Binary Image Analysis

Selim Aksoy

Department of Computer Engineering

Bilkent University

saksoy@cs.bilkent.edu.tr

Outline

- Introduction to binary image analysis
- Thresholding
- Mathematical morphology
- Pixels and neighborhoods
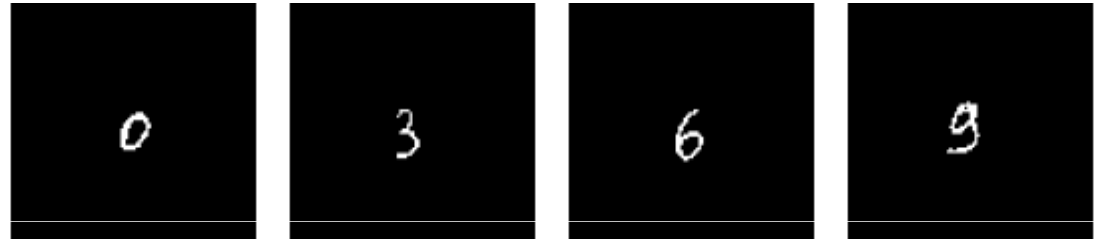- Connected components analysis

Binary image analysis

- Binary image analysis consists of a set of operations that are used to produce or process binary images, usually images of 0's and 1's where
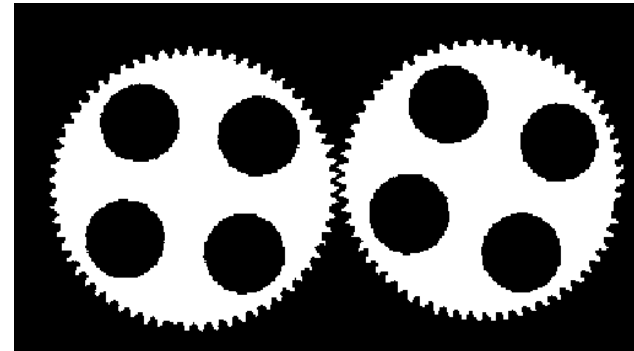 - 0 represents the background,
  - 1 represents the foreground.

```
000100100001000
000111100001000
000100100001000
```

Application areas

■ **Document analysis**

■ **Industrial inspection**
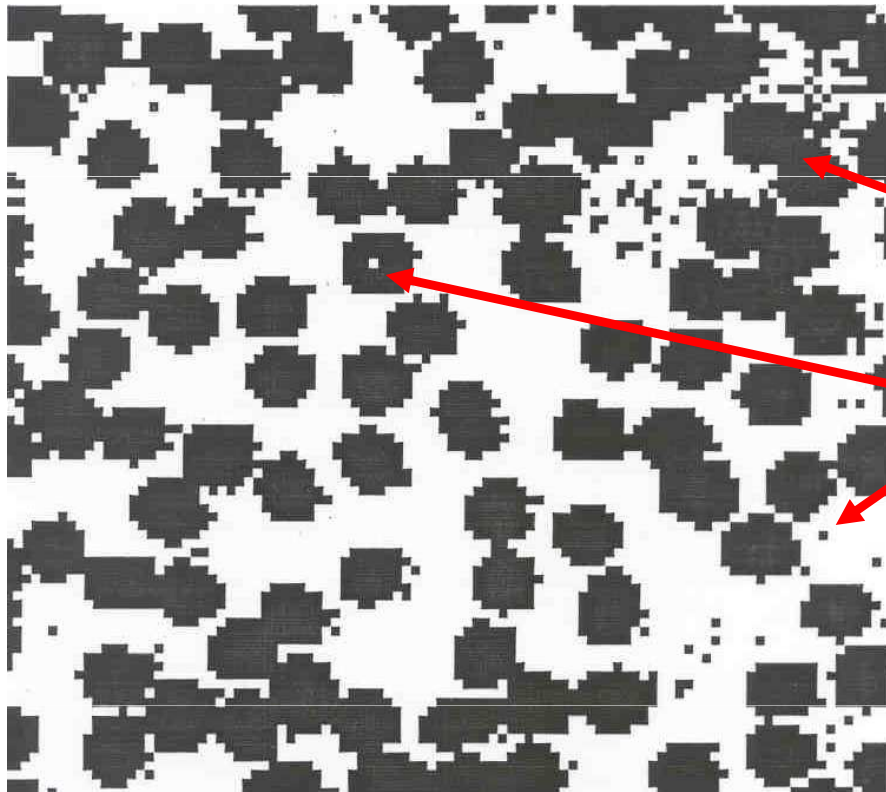

■ **Medical imaging**

Adapted from Shapiro and Stockman

Operations

- Separate objects from background and from one another.

- Aggregate pixels for each object.

- Compute features for each object.

Example: red blood cell image

- Many blood cells are separate objects.

- Many touch each other → bad!

- Salt and pepper noise is present.

- How useful is this data?

- 63 separate objects are detected.
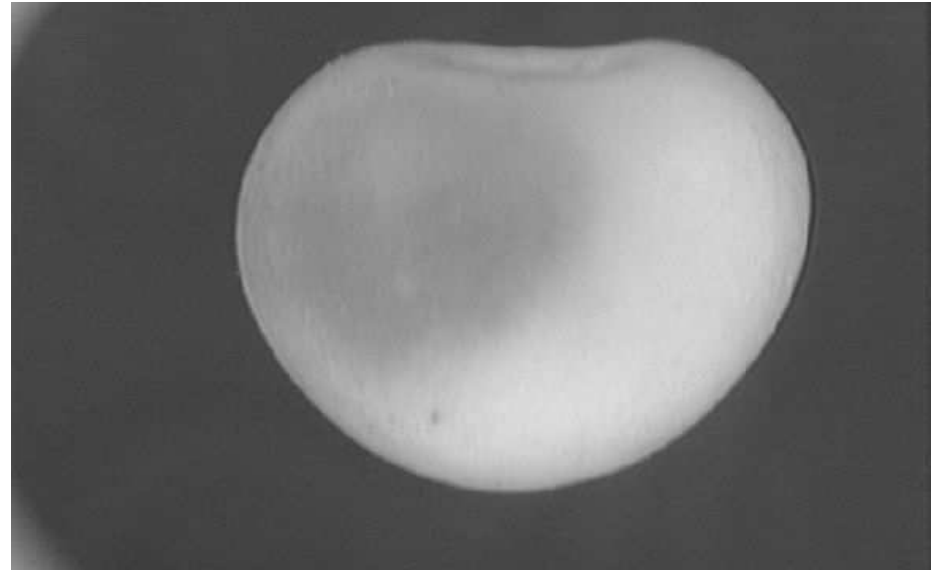
- Single cells have area of about 50 pixels.

Adapted from Linda Shapiro, U of Washington

Thresholding

- Binary images can be obtained from gray level images by thresholding.

- Assumptions for thresholding:
  - Object region of interest has intensity distribution different from background.
  - Object pixels likely to be identified by intensity alone:
    - intensity > a
    - intensity < b
    - a < intensity < b

- Works OK with flat-shaded scenes or engineered scenes.

- Does not work well with natural scenes.
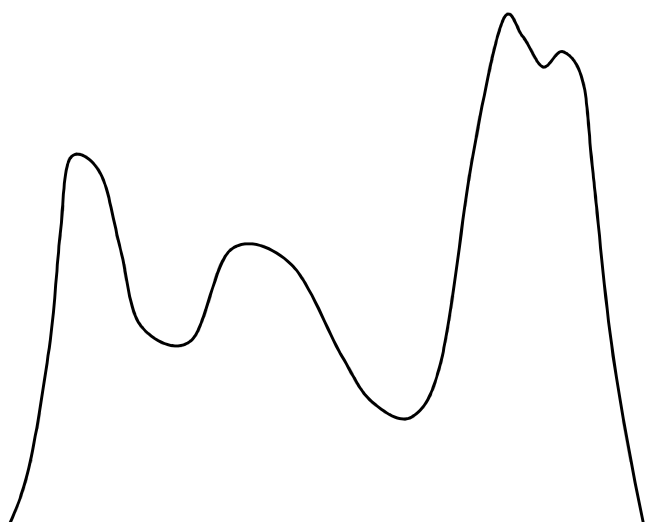
Use of histograms for thresholding

- Background is black.

- Healthy cherry is bright.

- Bruise is medium dark.

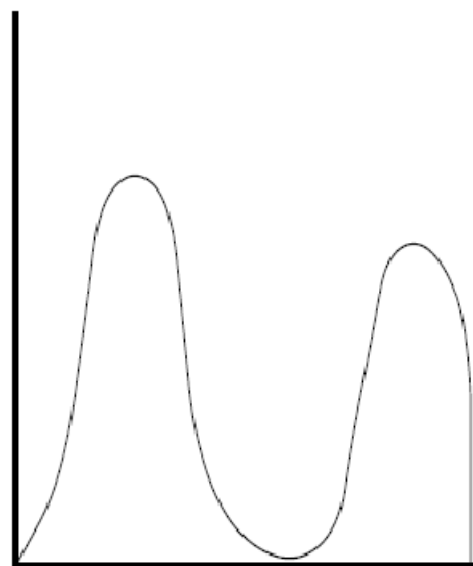- Histogram shows two cherry regions (black background has been removed).
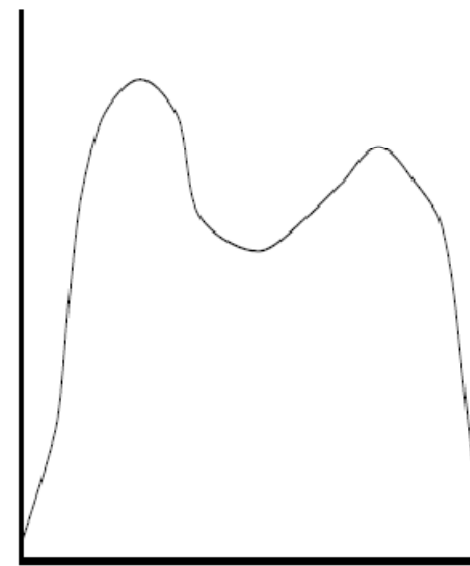
Automatic thresholding

- How can we use a histogram to separate an image into 2 (or several) different regions?

Is there a single clear threshold? 2? 3?

Two distinct modes

Overlapped modes

Adapted from Shapiro and Stockman

Automatic thresholding: Otsu's method

- Assumption: the histogram is bimodal.

- Method: find the threshold t that minimizes the <u>weighted sum of within-group variances</u> for the two groups that result from separating the gray levels at value t.

- The best threshold t can be determined by a simple sequential search through all possible values of t.

