Probabilistic Graphical Models
Part III: Example Applications

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Introduction

- We will look at example uses of Bayesian networks and Markov networks for the following applications:
  - Alarm network for monitoring intensive care patients — Bayesian networks
  - Recommendation system — Bayesian networks
  - Diagnostic systems — Bayesian networks
  - Statistical text analysis — probabilistic latent semantic analysis
  - Statistical text analysis — latent Dirichlet allocation
  - Scene classification — probabilistic latent semantic analysis
  - Object detection — probabilistic latent semantic analysis
  - Image segmentation — Markov random fields
  - Contextual classification — conditional random fields

Intensive Care Monitoring

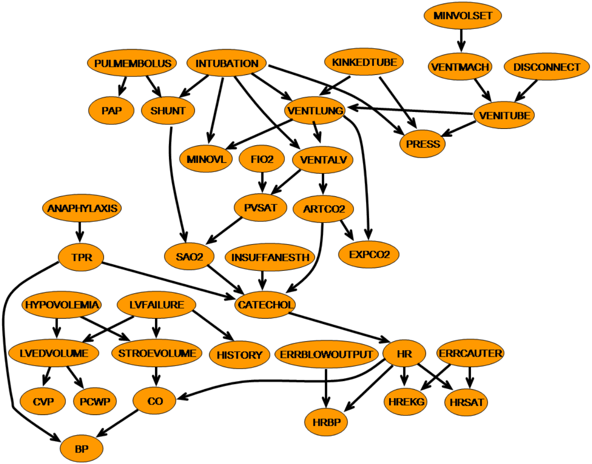


Figure 1: The “alarm” network for monitoring intensive care patients. The network has 37 variables and 509 parameters (full joint has 2^{37}). (Figure from N. Friedman)

Recommendation Systems

- Given user preferences, the system can suggest recommendations.
- Input: movie preferences of many users.
- Output: model correlations between movie features.
  - Users that like comedy, often like drama.
  - Users that like action, often do not like cartoons.
  - Users that like Robert De Niro films, often like Al Pacino films.
  - Given user preferences, the system can predict the probability that new movies match preferences.

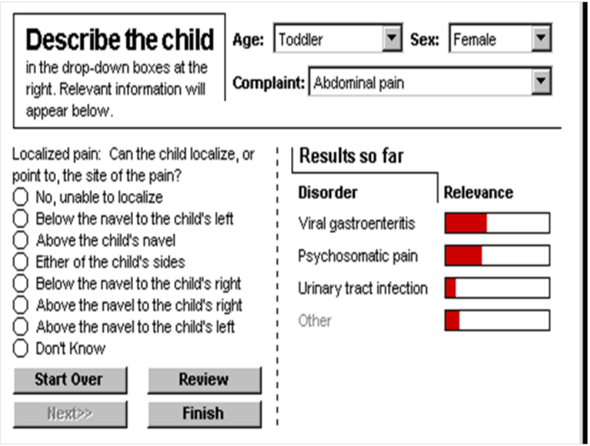


Figure 2: Diagnostic indexing for home health site at Microsoft. Users can enter symptoms and can get recommendations.

Statistical Text Analysis

- Input: An unorganized collection of documents
- Output: An organized collection, and a description of how

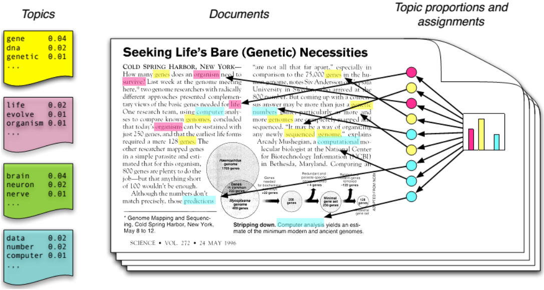


Figure 3: We assume that some number of "topics", which are distributions over words, exist for the whole collection. Each document is assumed to be generated as follows. First, choose a distribution over the topics; then, for each word, choose a topic assignment, and choose the word from the corresponding topic. (Figure from D. Blei)

Statistical Text Analysis

- T. Hofmann, "Unsupervised learning by probabilistic latent semantic analysis," *Machine Learning*, vol. 42, no. 1–2, pp. 177–196, January–February 2001.
- The probabilistic latent semantic analysis (PLSA) algorithm has been originally developed for statistical text analysis to discover topics in a collection of documents that are represented using the frequencies of words from a vocabulary.

