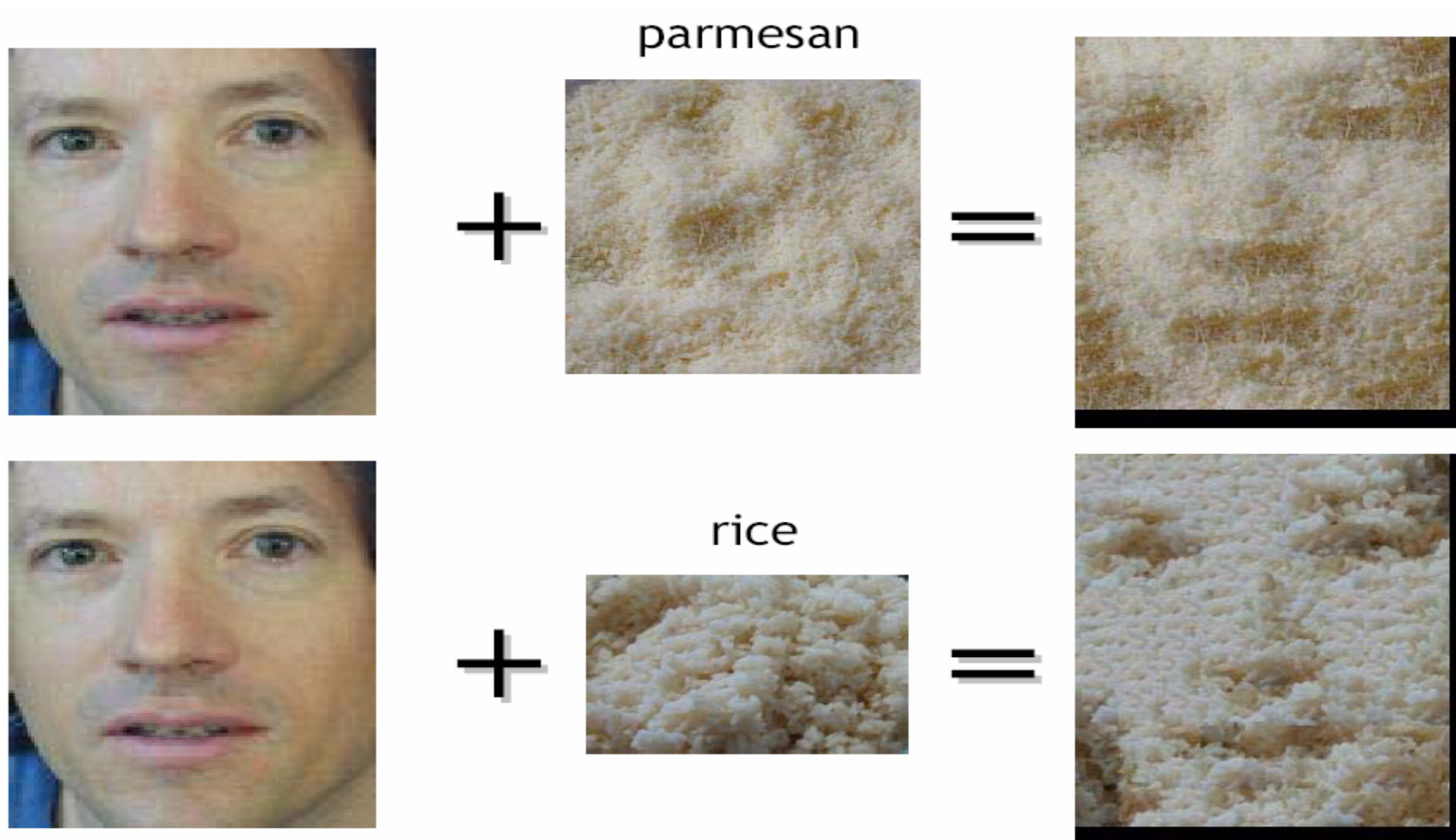# Examples
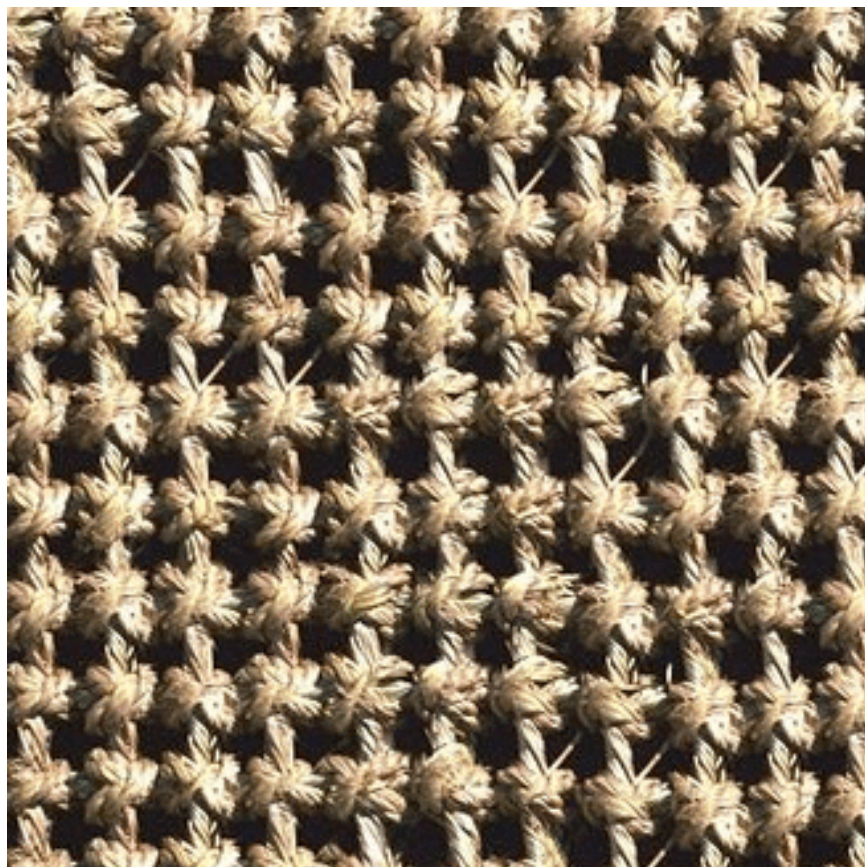
Examples: political synthesis