Group 1 | Group 2

t

- If the gray levels are strongly dependent on the location within the image, local or dynamic thresholds can also be used.
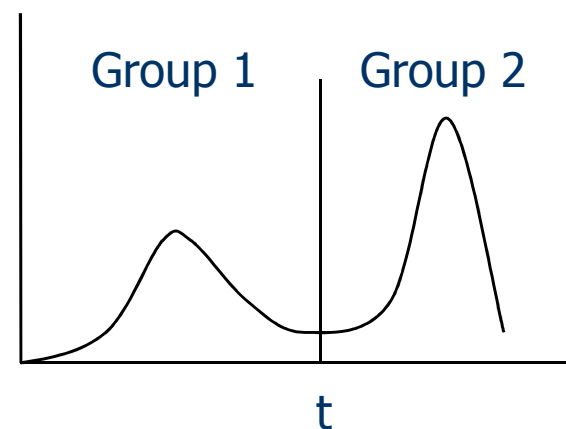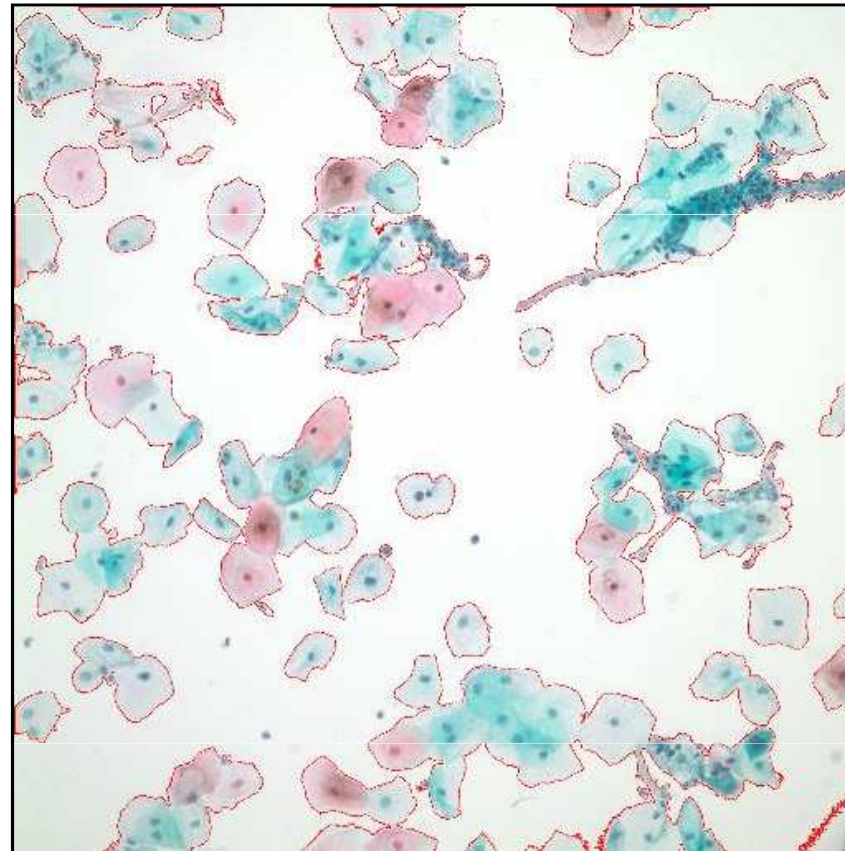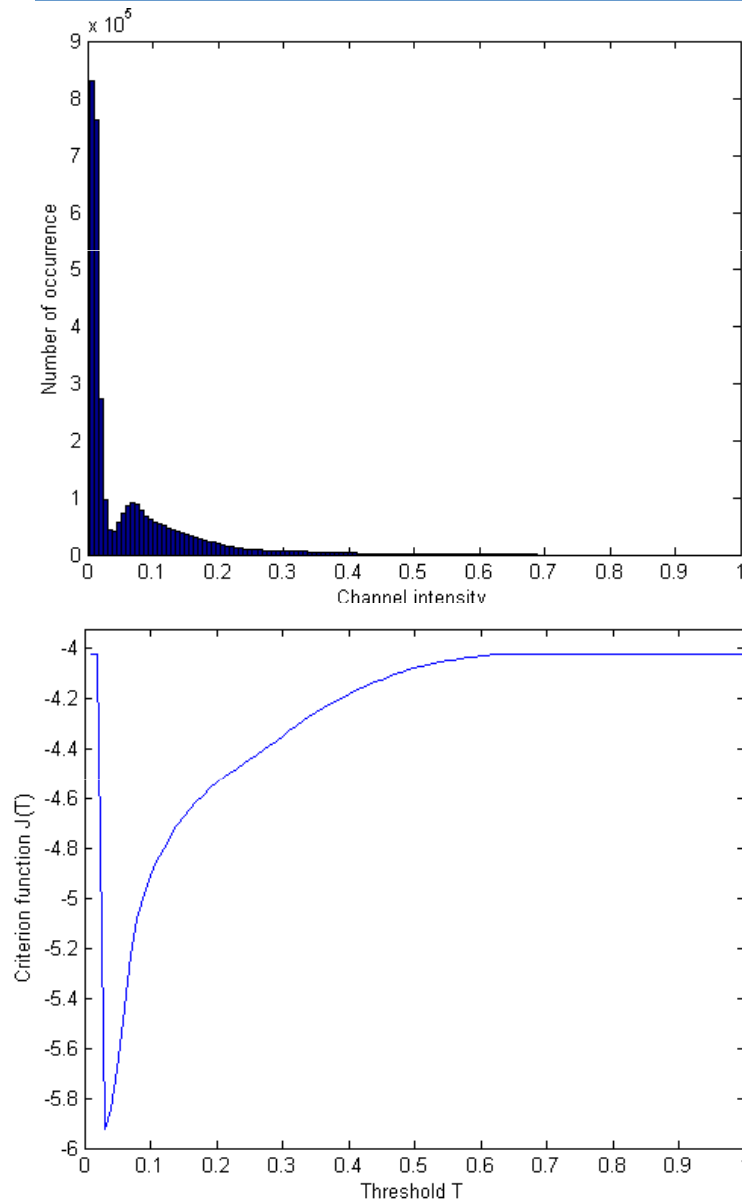
Automatic thresholding
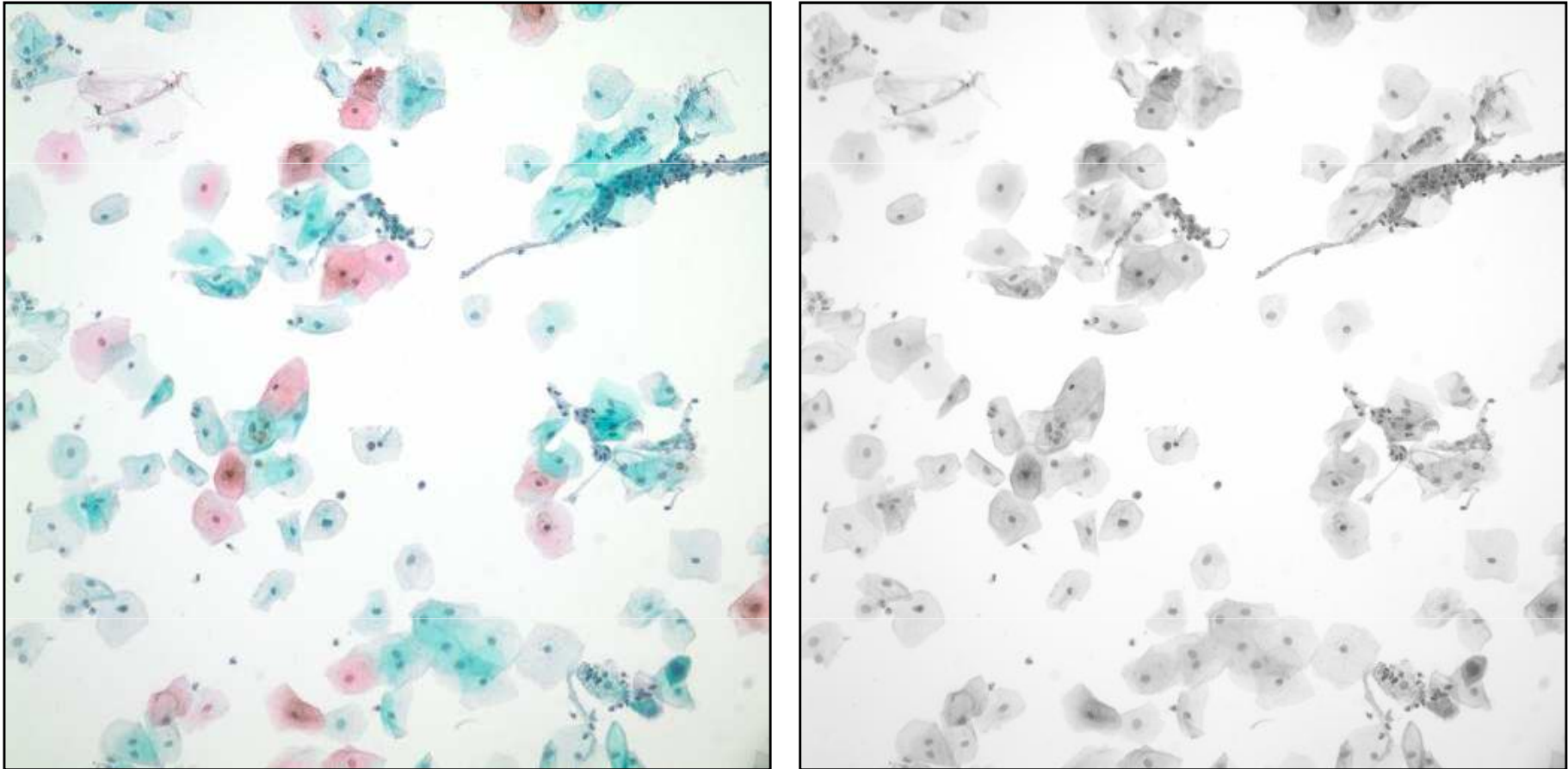
A Pap smear image example: RGB image (left) and grayscale image (right).

Automatic thresholding

Histogram of the negative image (top-left), sum of within-group variances versus the threshold (bottom-left), resulting mask overlayed as red on the original image (top).

Mathematical morphology

- The word morphology refers to form and structure.

- In computer vision, it is used to refer to the shape of a region.

- The language of mathematical morphology is set theory where sets represent objects in an image.

- We will discuss morphological operations on binary images whose components are sets in the 2D integer space Z^2.

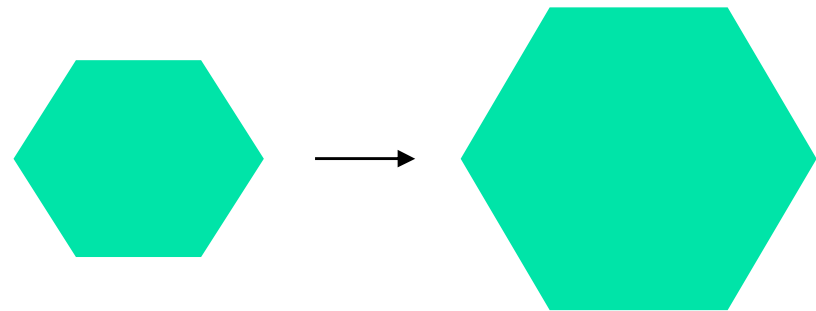# Mathematical morphology

- Mathematical morphology consists of two basic operations
  - dilation
  - erosion

  and several composite relations
  - opening
  - closing
  - conditional dilation
  - ...

Dilation

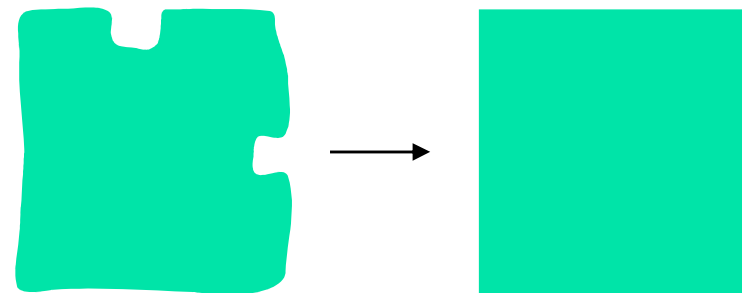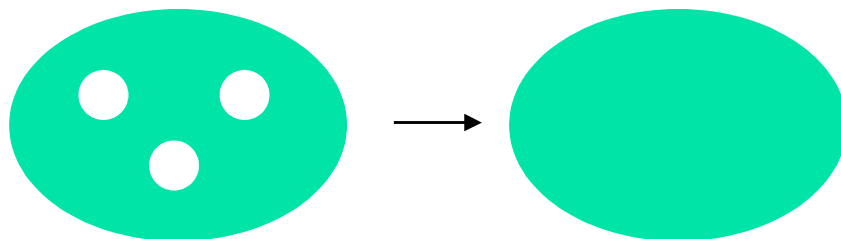- Dilation expands the connected sets of 1s of a binary image.

- It can be used for

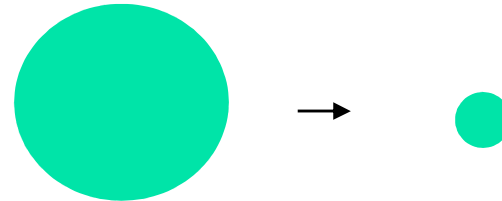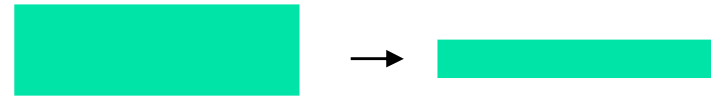 - growing features

  - filling holes and gaps

Adapted from Linda Shapiro, U of Washington

Erosion

- Erosion shrinks the connected sets of 1s of a binary image.

- It can be used for

  - shrinking features

  - removing bridges, branches and small protrusions

Adapted from Linda Shapiro, U of Washington
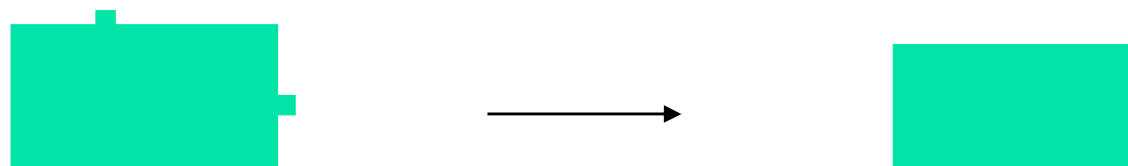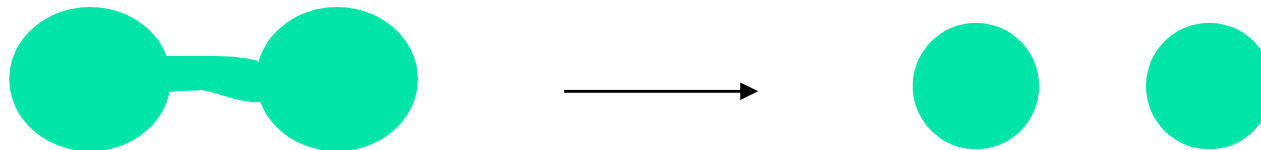
Basic concepts from set theory

- Let A be a set in Z^2. If $a = (a_1, a_2)$ is an element of A, we write $a \in A$; otherwise, we write $a \notin A$.

- Set A being a *subset* of set B is denoted by $A \subseteq B$.

- The *union* of two sets A and B is denoted by $A \cup B$.

- The *intersection* of two sets A and B is denoted by $A \cap B$.

- The *complement* of a set A is the set of elements not contained in A:

$$A^c = \{w \mid w \notin A\}.$$

- The *difference* of two sets A and B, denoted by $A - B$, is defined as

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c.$$

Basic concepts from set theory

- The *reflection* of set B, denoted by $\check{B}$, is defined as

$$\check{B} = \{w | w = -b, \forall b \in B\}.$$

- The *translation* of set A by point $z = (z_1, z_2)$, denoted by A_z, is defined as

$$A_z = \{w | w = a + z, \forall a \in A\}.$$
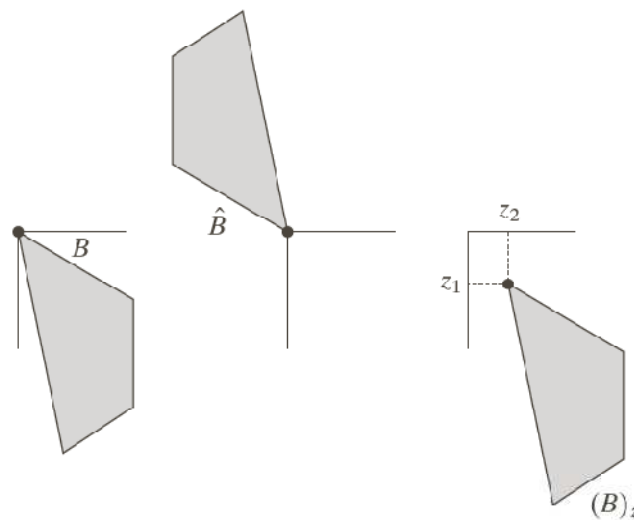
a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

Structuring elements

- Structuring elements are small binary images used as shape masks in basic morphological operations.

- They can be any shape and size that is digitally representable.

- One pixel of the structuring element is denoted as its origin.

- Origin is often the central pixel of a symmetric structuring element but may in principle be any chosen pixel.

Structuring elements

a) BOX(3,5)

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

b) DISK(5)

	1	1	1	
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
	1	1	1	

c) RING(5)

	1	1	1	
1				1
1				1
1				1
	1	1	1	

d)

1	1		
1	1		
1	1	1	1
1	1	1	1

e)

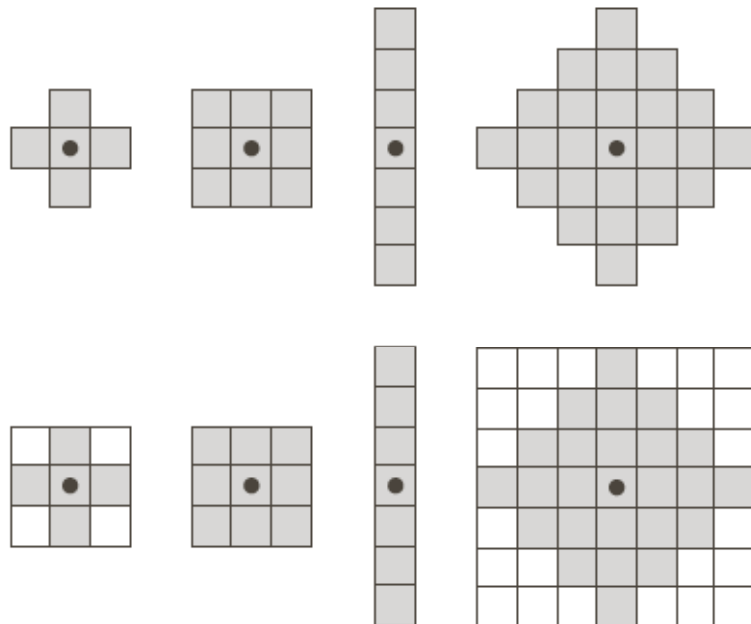| 1	1	1	1	1	1
1		1	1		1
1		1	1		1
1		1	1		1

f)