Statistical Text Analysis

- PLSA uses a graphical model for the joint probability of the documents and their words in terms of the probability of observing a word given a topic (aspect) and the probability of a topic given a document.
- Suppose there are N documents having content coming from a vocabulary with M words.
- The collection of documents is summarized in an N-by-M co-occurrence table n where $n(d_i, w_j)$ stores the number of occurrences of word w_j in document d_i.
- In addition, there is a latent topic variable z_k associated with each observation, an observation being the occurrence of a word in a particular document.

Statistical Text Analysis

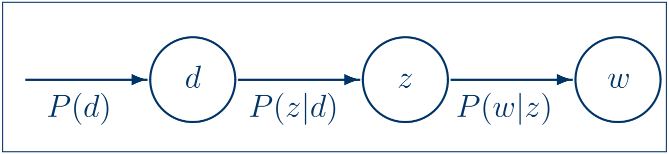


Figure 4: The graphical model used by PLSA for modeling the joint probability $P(w_j, d_i, z_k)$.

Statistical Text Analysis

- The generative model $P(d_i, w_j) = P(d_i)P(w_j|d_i)$ for word content of documents can be computed using the conditional probability

$$P(w_j|d_i) = \sum_{k=1}^{K} P(w_j|z_k)P(z_k|d_i).$$

- $P(w_j|z_k)$ denotes the topic-conditional probability of word w_j occurring in topic z_k.
- $P(z_k|d_i)$ denotes the probability of topic z_k observed in document d_i.
- K is the number of topics.

Statistical Text Analysis

- Then, the topic specific word distribution $P(w_j|z_k)$ and the document specific word distribution $P(w_j|d_i)$ can be used to determine similarities between topics and documents.
- In PLSA, the goal is to identify the probabilities $P(w_j|z_k)$ and $P(z_k|d_i)$.
- These probabilities are learned using the EM algorithm.

Statistical Text Analysis

- In the E-step, the posterior probability of the latent variables are computed based on the current estimates of the parameters as

$$P(z_k|d_i, w_j) = \frac{P(w_j|z_k)P(z_k|d_i)}{\sum_{l=1}^{K} P(w_j|z_l)P(z_l|d_i)}.$$

- In the M-step, the parameters are updated to maximize the expected complete data log-likelihood as

$$P(w_j|z_k) = \frac{\sum_{i=1}^{N} n(d_i, w_j)P(z_k|d_i, w_j)}{\sum_{m=1}^{M} \sum_{i=1}^{N} n(d_i, w_m)P(z_k|d_i, w_m)},$$

$$P(z_k|d_i) = \frac{\sum_{j=1}^{M} n(d_i, w_j)P(z_k|d_i, w_j)}{\sum_{j=1}^{M} n(d_i, w_j)}.$$

Statistical Text Analysis

Aspect 1	Aspect 2	Aspect 3	Aspect 4
imag	video	region	speaker
SEGMENT	sequenc	contour	speech
textur	motion	boundari	recogni
color	frame	descrip	signal
tissu	scene	imag	train
brain	SEGMENT	SEGMENT	hmm
slice	shot	precis	sourc
cluster	imag	estim	speakerindepend
mri	cluster	pixel	SEGMENT
algorithm	visual	paramet	sound

Figure 5: Four aspects (topics) to most likely generate the word "segment", derived from a $K = 128$ aspects model of a document collection consisting of abstracts of 1568 documents on clustering. The displayed word stems are the most probable words in the class-conditional distribution $P(w_j|z_k)$, from top to bottom in descending order.

