Probabilistic Graphical Models
Part III: Example Applications

Selim Aksoy

Department of Computer Engineering
Bilkent University
saksoy@cs.bilkent.edu.tr

CS 551, Fall 2012
We will look at example uses of Bayesian networks and Markov networks for the following applications:

- Alarm network for monitoring intensive care patients — Bayesian networks
- Recommendation system — Bayesian networks
- Diagnostic systems — Bayesian networks
- Statistical text analysis — probabilistic latent semantic analysis
- Scene classification — probabilistic latent semantic analysis
- Object detection — probabilistic latent semantic analysis
- Image segmentation — Markov random fields
- Contextual classification — conditional random fields
Figure 1: The “alarm” network for monitoring intensive care patients. The network has 37 variables and 509 parameters (full joint has $2^{37}$).
Recommendation Systems

- Given user preferences, the system can suggest recommendations.
- Input: movie preferences of many users.
- Output: model correlations between movie features.
  - Users that like comedy, often like drama.
  - Users that like action, often do not like cartoons.
  - Users that like Robert De Niro films, often like Al Pacino films.
- Given user preferences, the system can predict the probability that new movies match preferences.
Figure 2: Diagnostic indexing for home health site at Microsoft. Users can enter symptoms and can get recommendations.
T. Hofmann, “Unsupervised learning by probabilistic latent 
semantic analysis,” *Machine Learning*, vol. 42, no. 1–2, 

The probabilistic latent semantic analysis (PLSA) algorithm 
has been originally developed for statistical text analysis to 
discover topics in a collection of documents that are 
represented using the frequencies of words from a 
vocabulary.
PLSA uses a graphical model for the joint probability of the
documents and their words in terms of the probability of
observing a word given a topic (aspect) and the probability
of a topic given a document.

Suppose there are $N$ documents having content coming
from a vocabulary with $M$ words.

The collection of documents is summarized in an $N$-by-$M$
co-occurrence table $n$ where $n(d_i, w_j)$ stores the number of
occurrences of word $w_j$ in document $d_i$.

In addition, there is a latent topic variable $z_k$ associated with
each observation, an observation being the occurrence of a
word in a particular document.
Figure 3: The graphical model used by PLSA for modeling the joint probability $P(w_j, d_i, z_k)$. 
The generative model $P(d_i, w_j) = P(d_i)P(w_j|d_i)$ for word content of documents can be computed using the conditional probability

$$P(w_j|d_i) = \sum_{k=1}^{K} P(w_j|z_k)P(z_k|d_i).$$

- $P(w_j|z_k)$ denotes the topic-conditional probability of word $w_j$ occurring in topic $z_k$.
- $P(z_k|d_i)$ denotes the probability of topic $z_k$ observed in document $d_i$.
- $K$ is the number of topics.
Then, the topic specific word distribution $P(w_j|z_k)$ and the document specific word distribution $P(w_j|d_i)$ can be used to determine similarities between topics and documents.

In PLSA, the goal is to identify the probabilities $P(w_j|z_k)$ and $P(z_k|d_i)$.

These probabilities are learned using the EM algorithm.
In the E-step, the posterior probability of the latent variables are computed based on the current estimates of the parameters as

\[ P(z_k|d_i, w_j) = \frac{P(w_j|z_k)P(z_k|d_i)}{\sum_{l=1}^{K}P(w_j|z_l)P(z_l|d_i)}. \]

In the M-step, the parameters are updated to maximize the expected complete data log-likelihood as

\[ P(w_j|z_k) = \frac{\sum_{i=1}^{N} n(d_i, w_j)P(z_k|d_i, w_j)}{\sum_{m=1}^{M}\sum_{i=1}^{N} n(d_i, w_m)P(z_k|d_i, w_m)}, \]

\[ P(z_k|d_i) = \frac{\sum_{j=1}^{M} n(d_i, w_j)P(z_k|d_i, w_j)}{\sum_{j=1}^{M} n(d_i, w_j)}. \]
Figure 4: Four aspects (topics) to most likely generate the word “segment”, derived from a $K = 128$ aspects model of a document collection consisting of abstracts of 1568 documents on clustering. The displayed word stems are the most probable words in the class-conditional distribution $P(w_j|z_k)$, from top to bottom in descending order.
Figure 5: Abstracts of four exemplary documents from the collection along with latent class posterior probabilities $P(z_k | d_i, W = \text{"segment"})$ and word probabilities $P(w = \text{"segment"}|d_i)$. 