1
1
1
1

FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.
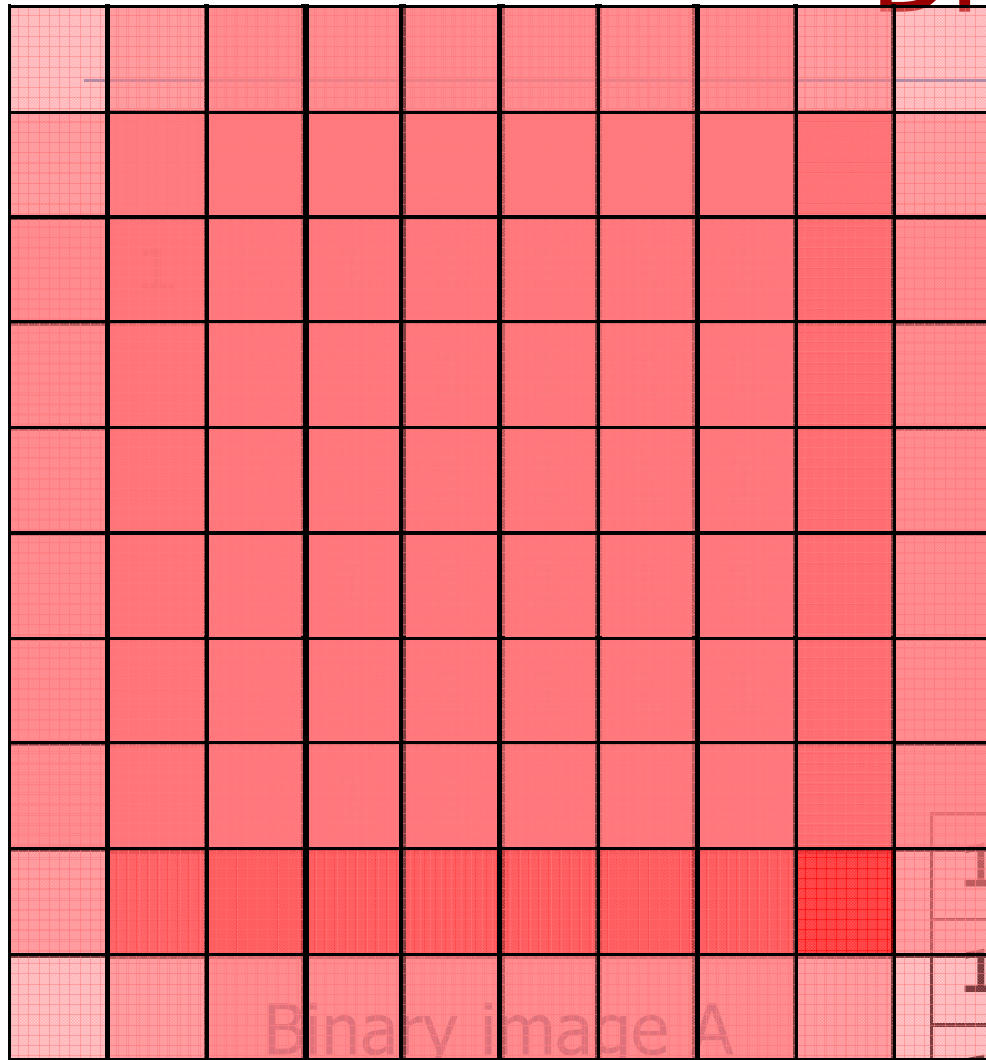
Dilation

- The *dilation* of binary image A by structuring element B is denoted by $A \oplus B$ and is defined by

$$A \oplus B = \{z | \check{B}_z \cap A \neq \emptyset\},$$
$$= \bigcup_{a \in A} B_a.$$

  ▶ First definition: The dilation is the set of all displacements z such that $\check{B}_z$ and A overlap by at least one element.

  ▶ Second definition: The structuring element is swept over the image. Each time the origin of the structuring element touches a binary 1-pixel, the entire translated structuring element is ORed to the output image, which was initialized to all zeros.

Dilation

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1
0	1	1	1	1	0	0	0

Binary image A

1	1	1
1	1	1
1	1	1

Dilation result

Structuring element B

(1st definition)

Dilation

Binary image A

1	1	1
1	1	1
1	1	1

Structuring element B

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1			

Dilation result

(2nd definition)
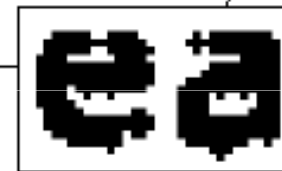
Dilation

Structuring Element
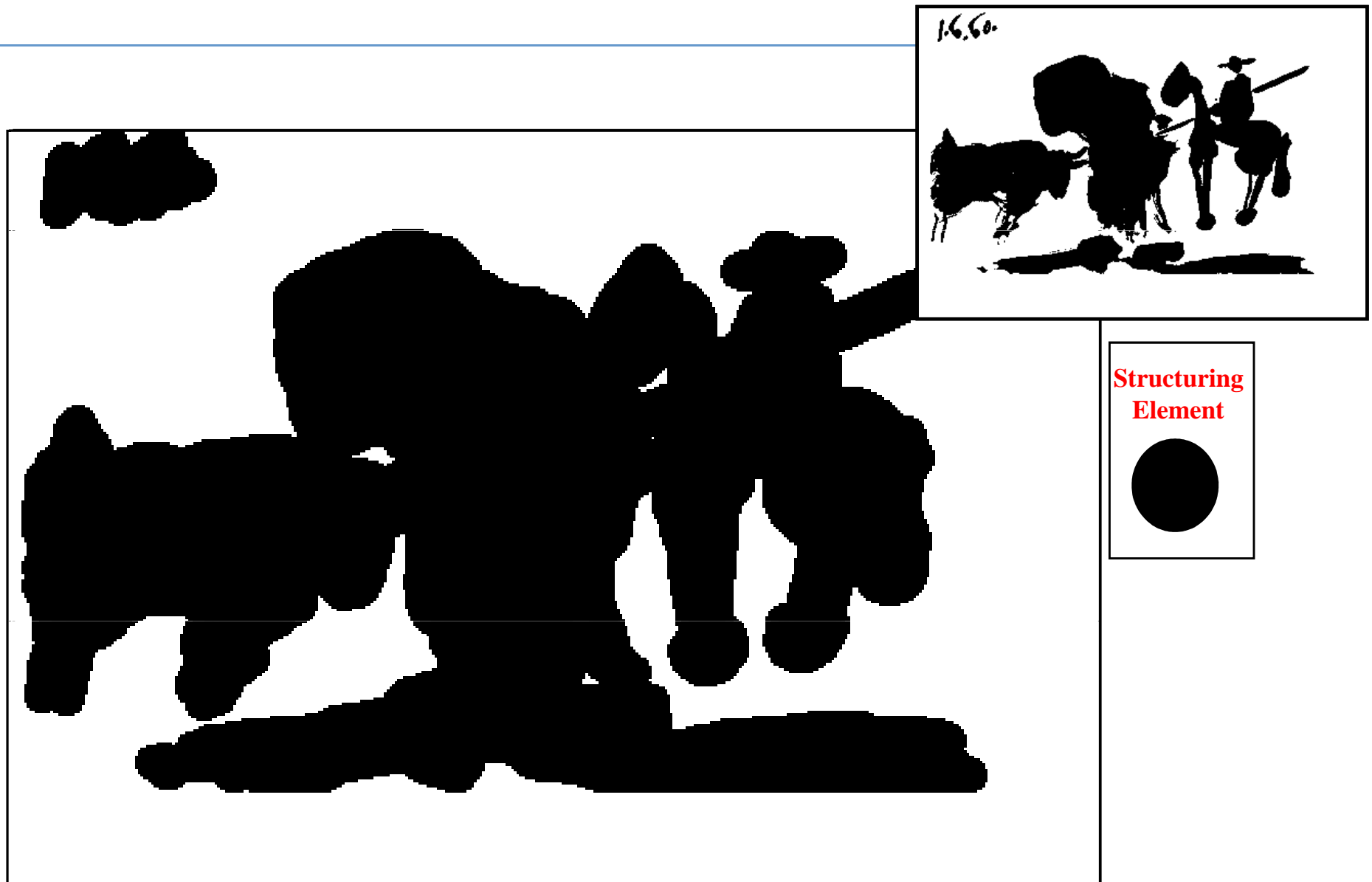
Adapted from John Goutsias, Johns Hopkins Univ.

Pablo Picasso, *Pass with the Cape*, 1960

Dilation

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.
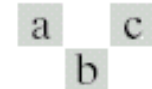
a   c
   b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
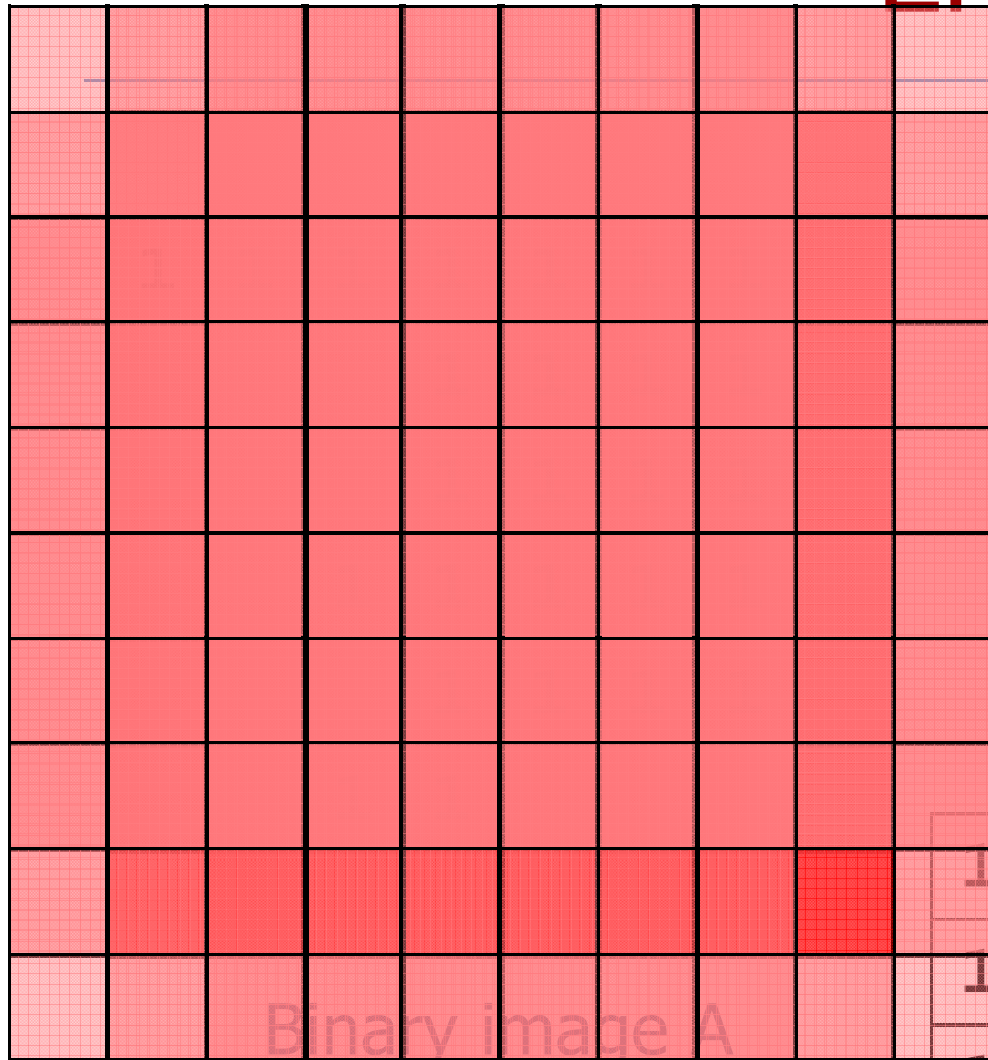(c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0

Adapted from Gonzales and Woods

Erosion

- The *erosion* of binary image A by structuring element B is denoted by $A \ominus B$ and is defined by

$$A \ominus B = \{z | B_z \subseteq A\},$$
$$= \{a | a + b \in A, \forall b \in B\}.$$

  - First definition: The erosion is the set of all points z such that B, translated by z, is contained in A.
  - Second definition: The structuring element is swept over the image. At each position where every 1-pixel of the structuring element covers a 1-pixel of the binary image, the binary image pixel corresponding to the origin of the structuring element is ORed to the output image.

Erosion

Binary image A

Structuring element B
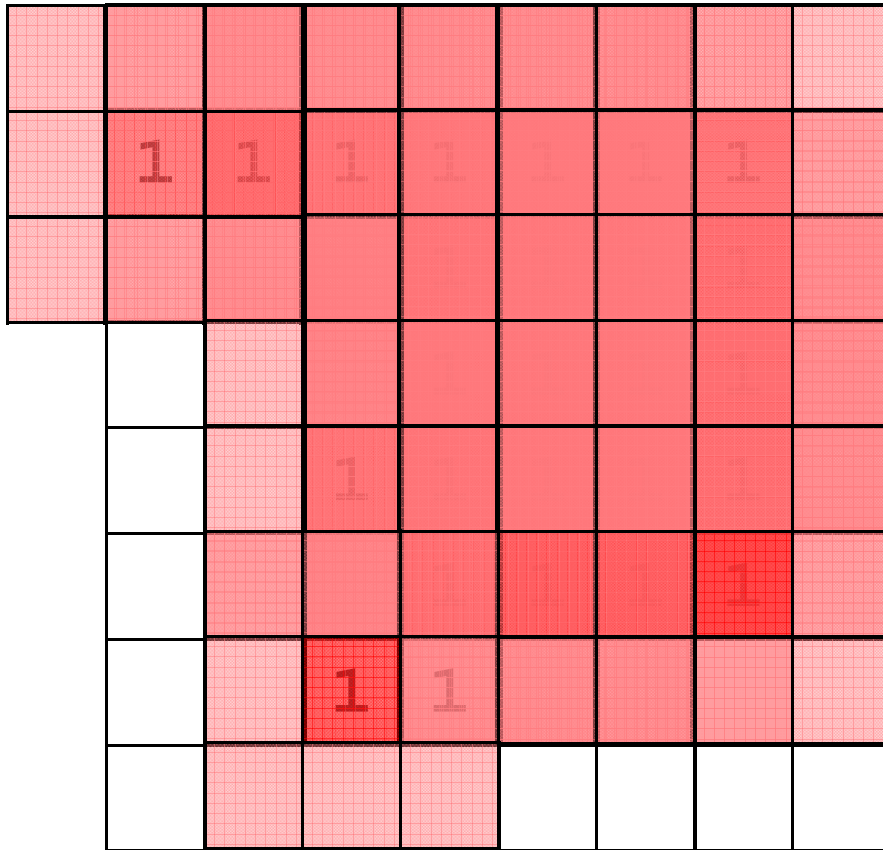
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0
0	0	0	0	1	1	0	0
0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Erosion result

(1st definition)

Erosion

Binary image A

Structuring element B

Erosion result

(2nd definition)