Examples: texture transfer
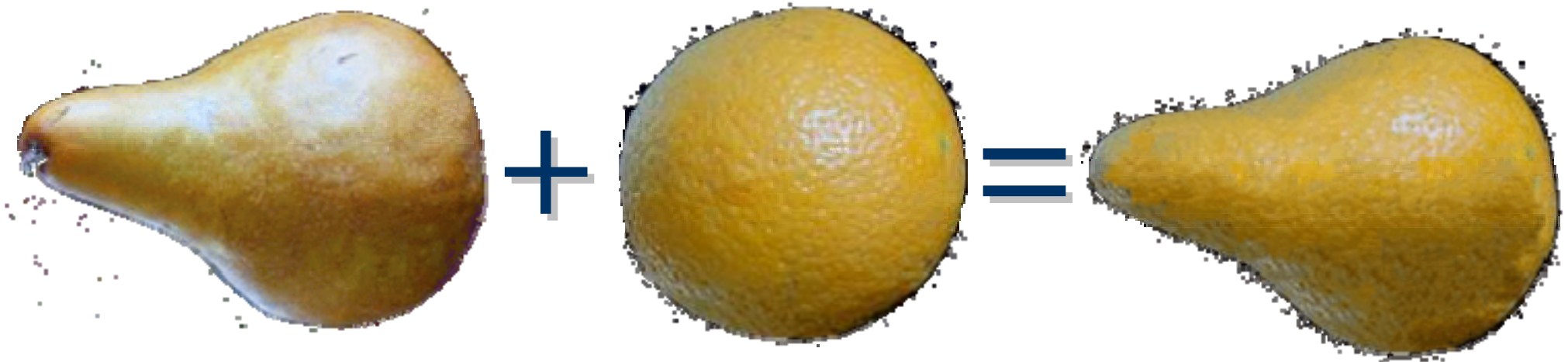
Examples: combining images

Examples: texture transfer
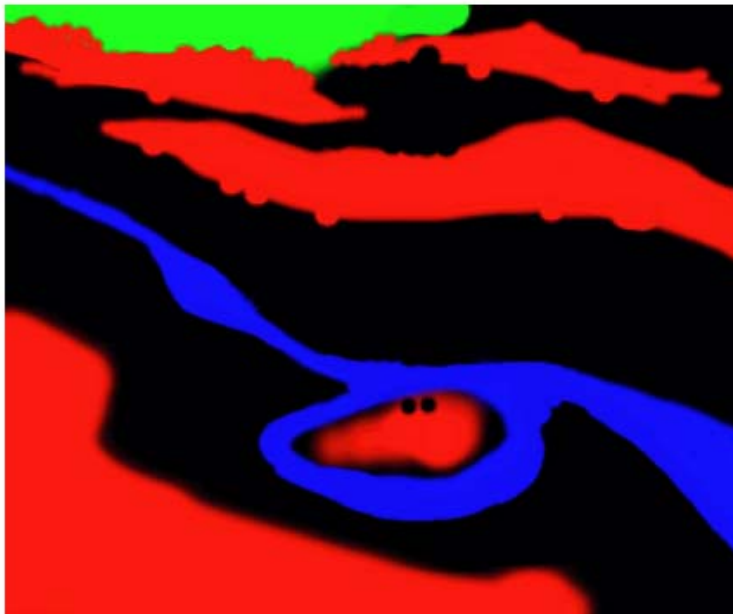

parmesan

rice

Examples: image analogies

Unfiltered source (A)
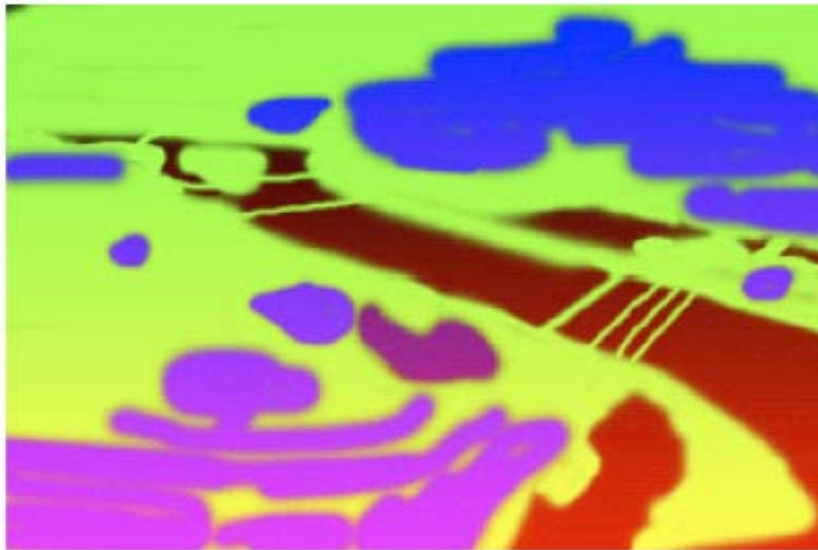
Filtered source (A')

Unfiltered (B)

Filtered (B')

Examples: image analogies

Unfiltered source (A)

Filtered source (A')