Document 1, $P(z_k | d_1, W = \text{"segment"}) = (0.951, 0.002, 0.001, 0.0001, \ldots)$  
$P(W = \text{"segment"}|d_1) = 0.06$

**SEGMENT** medici imag challen imag field imag analys diagnos base propr **SEGMENT** digit imag **SEGMENT** medici imag need applic invol estima boundari object classif tissu abnorm shap analys contain detect textur **SEGMENT** despit exist techniqu **SEGMENT** specif medici imag remain crucial problem [...]

Document 2, $P(z_k | d_2, W = \text{"segment"}) = (0.025, 0.956, 0.0002, 0.0002, \ldots)$  
$P(W = \text{"segment"}|d_2) = 0.014$

describ new techniqu extract hierarch decompos complex video secur brows purpous techniqu combin visual tempor inform captur import relat scene scene video allow analysi underli stori structur priori knowledg content defin gener model hierarch scene transition graph appli model implement brows video shot identifi collect kei frame repres video **SEGMENT** collec classifi accord gross visual inform [...]

Document 3, $P(z_k | d_3, W = \text{"segment"}) = (0.025, 0.003, 0.897, 0.016, \ldots)$  
$P(W = \text{"segment"}|d_3) = 0.010$

paper describ contou extrac scheme refin roughli estim initi contou outlin precis object boundari author approach mixtur densit descrip parametr describ decompos subregio obtain region cluster descrip likelihood pixel belong object background evalua unlik activ contou extrac scheme region edgebas estim scheme integ energi minim process [...]

Document 4, $P(z_k | d_4, W = \text{"segment"}) = (0.025, 0.076, 0.001, 0.867, \ldots)$  
$P(W = \text{"segment"}|d_4) = 0.010$

consid signal origin sequenc sour specif problem **SEGMENT** signal relat **SEGMENT** sour address issu wide applic field report describ resol method ergod hidden markov model hmm hmm state correspond signal sour signal sour sequenc determin decod procedur viterbi algorithm forward algorithm observ sequenc baumwelch train estim hmm paramet train materi applic multipl signal sour identif problem expert perform unknown speaker identif [...]

Figure 5: Abstracts of four exemplary documents from the collection along with latent class posterior probabilities $P(z_k | d_i, w = \text{"segment"})$ and word probabilities $P(w = \text{"segment"}|d_i)$. 

CS 551, Fall 2012 ©2012, Selim Aksoy (Bilkent University) 13 / 38
Scene Classification


- The PLSA model is used for scene classification by modeling images using visual words (visterms).

- The topic (aspect) probabilities are used as features as an alternative representation to the word histograms.
Figure 6: Image representation as a collection of visual words (visterms).
Figure 7: 10 most probable images from a data set consisting of city and landscape images for seven topics (aspects) out of 20.

We used the PLSA technique for object detection to model the joint probability of the segments and their features in terms of the probability of observing a feature given an object and the probability of an object given the segment.
Figure 8: After image segmentation, each segment is modeled using the statistical summary of its pixel content (e.g., quantized spectral values).
Figure 9: (a) PLSA graphical model. The filled nodes indicate observed random variables whereas the unfilled node is unobserved. The red arrows show examples for the measurements represented at each node. (b) In PLSA, the object specific feature probability, $P(x_j | t_k)$, and the segment specific object probability, $P(t_k | s_i)$, are used to compute the segment specific feature probability, $P(x_j | s_i)$. 
Object Detection

- After learning the parameters of the model, we want to find good segments belonging to each object type.