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Erosion

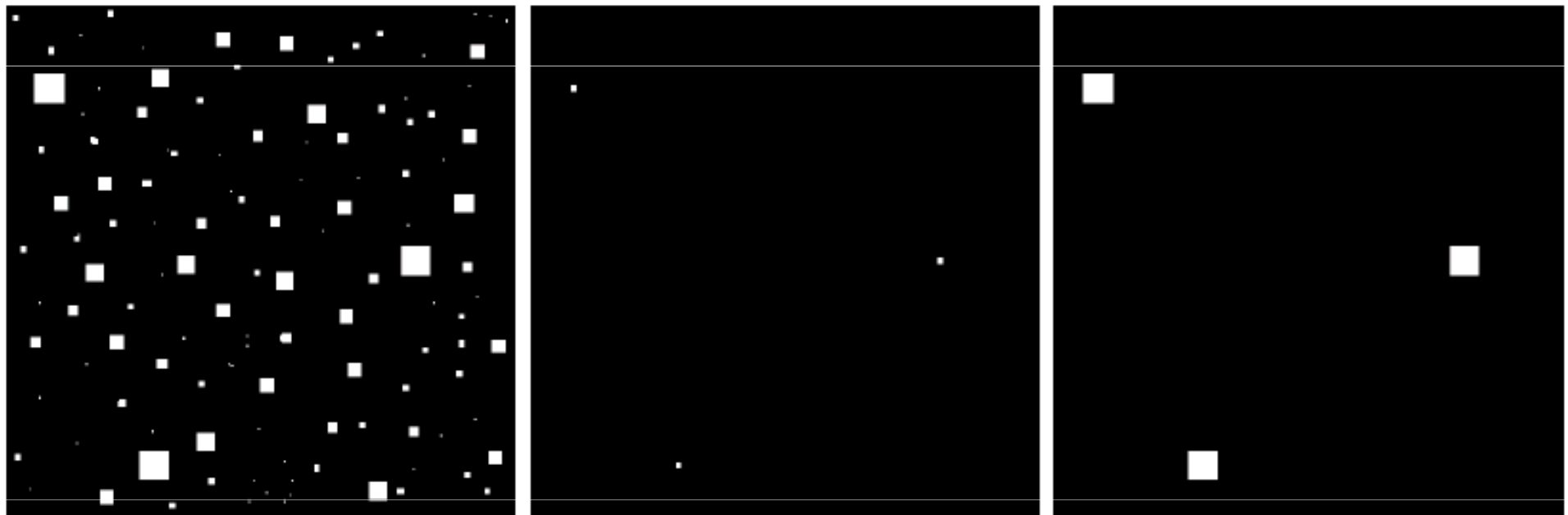**Structuring Element**
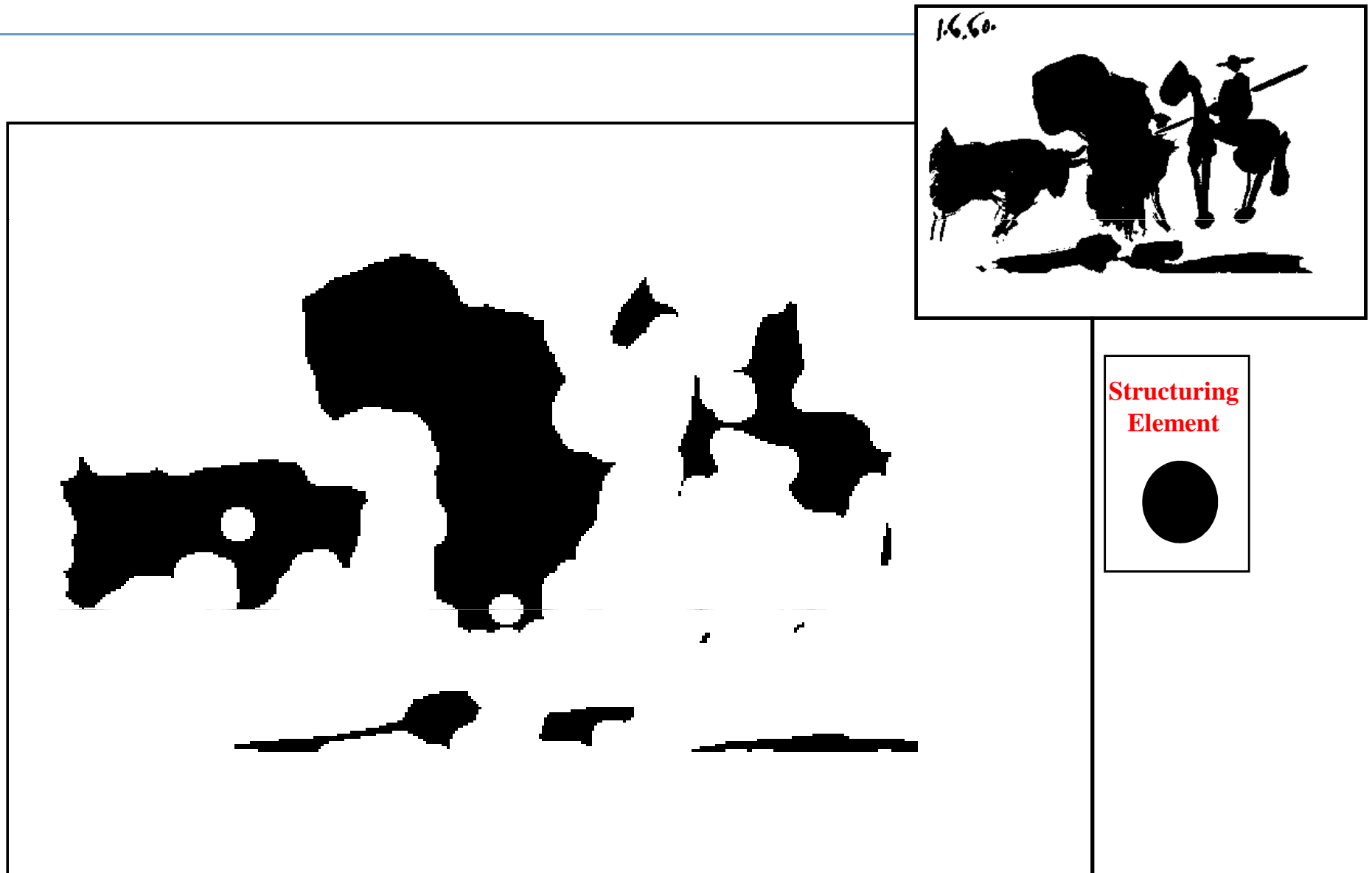
Pablo Picasso, *Pass with the Cape*, **1960**

Adapted from John Goutsias, Johns Hopkins Univ.

Erosion

a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening

- The *opening* of binary image A by structuring element B is denoted by $A \circ B$ and is defined by
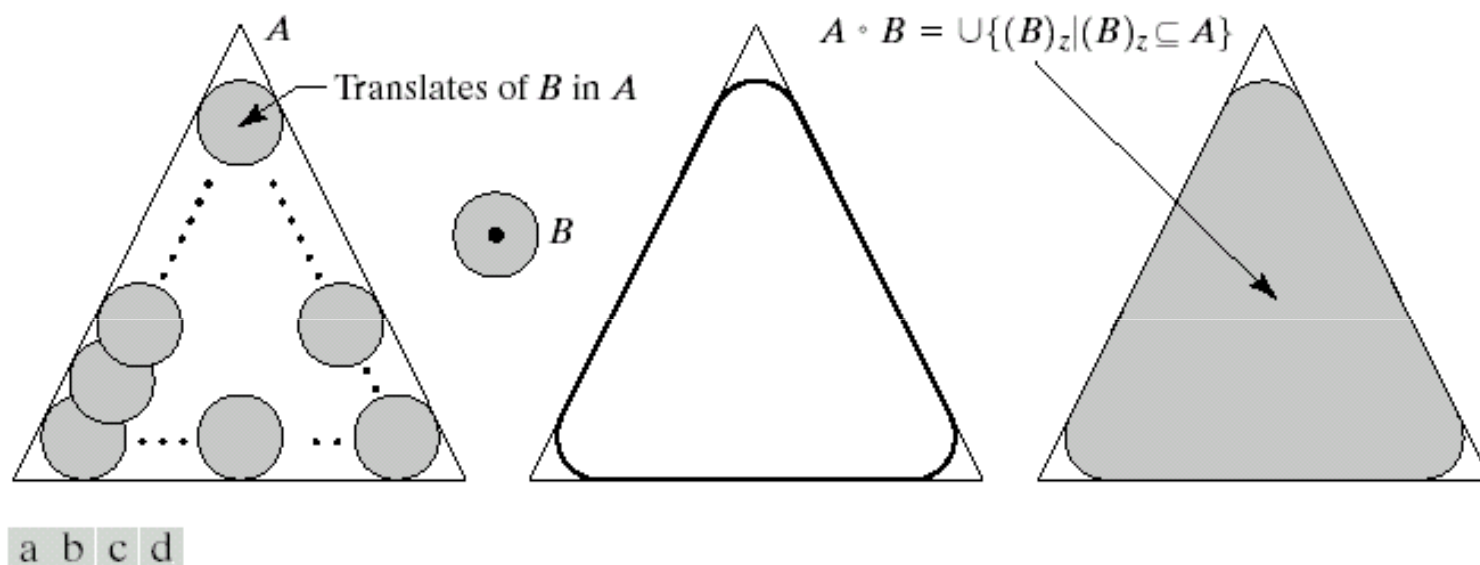
$$A \circ B = (A \ominus B) \oplus B.$$

a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Opening

Binary image A

Structuring element B

Opening result

Opening

Structuring Element

Adapted from John Goutsias, Johns Hopkins Univ.

Pablo Picasso, *Pass with the Cape*, **1960**

Closing

- The *closing* of binary image A by structuring element B is denoted by $A \bullet B$ and is defined by

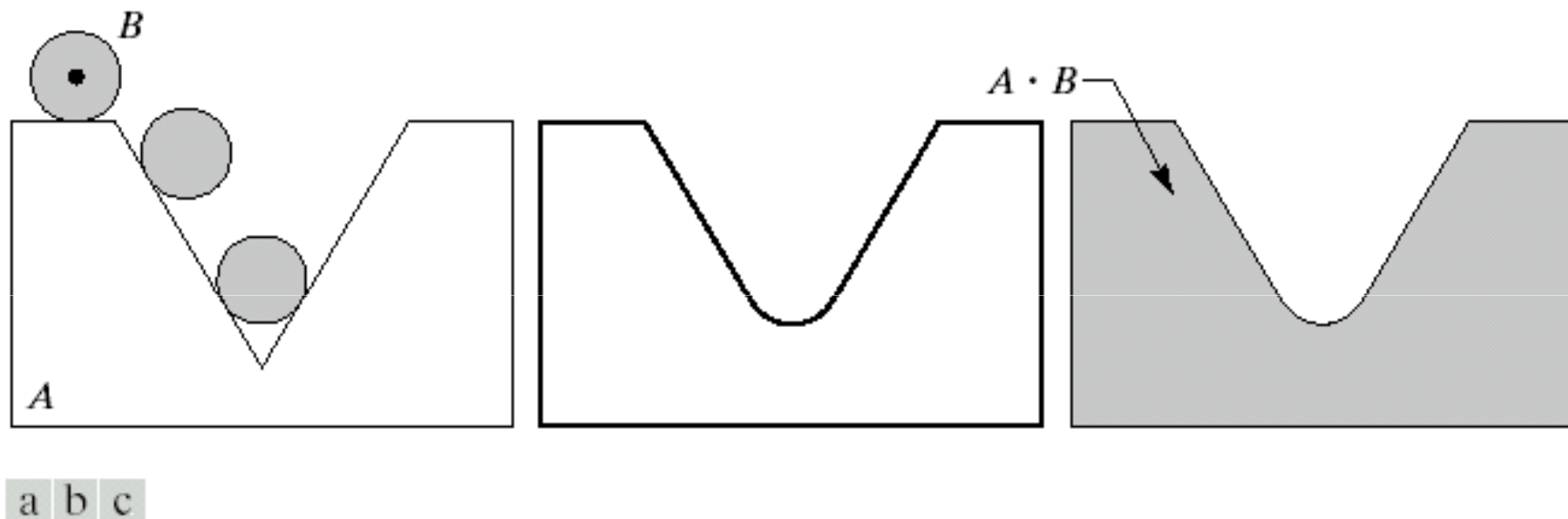$$A \bullet B = (A \oplus B) \ominus B.$$

a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Closing

Binary image A

Structuring element B

Closing result

Examples
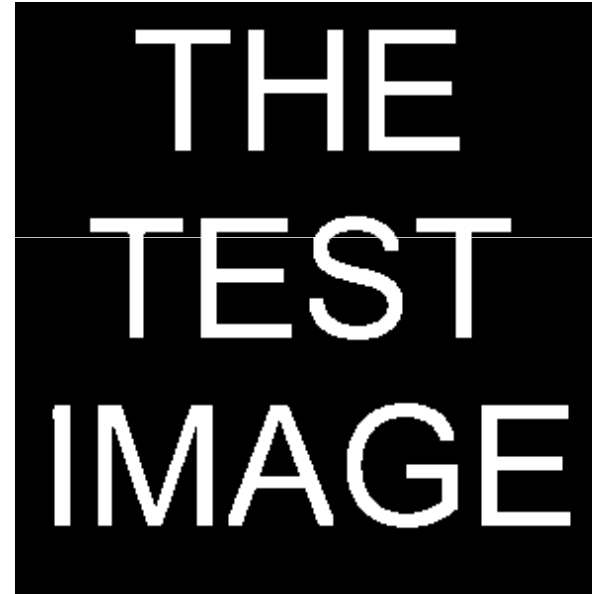

Original image   Eroded once   Eroded twice

Examples

Original image

Opened twice
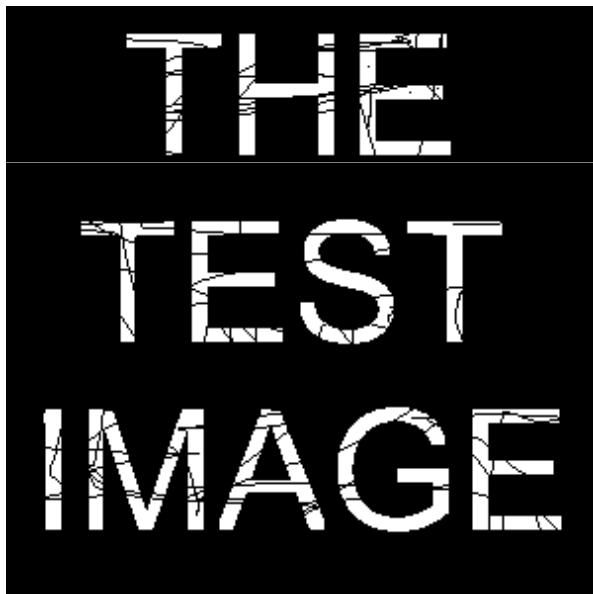
Original image
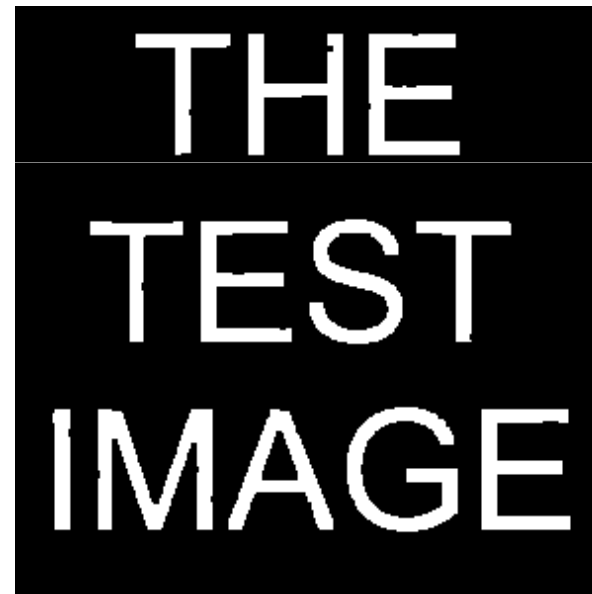
Closed once