Statistical Text Analysis

Document 1, $P\{z_k|d_1, W = \text{'segment'}\} = (0.951, 0.002, 0.001, 0.0001, \ldots)$
$P\{W = \text{'segment'}|d_1\} = 0.06$

SEGMENT medic imag challeng problem field imag analysi diagnost base proper **SEGMENT** digit imag **SEGMENT** medic imag need applic involv estim boundari object classif tissu abnorm shape analysi contour detec textur **SEGMENT** despit exist techniqu **SEGMENT** specif medic imag remain crucial problem [...]

Document 2, $P\{z_k|d_2, W = \text{'segment'}\} = (0.025, 0.956, 0.0002, 0.0002, \ldots)$
$P\{W = \text{'segment'}|d_2\} = 0.014$

describ new techniqu extract hierarch decomposi complex video selec brows purpos techniqu combin visual tempor inform captur import relat scene scene video allow analysi underli stori structur priori knowledg content defin gener model hierarch scene transition graph appli model implement brows video shot identifi collec kei frame repres video **SEGMENT** collec classifi accord gross visual inform [...]

Document 3, $P\{z_k|d_3, W = \text{'segment'}\} = (0.025, 0.003, 0.897, 0.016, \ldots)$
$P\{W = \text{'segment'}|d_3\} = 0.010$

paper describ contour extrac scheme refin roughli estim initi contour outlin precis object boundari author approach mixtur densiti descrip parametr descrip decompos subregion obtain region cluster descrip likelihood pixel belong object background evalu unlik activ contour extrac scheme region edgebas estim scheme integr energi minim process [...]

Document 4, $P\{z_k|d_3, W = \text{'segment'}\} = (0.025, 0.076, 0.001, 0.867, \ldots)$
$P\{W = \text{'segment'}|d_4\} = 0.010$

consid signal origin sequenc sourc specif problem **SEGMENT** signal relat **SEGMENT** sourc address issu wide applic field report describ resolu method ergod hidden markov model hmm hmm state correspond signal sourc signal sourc sequenc determin decod procedur viterbi algorithm forward algorithm observ sequenc baumwelch train estim hmm paramet train materi applic multipl signal sourc identif problem experi perform unknown speaker identif [...]

Figure 6: Abstracts of four examplary documents from the collection along with latent class posterior probabilities $P(z_k|d_i, w = \text{"segment"})$ and word probabilities $P(w = \text{"segment"}|d_i)$.

Statistical Text Analysis

- D. M. Blei, A. Y. Ng, M. I. Jordan, "Latent Dirichlet Allocation," *Journal of Machine Learning Research*, vol. 3, pp. 993–1022, January 2003.
- D. M. Blei, "Probabilistic Topic Models," *Communications of the ACM*, vol. 55, no. 4, pp. 77–84, April 2012.
- Latent Dirichlet allocation (LDA) is a similar topic model with the addition of a prior on the topic distribution of a document.

Statistical Text Analysis

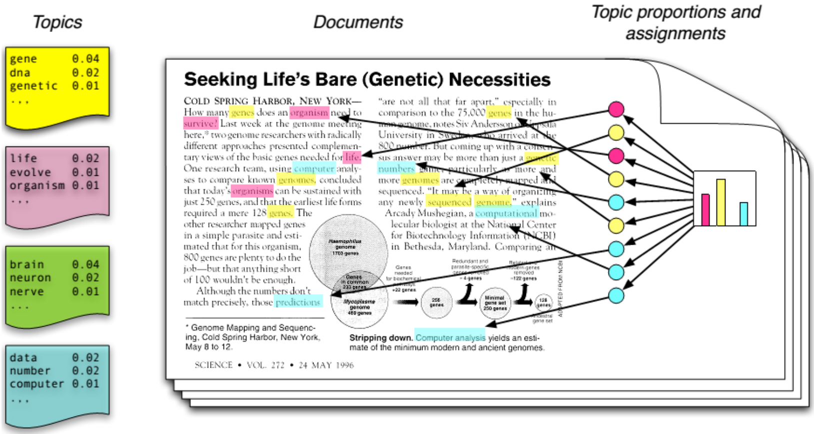


Figure 7: Each topic is a distribution over words. Each document is a mixture of corpus-wide topics. Each word is drawn from one of those topics.