- This is done by comparing the object specific feature distribution $P(x|t)$ and the segment specific feature distribution $P(x|s)$.

- The similarity between two distributions can be measured using the Kullback-Leibler (KL) divergence $D(p(x|s)\|p(x|t))$.

- Then, for each object type, the segments can be sorted according to their KL divergence scores, and the most representative ones for that object type can be selected.
Object Detection

Figure 10: Examples of object detection.
Object Detection

(a) Image  (b) Buildings  (c) Roads  (d) Vegetation

Figure 11: Examples of object detection.

Markov random fields are used as a neighborhood model for image segmentation by classifying pixels into different pixel classes.
The goal is to assign each pixel into a set of labels $w \in \Omega$.

- Pixels are modeled using color and texture features.
- Pixel features are modeled using multivariate Gaussians, $p(x|w)$.
- A first-order neighborhood system is used as the prior for the labeling process.
Figure 12: The Markov random field used as the first-order neighborhood model for the labeling process.
Image Segmentation

- The prior is modeled as

\[ p(w) = \frac{1}{Z} \exp \left( - \sum_{c \in \mathcal{C}} V_c(w_c) \right) \]

where \( V_c \) denotes the clique potential of clique \( c \in \mathcal{C} \) having the label configuration \( w_c \).

- Each clique corresponds to a pair of neighboring pixels.

- The potentials favor similar classes in neighboring pixels as

\[ V_c = \delta(w_s, w_r) = \begin{cases} 
+1 & \text{if } w_s \neq w_r, \\
-1 & \text{otherwise.} 
\end{cases} \]
The prior is proportional to the length of the region boundaries. Thus, homogeneous segmentations will get a higher probability.

The final labeling for each pixel is done by maximizing the posterior probability

\[ p(w|x) \propto p(x|w)p(w). \]
Figure 13: Example segmentation results.
Figure 14: Example Markov random field models used in the literature. (a) First-order neighborhood system. (b) Non-regular planar graph associated to an image partition. (c) Quad-tree.
Contextual Classification


- Semantic context among objects is used for improving object categorization.
Figure 15: Idealized context based object categorization system: an original image is perfectly segmented into objects; each object is categorized; and object’s labels are refined with respect to semantic context in the image.
Figure 16: Object categorization framework: $S_1, \ldots, S_k$ is the set of $k$ segments for an image; $L_1, \ldots, L_n$ is a ranked list of $n$ labels for each segment; $O_1, \ldots, O_m$ is a set of $m$ object categories in the image.
A conditional random field (CRF) framework is used to incorporate semantic context into the object categorization. Given an image $I$ and its segmentation $S_1, \ldots, S_k$, the goal is to find segment labels $c_1, \ldots, c_k$ such that they agree with the segment contents and are in contextual agreement with each other.
This interaction is modeled as a probability distribution

\[ p(c_1, \ldots, c_k | S_1, \ldots, S_k) = \frac{B(c_1, \ldots, c_k) \prod_{i=1}^{k} A(i)}{Z(\phi, S_1, \ldots, S_k)} \]

with

\[ A(i) = p(c_i | S_i) \text{ and } B(c_1, \ldots, c_k) = \exp \left( \sum_{i,j=1}^{k} \phi(c_i, c_j) \right), \]

where \( Z(\cdot) \) is the partition function.

The semantic context information is modeled using context matrices that are symmetric, nonnegative matrices that contain the co-occurrence frequency among object labels in the training set.
Figure 17: An example conditional random field. Squares indicate feature functions and circles indicate variable nodes. Arrows represent single node potentials due to feature functions, and undirected edges represent pairwise potentials. Global context is represented by $h$. 
Figure 18: An example context matrix.
Figure 19: Example results where context improved the categorization accuracy. Left to right: original segmentation, categorization w/o contextual constraints, categorization w/ contextual constraints, ground truth.
Contextual Classification

Figure 20: Example results where context reduced the categorization accuracy. Left to right: original segmentation, categorization w/o contextual constraints, categorization w/ contextual constraints, ground truth.