Examples

$A \ominus B$

$(A \ominus B) \oplus B = A \circ B$

$(A \circ B) \oplus B$

$[(A \circ B) \oplus B] \ominus B = (A \circ B) \cdot B$
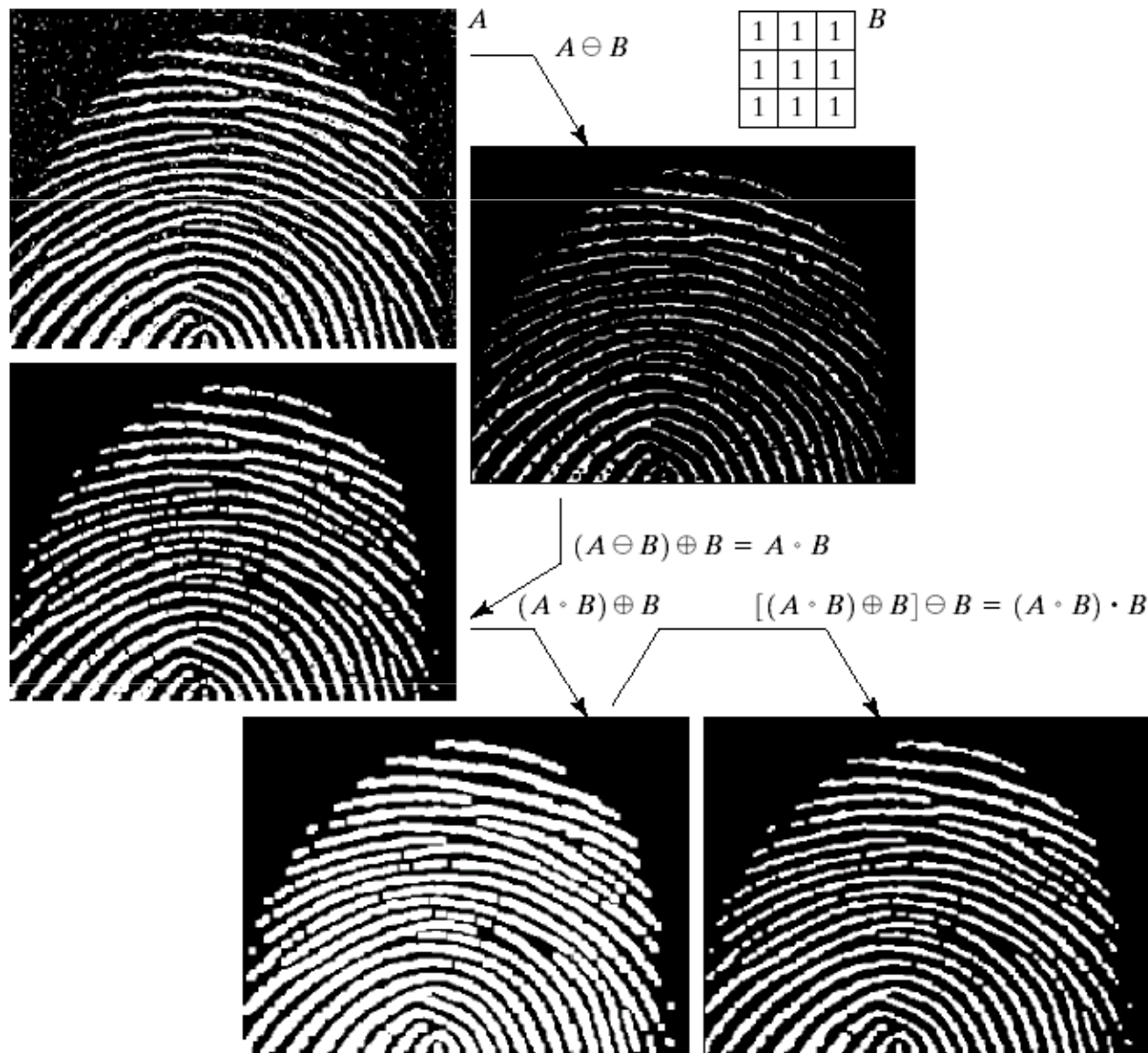
a	b
d	c
	e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Properties

- Dilation and erosion are duals of each other with respect to set complementation and reflection, i.e.,

$$(A \ominus B)^c = A^c \oplus \check{B}.$$

- Opening and closing are duals of each other with respect to set complementation and reflection, i.e.,

$$(A \bullet B)^c = A^c \circ \check{B}.$$

Properties

- Opening satisfies the following properties:
  - ▶ $A \circ B$ is a subset of A.
  - ▶ If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
  - ▶ $(A \circ B) \circ B = A \circ B$.
- Closing satisfies the following properties:
  - ▶ A is a subset of $A \bullet B$.
  - ▶ If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
  - ▶ $(A \bullet B) \bullet B = A \bullet B$.

Boundary extraction

- The *boundary* of a set A can be obtained by first eroding A by B and then performing the set difference between A and its erosion, i.e.,

$$\text{boundary}(A) = A - (A \ominus B)$$

where B is a suitable structuring element.

a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Conditional dilation

- Given an original binary image B, a processed binary image C, and a structuring element A, let $C_0 = C$ and $C_n = (C_{n-1} \oplus A) \cap B$. The *conditional dilation* of C by A with respect to B is defined by

$$C \oplus |_B A = C_m$$

  where the index m is the smallest index satisfying $C_m = C_{m-1}$.

- Given a structuring element that when applied to a binary image
  - ▶ removes the components that do not satisfy certain shape and size constraints, and
  - ▶ leaves a few 1-pixels of those components that do satisfy the constraints,

  conditional dilation recovers the latter components using what remains of them after the erosion.

Region filling

- Given set A containing the boundary points of a region, and a point p inside the boundary, the following procedure *fills the region* with 1's:

$$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots$$

where $X_0 = p$ and B is the cross structuring element. The procedure terminates at iteration step k if $X_k = X_{k-1}$.

- The set union of X_k and A contains the filled set and its boundary.

Region filling

a b c
d e f
g h i
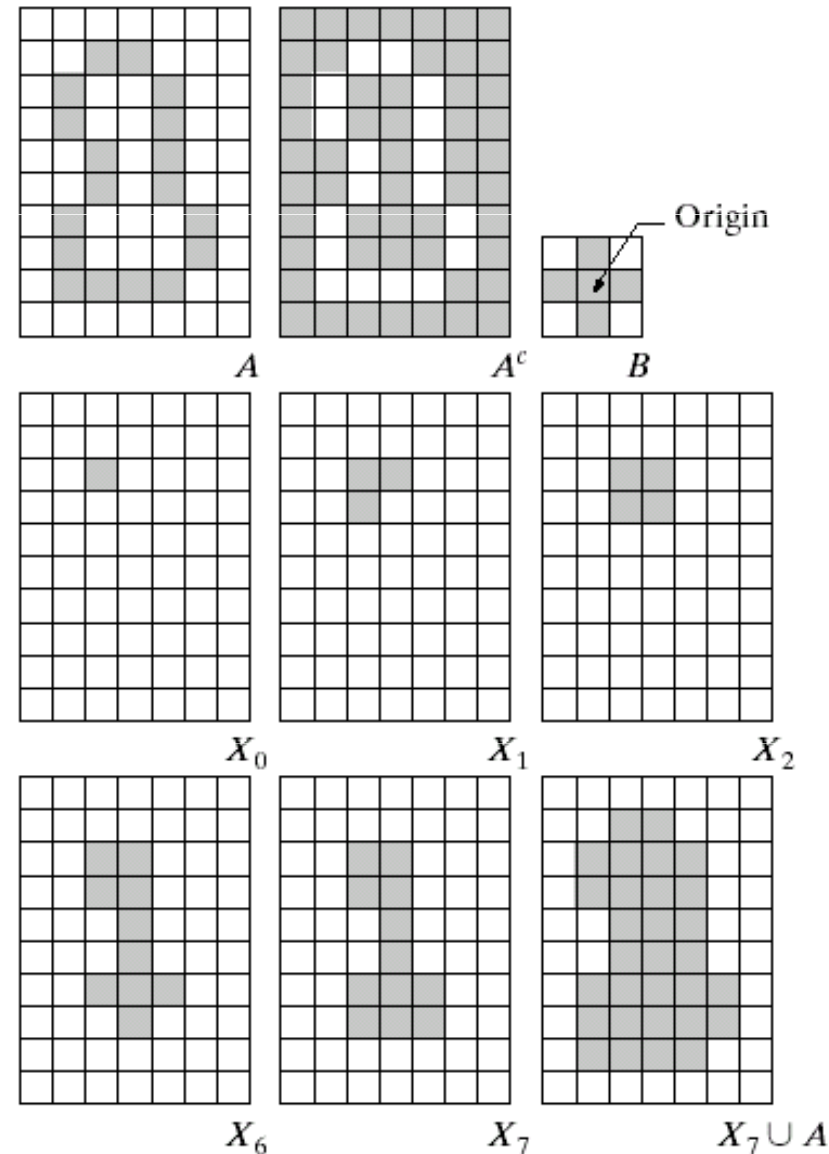
FIGURE 9.15
Region filling.
(a) Set A.
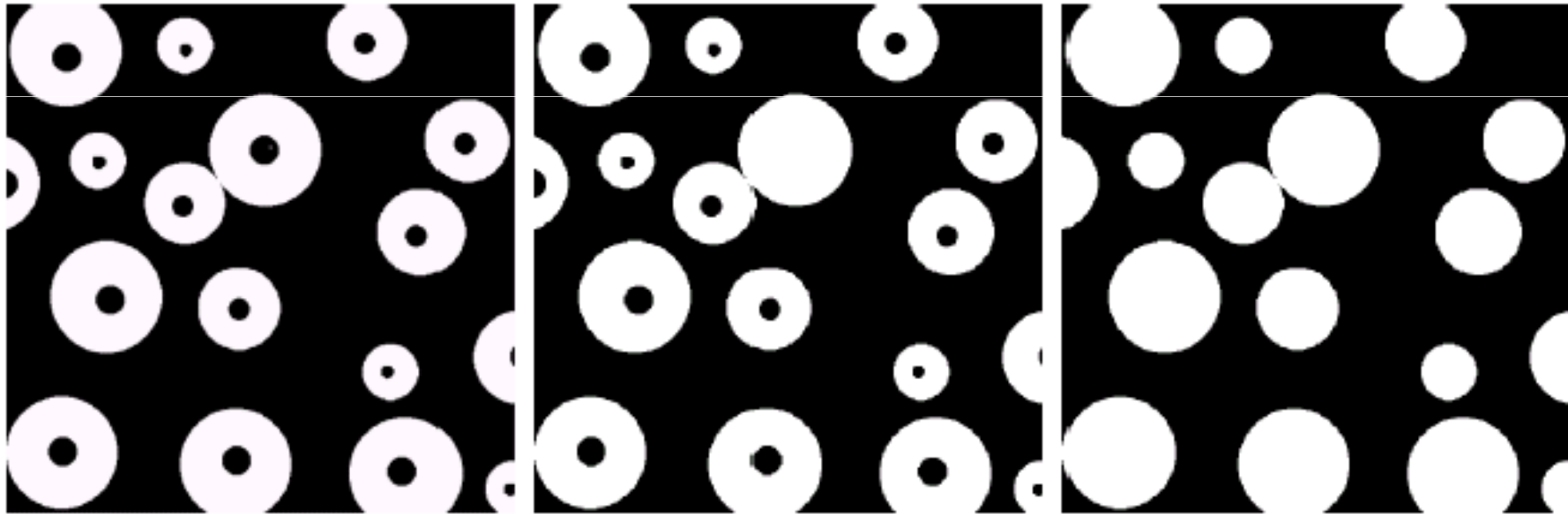(b) Complement
of A.
(c) Structuring
element B.
(d) Initial point
inside the
boundary.
(e)–(h) Various
steps of
Eq. (9.5-2).
(i) Final result
[union of (a) and
(h)].

A A^c B Origin

X_0 X_1 X_2

X_6 X_7 $X_7 \cup A$

Region filling

a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Hit-or-miss transform

- Let $B = (B_1, B_2)$ denote the set composed of an object and its local background where B_1 is the set formed from elements of B associated with the object and B_2 is the set of elements of B associated with the background.

- The *hit-or-miss transform* is denoted by $A \circledast B$ and is defined by

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2).$$

- The set $A \circledast B$ contains all the (origin) points at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
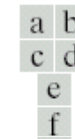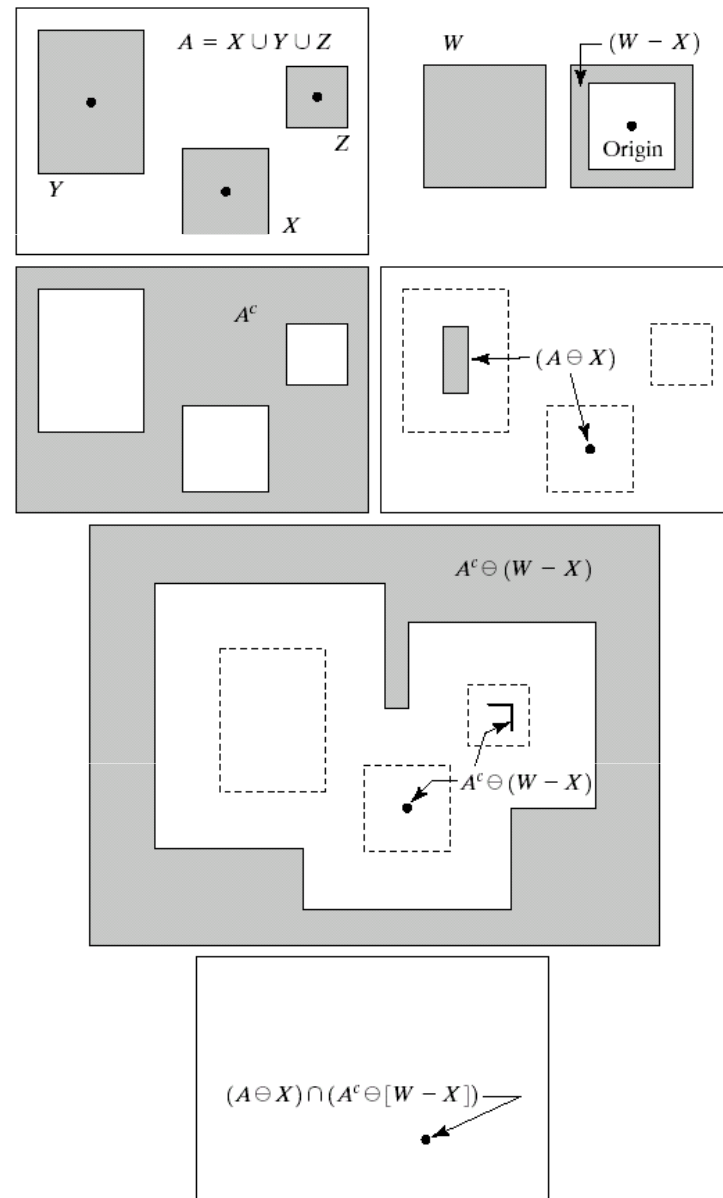
Hit-or-miss transform

a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$.
(c) Complement of A. (d) Erosion of A by X.
(e) Erosion of A^c by $(W - X)$.
(f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

Thinning

- The *thinning* of a set A by a structuring element B, denoted by $A \otimes B$, can be defined in terms of the hit-or-miss transform:

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c.$$

- Thinning is done using a sequence of structuring elements where each one is the rotated version of the previous one in the sequence.
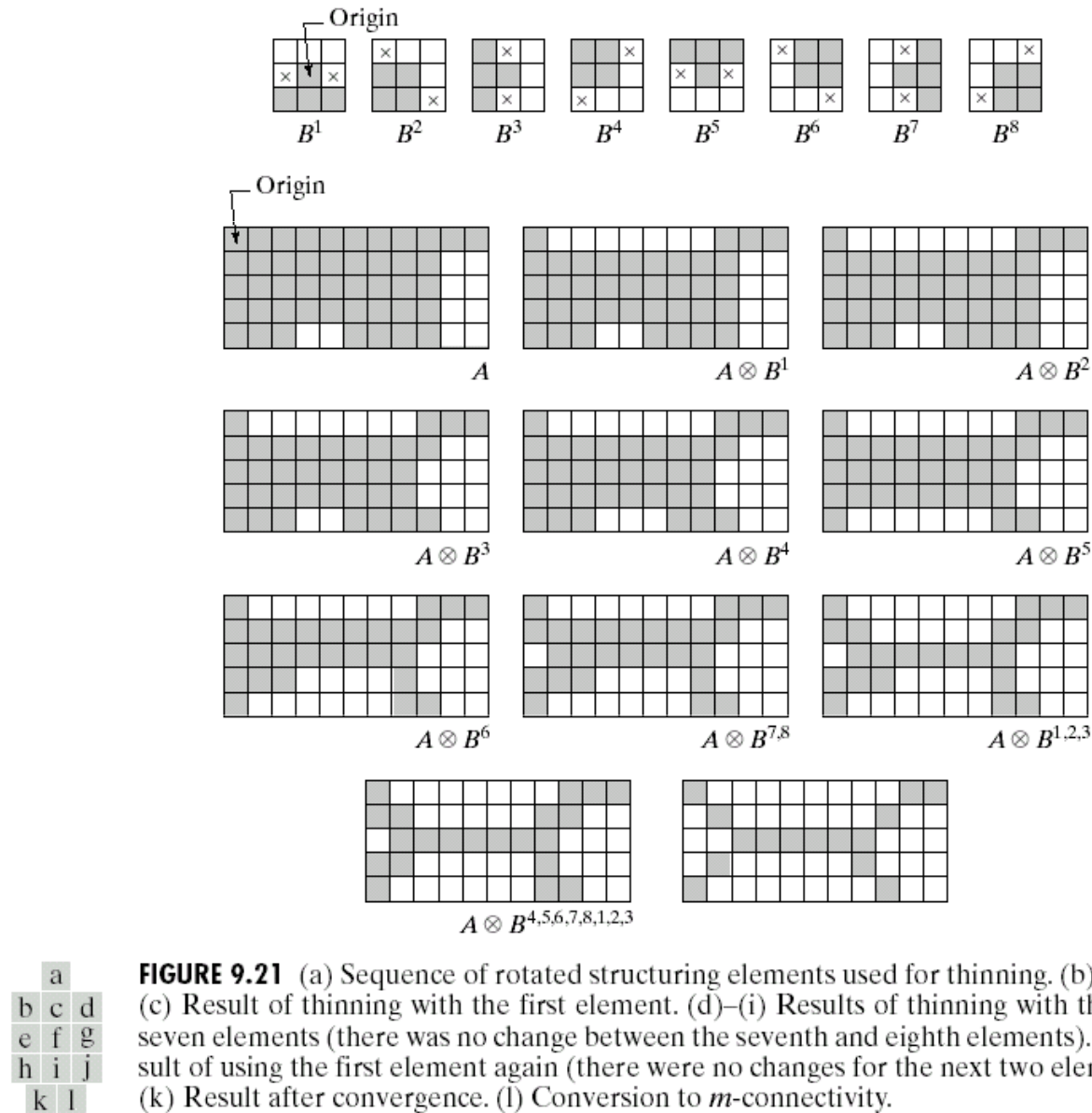
Thinning

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

Thickening

- *Thickening* is the morphological dual of thinning and is defined by

$$A \odot B = A \cup (A \circledast B)$$

where B is a structuring element suitable for thickening.

- A sequence of structuring elements of the same form as in thinning but with all 1's and 0's interchanged can be used.
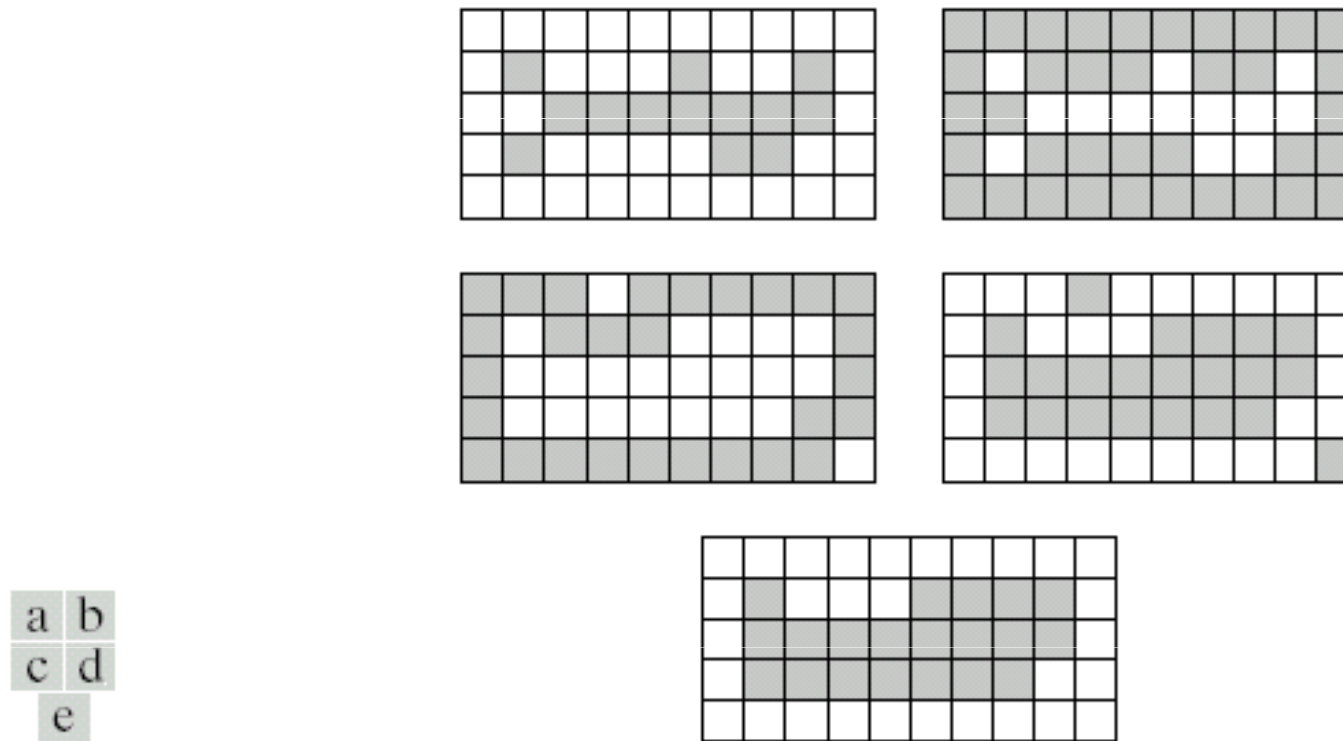
Thickening

a b
c d
e

FIGURE 9.22 (a) Set *A*. (b) Complement of *A*. (c) Result of thinning the complement of *A*. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Examples

Detecting runways in satellite airport imagery

http://www.mmorph.com/mxmorph/html/mmdemos/mmdairport.html

Examples

Segmenting letters, words and paragraphs

http://www.mmorph.com/mxmorph/html/mmdemos/mmdlabeltext.html
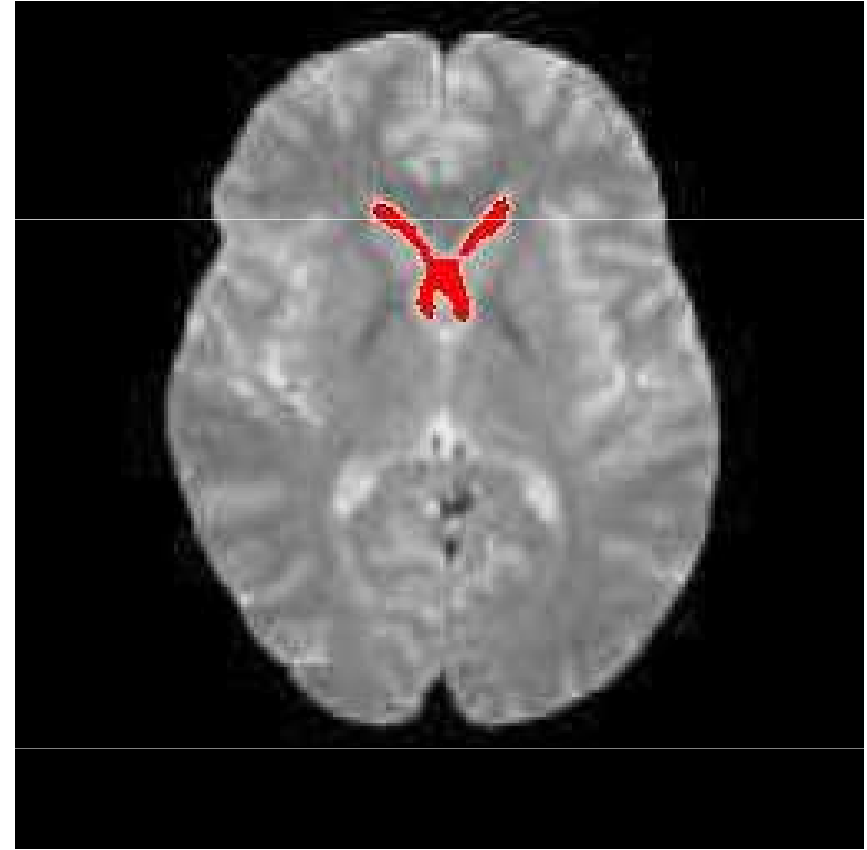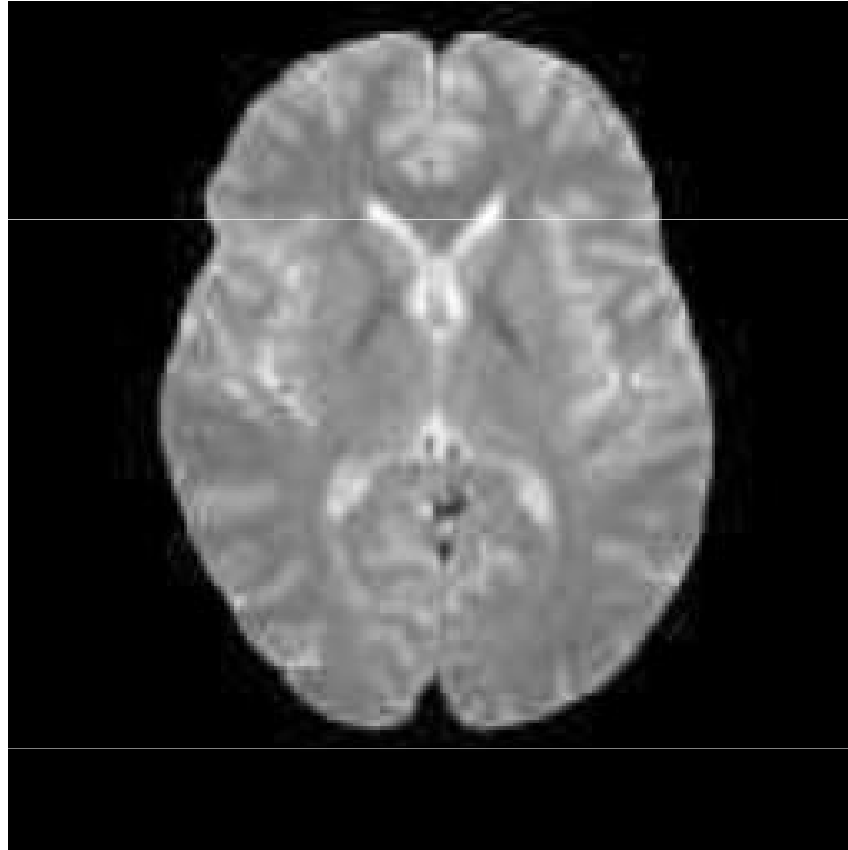
Examples

Extracting the lateral ventricle from an MRI image of the brain

http://www.mmorph.com/mxmorph/html/mmdemos/mmdbrain.html
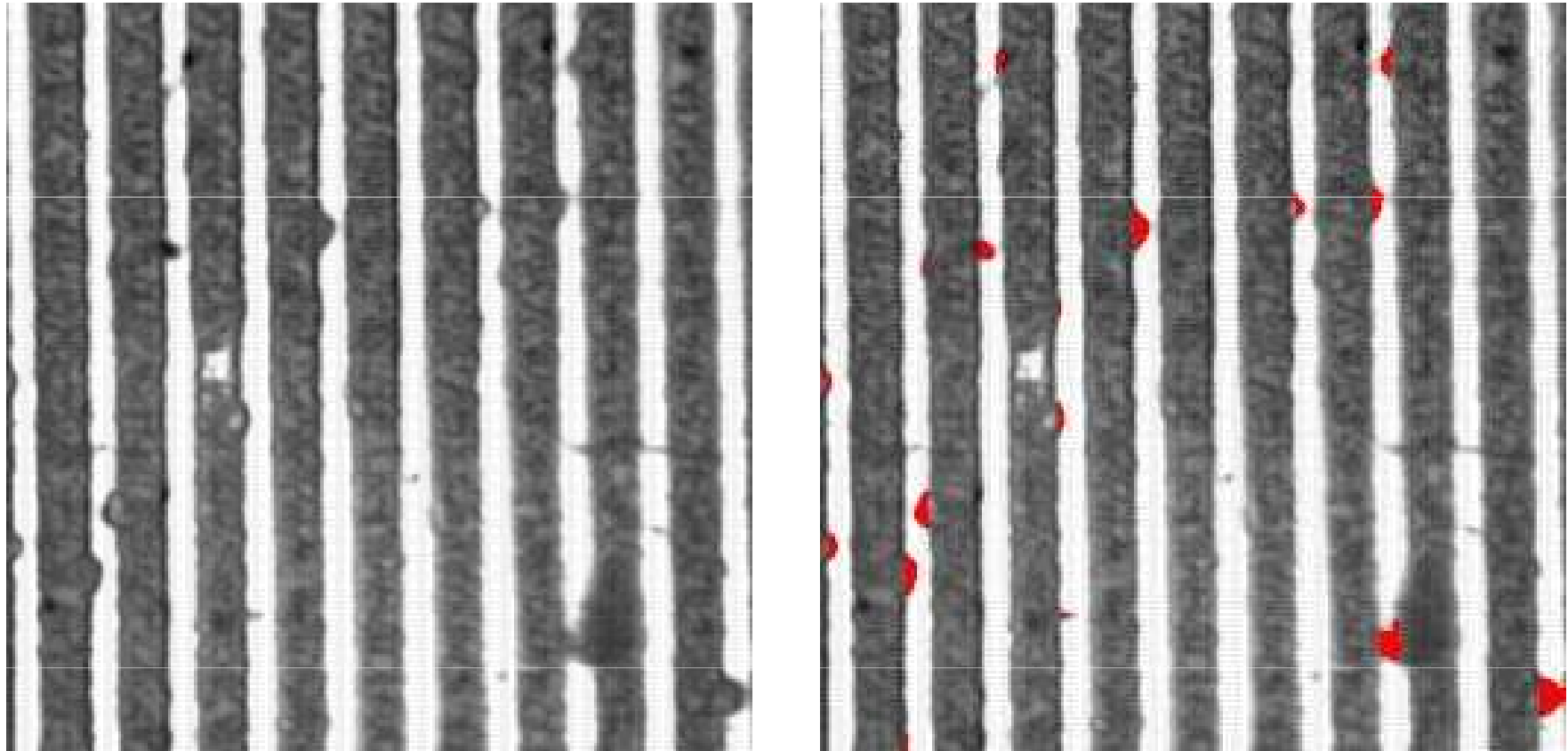
Examples