Statistical Text Analysis

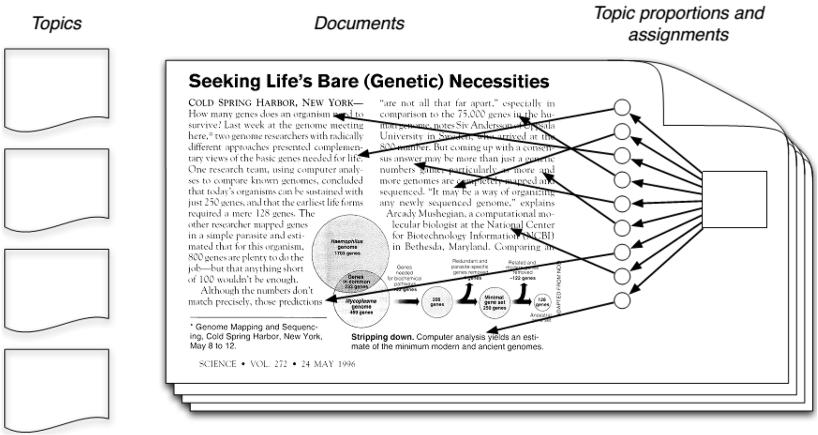
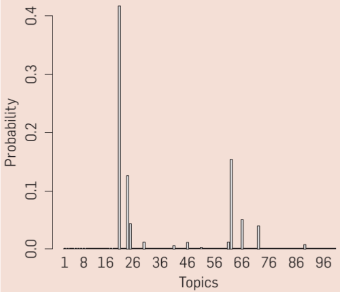


Figure 8: In reality we only observe the documents. The other structure are hidden variables. Our goal is to infer these variables, i.e., compute their posterior distribution conditioned on the documents.

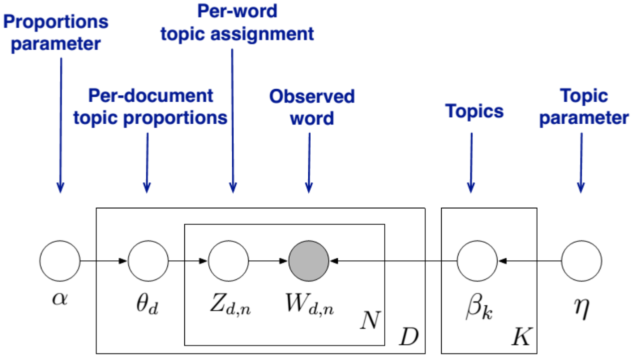
Statistical Text Analysis



"Genetics"	"Evolution"	"Disease"	"Computers"
human	evolution	disease	computer
genome	evolutionary	host	models
dna	species	bacteria	information
genetic	organisms	diseases	data
genes	life	resistance	computers
sequence	origin	bacterial	system
gene	biology	new	network
molecular	groups	strains	systems
sequencing	phylogenetic	control	model
map	living	infectious	parallel
information	diversity	malaria	methods
genetics	group	parasite	networks
mapping	new	parasites	software
project	two	united	new
sequences	common	tuberculosis	simulations

Figure 9: A 100-topic LDA model is fit to 17000 articles from the journal Science. (left) The inferred topic proportions for the article in the previous figure. (right) Top 15 most frequent words from the most frequent topics found in this article.