Detecting defects in a microelectronic circuit

http://www.mmorph.com/mxmorph/html/mmdemos/mmdlith.html
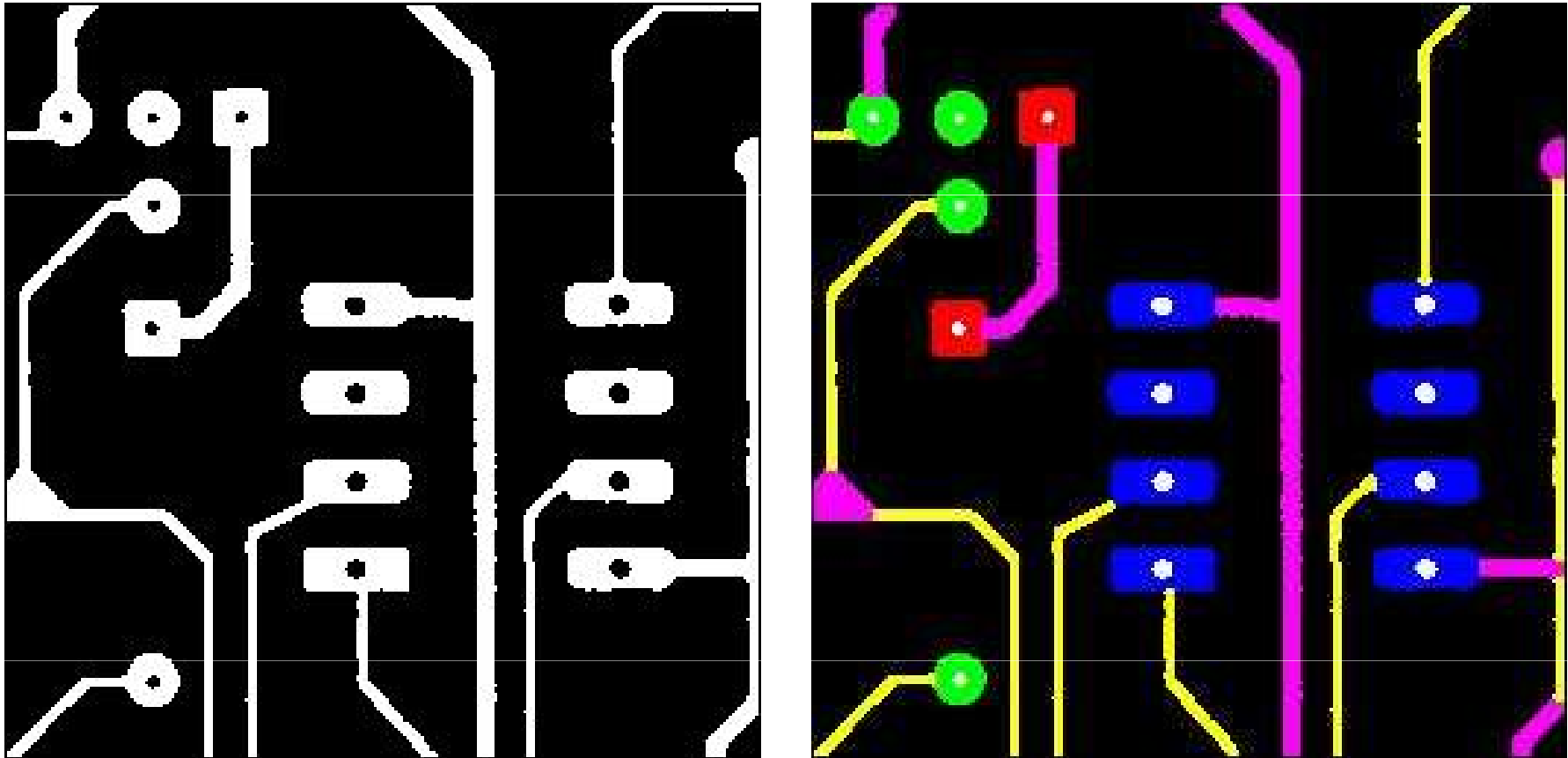
Examples

Decomposing a printed circuit board in its main parts

http://www.mmorph.com/mxmorph/html/mmdemos/mmdpcb.html
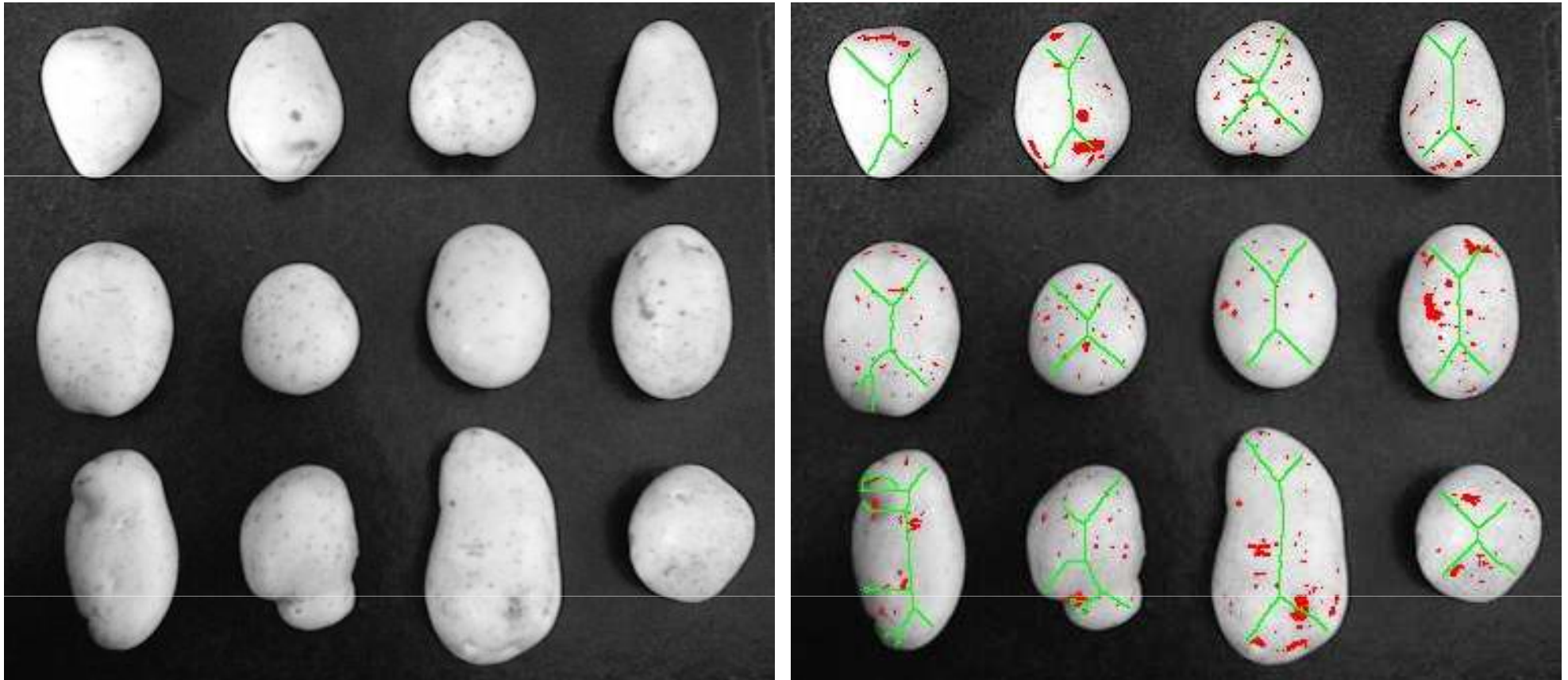
Examples

Grading potato quality by shape and skin spots

http://www.mmorph.com/mxmorph/html/mmdemos/mmdpotatoes.html
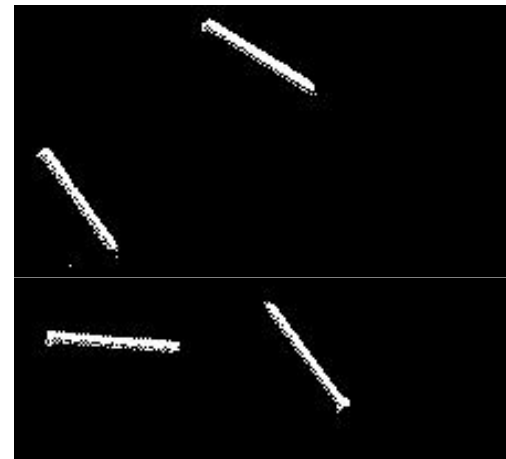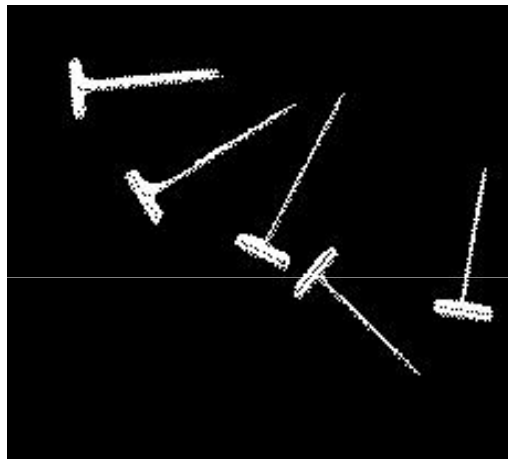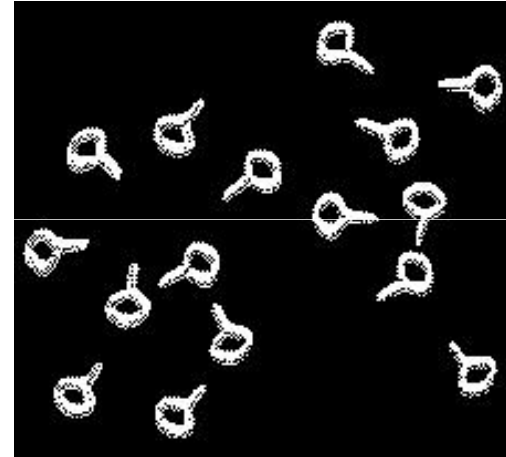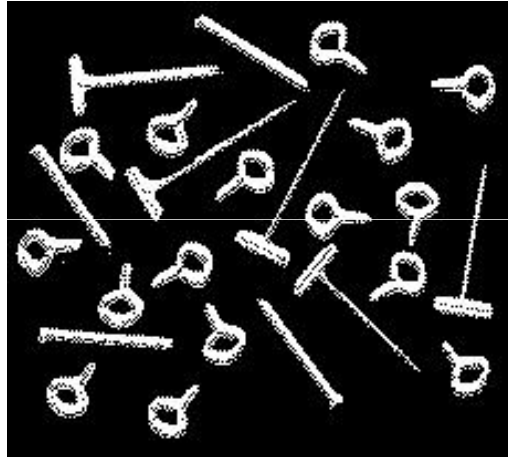
Examples

Classifying two dimensional pieces

http://www.mmorph.com/mxmorph/html/mmdemos/mmdpieces.html

Examples

Traffic scene   Temporal average   Average of differences

Lane detection example
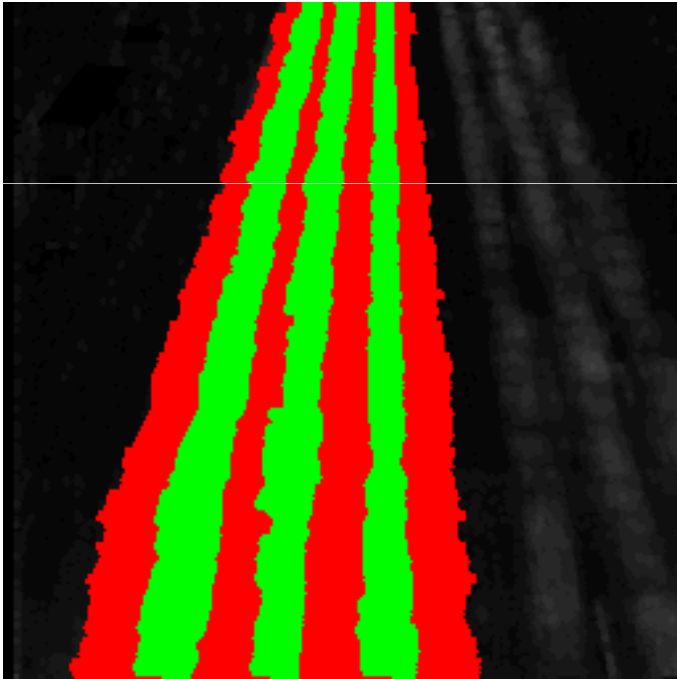
Examples

Threshold and dilation to detect lane markers
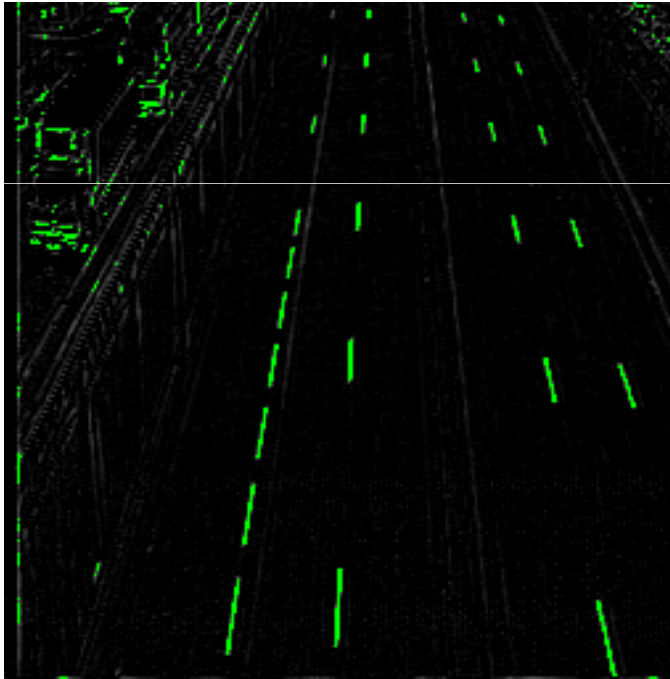
White line detection (top hat)
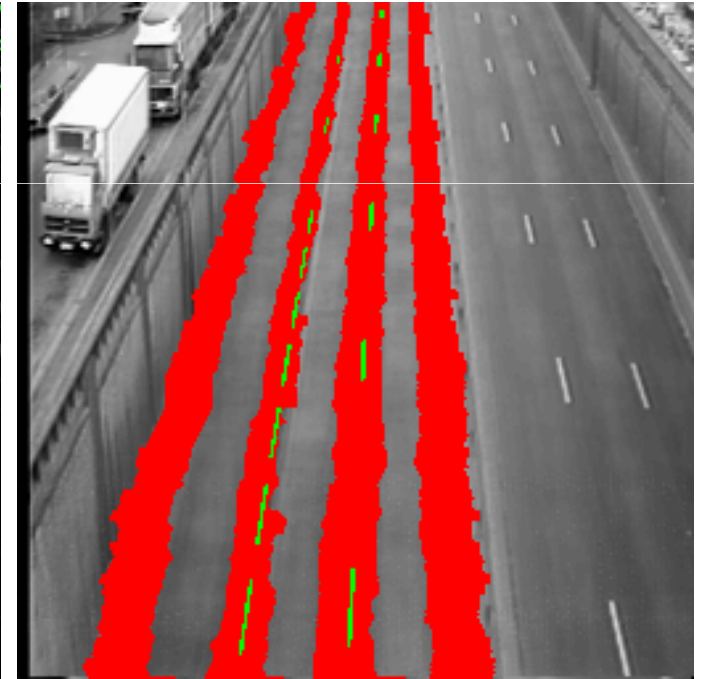
Detected lanes

Lane detection example

Pixels and neighborhoods

- In many algorithms, not only the value of a particular pixel, but also the values of its neighbors are used when processing that pixel.

- The two most common definitions for neighbors are the 4-neighbors and the 8-neighbors of a pixel.

	N	
W	*	E
	S	

NW	N	NE
W	*	E
SW	S	SE

a) four-neighborhood N_4 b) eight-neighborhood N_8

Figure 3.2: The two most common neighborhoods of a pixel.

Pixels and neighborhoods

- Given pixels $p = (x_1, y_1)$ and $q = (x_2, y_2)$:

  ▶ The *Euclidean distance* between p and q is defined as

  $$D_e(p, q) = \left((x_1 - x_2)^2 + (y_1 - y_2)^2\right)^{1/2}.$$

  ▶ The D_4 distance (also called the *city-block distance*) between p and q is defined as

  $$D_4(p, q) = |x_1 - x_2| + |y_1 - y_2|.$$

  ▶ The D_8 distance (also called the *chessboard distance*) between p and q is defined as

  $$D_8(p, q) = \max\{|x_1 - x_2|, |y_1 - y_2|\}.$$

Pixels and neighborhoods

$$2$$
$$2 \quad 1 \quad 2$$
$$2 \quad 1 \quad 0 \quad 1 \quad 2$$
$$2 \quad 1 \quad 2$$
$$2$$

Pixels with $D_4 \leq 2$ from the center pixel.

$$2 \quad 2 \quad 2 \quad 2 \quad 2$$
$$2 \quad 1 \quad 1 \quad 1 \quad 2$$
$$2 \quad 1 \quad 0 \quad 1 \quad 2$$
$$2 \quad 1 \quad 1 \quad 1 \quad 2$$
$$2 \quad 2 \quad 2 \quad 2 \quad 2$$

Pixels with $D_8 \leq 2$ from the center pixel.

Connected components analysis

- Once you have a binary image, you can identify and then analyze each connected set of pixels.

- The connected components operation takes in a binary image and produces a labeled image in which each pixel has the integer label of either the background (0) or a component.
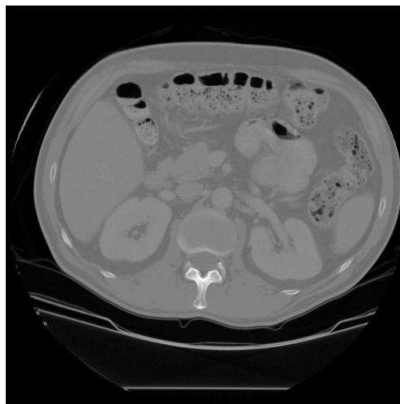
Original image   Thresholded image   After morphology   Connected components
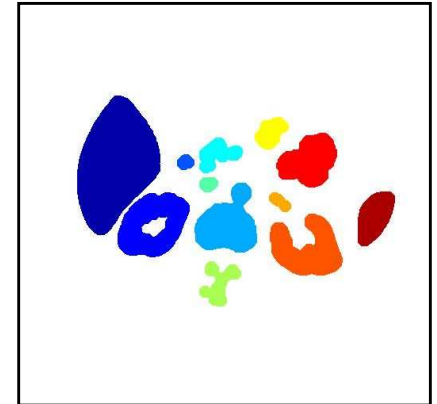
Connected components analysis

- **Methods for connected components analysis:**
  - Recursive tracking (almost never used)
  - Parallel growing (needs parallel hardware)
  - Row-by-row (most common)
    - Classical algorithm
    - Run-length algorithm (see Shapiro-Stockman)

1	1	0	1	1	1	0	1
1	1	0	1	0	1	0	1
1	1	1	1	0	0	0	1
0	0	0	0	0	0	0	1
1	1	1	1	0	1	0	1
0	0	0	1	0	1	0	1
1	1	0	1	0	0	0	1
1	1	0	1	0	1	1	1

a) binary image

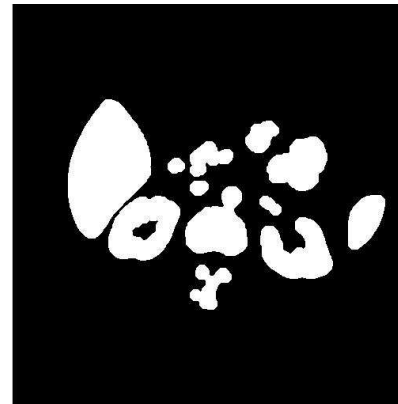|							
1	1	0	1	1	1	0	2
1	1	0	1	0	1	0	2
1	1	1	1	0	0	0	2
0	0	0	0	0	0	0	2
3	3	3	3	0	4	0	2
0	0	0	3	0	4	0	2
5	5	0	3	0	0	0	2
5	5	0	3	0	2	2	2

b) connected components labeling
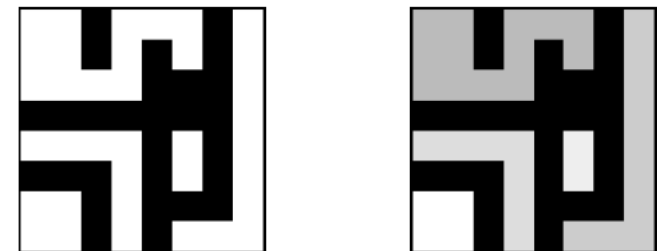
c) binary image and labeling, expanded for viewing

Connected components analysis

- Recursive labeling algorithm:
    1. Negate the binary image so that all 1s become -1s.
    2. Find a pixel whose value is -1, assign it a new label, call procedure *search* to find its neighbors that have values -1, and recursively repeat the process for these neighbors.

	1	
2	*	3
	4	

a) four-neighborhood

1	2	3
4	*	5
6	7	8

b) eight-neighborhood

Figure 3.7: Scan-line order for returning the neighbors of a pixel.

Compute the connected components of a binary image.
B is the original binary image.
LB will be the labeled connected component image.

```
procedure recursive_connected_components(B, LB);
{
LB := negate(B);
label := 0;
find_components(LB, label);
print(LB);
}

procedure find_components(LB, label);
{
for L := 0 to MaxRow
    for P := 0 to MaxCol
        if LB[L,P] == -1 then
            {
            label := label + 1;
            search(LB, label, L, P);
            }
}

procedure search(LB, label, L, P);
{
LB[L,P] := label;
Nset := neighbors(L, P);
for each (L',P') in Nset
    {
    if LB[L',P'] == -1
    then search(LB, label, L', P');
    }
}
```

Algorithm 2: Recursive Connected Components

Connected components analysis

- Row-by-row labeling algorithm:
    1. The first pass propagates a pixel's label to its neighbors to the right and below it. (Whenever two different labels can propagate to the same pixel, these labels are recorded as an equivalence class.)
    2. The second pass performs a translation, assigning to each pixel the label of its equivalence class.

- A union-find data structure is used for efficient construction and manipulation of equivalence classes represented by tree structures.

Connected components analysis

PARENT

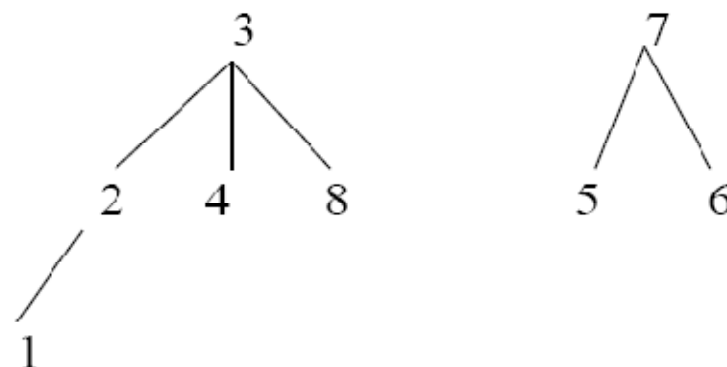| 1	2	3	4	5	6	7	8
2	3	0	3	7	7	0	3

Figure 3.9: The union-find data structure for two sets of labels. The first set contains the labels { 1,2,3,4,8 } , and the second set contains labels { 5,6,7 } . For each integer label i, the value of $PARENT[i]$ is the label of the parent of i or zero if i is a root node and has no parent.

Connected components analysis

```
Find the parent label of a set.
X is a label of the set.
PARENT is the array containing the union-find data structure.


    procedure find(X, PARENT);
    {
    j := X;
    while PARENT[j] <> 0
      j := PARENT[j];
    return(j);
    }
```

Algorithm 3: Find

```
Construct the union of two sets.
X is the label of the first set.
Y is the label of the second set.
PARENT is the array containing the union-find data structure.


    procedure union(X, Y, PARENT);
    {
    j := X;
    k := Y;
    while PARENT[j] <> 0
      j := PARENT[j];
    while PARENT[k] <> 0
      k := PARENT[k];
    if j <> k then PARENT[k] := j;
    }
```

Algorithm 4: Union

Adapted from Shapiro and Stockman

Connected components analysis

Initialize the data structures for classical connected components.

```
procedure initialize();
"Initialize global variable label and array PARENT."
{
"Initialize label."
label := 0;
"Initialize the union-find structure."
for i := 1 to MaxLab
    PARENT[i] := 0;
}
```

Algorithm 5: Initialization for Classical Connected Components
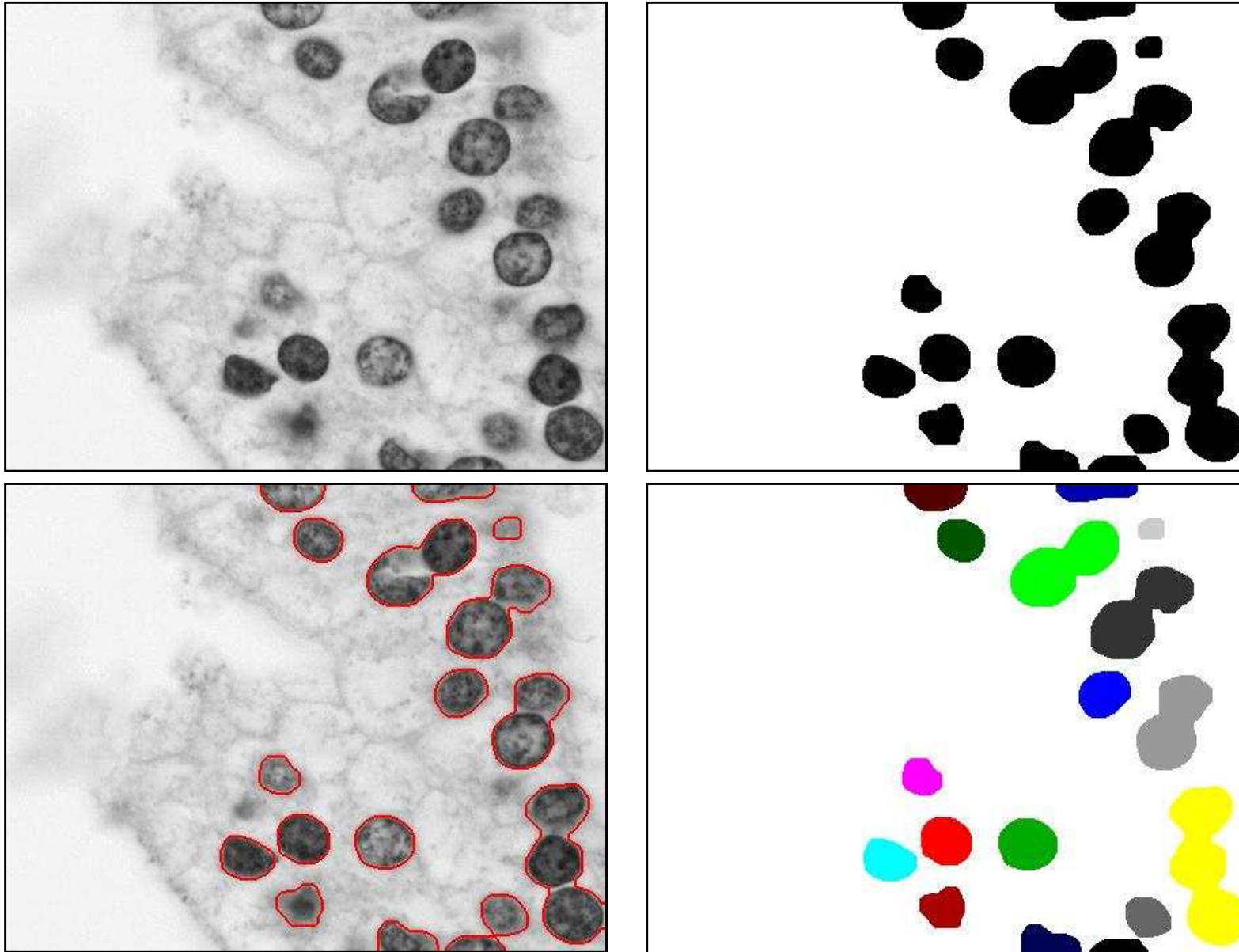
Adapted from Shapiro and Stockman

Compute the connected components of a binary image.
B is the original binary image.
LB will be the labeled connected component image.

```
procedure classical_with_union-find(B,LB);
{
"Initialize structures."
initialize();
"Pass 1 assigns initial labels to each row L of the image."
for L := 0 to MaxRow
  {
  "Initialize all labels on line L to zero"
  for P := 0 to MaxCol
    LB[L,P] := 0;
  "Process line L."
  for P := 0 to MaxCol
    if B[L,P] == 1 then
      {
      A := prior_neighbors(L,P);
      if isempty(A)
      then { M := label; label := label + 1; };
      else M := min(labels(A));
      LB[L,P] := M;
      for X in labels(A) and X <> M
        union(M, X, PARENT);
      }
  }
"Pass 2 replaces Pass 1 labels with equivalence class labels."
for L := 0 to MaxRow
  for P := 0 to MaxCol
    if B[L,P] == 1
    then LB[L,P] := find(LB[L,P],PARENT);
} ;
```

Algorithm 6: Classical Connected Components with Union-Find

Connected components analysis

Connected components analysis