Statistical Text Analysis



$$\prod_{i=1}^{K} p(\beta_i \mid \eta) \prod_{d=1}^{D} p(\theta_d \mid \alpha) \left(\prod_{n=1}^{N} p(z_{d,n} \mid \theta_d) p(w_{d,n} \mid \beta_{1:K}, z_{d,n}) \right)$$

Figure 10: The LDA model defines a factorization of the joint distribution.

Statistical Text Analysis

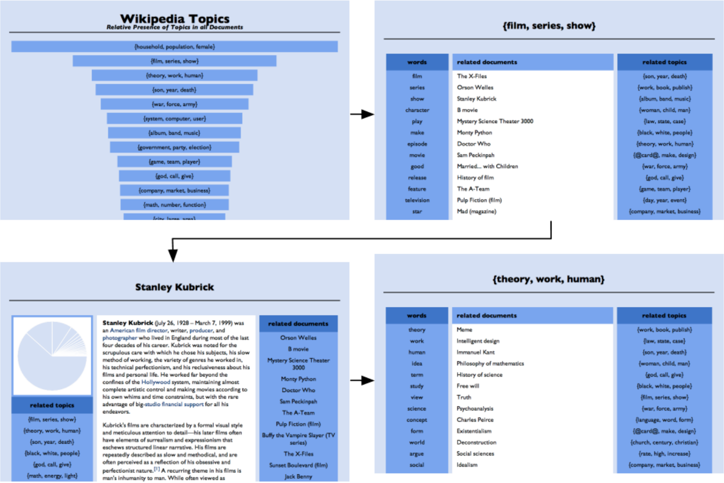


Figure 11: Example application: open source document browser.

Scene Classification

- P. Quelhas, F. Monay, J.-M. Odobez, D. Gatica-Perez, T. Tuytelaars, “A Thousand Words in a Scene,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 29, no. 9, pp. 1575–1589, September 2007.
- The PLSA model is used for scene classification by modeling images using visual words (visterms).
- The topic (aspect) probabilities are used as features as an alternative representation to the word histograms.

Scene Classification

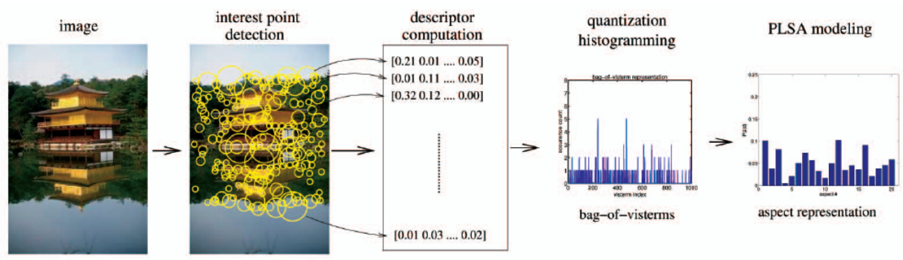


Figure 12: Image representation as a collection of visual words (visterms).

Scene Classification

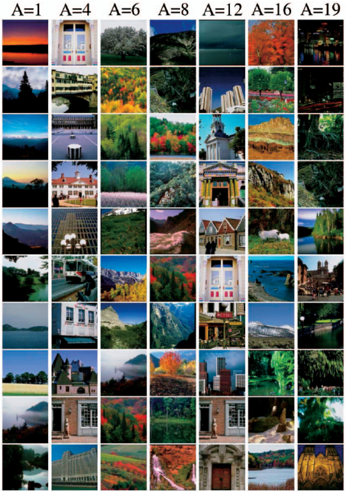


Figure 13: 10 most probable images from a data set consisting of city and landscape images for seven topics (aspects) out of 20.

Object Detection

- H. G. Akcay, S. Aksoy, "Automatic Detection of Geospatial Objects Using Multiple Hierarchical Segmentations," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 46, no. 7, pp. 2097–2111, July 2008.
- We used the PLSA technique for object detection to model the joint probability of the segments and their features in terms of the probability of observing a feature given an object and the probability of an object given the segment.

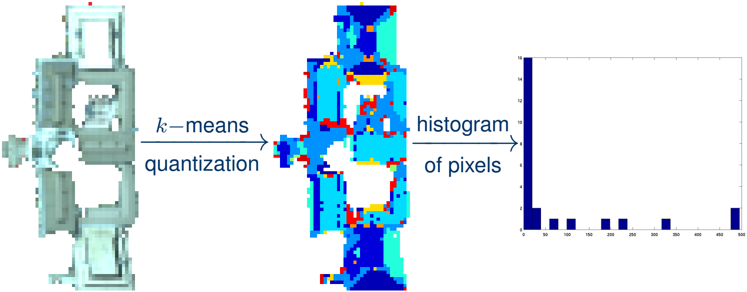


Figure 14: After image segmentation, each segment is modeled using the statistical summary of its pixel content (e.g., quantized spectral values).

Object Detection

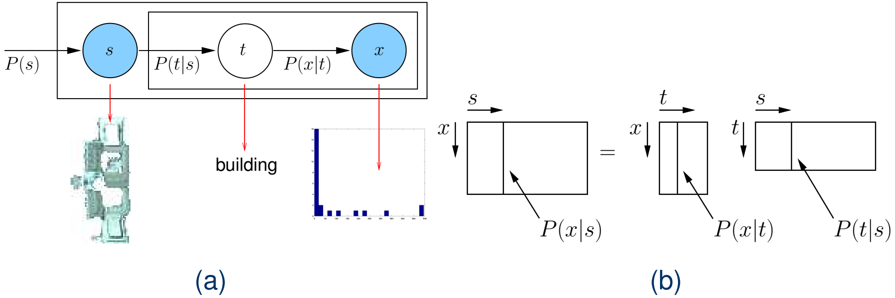
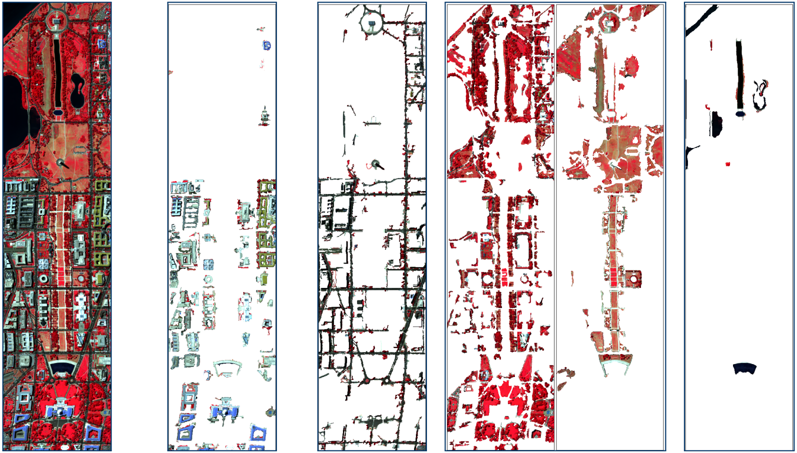


Figure 15: (a) PLSA graphical model. The filled nodes indicate observed random variables whereas the unfilled node is unobserved. The red arrows show examples for the measurements represented at each node. (b) In PLSA, the object specific feature probability, $P(x_j|t_k)$, and the segment specific object probability, $P(t_k|s_i)$, are used to compute the segment specific feature probability, $P(x_j|s_i)$.

Object Detection

- After learning the parameters of the model, we want to find good segments belonging to each object type.
- This is done by comparing the object specific feature distribution $P(x|t)$ and the segment specific feature distribution $P(x|s)$.
- The similarity between two distributions can be measured using the Kullback-Leibler (KL) divergence $D(p(x|s)\|p(x|t))$.
- Then, for each object type, the segments can be sorted according to their KL divergence scores, and the most representative ones for that object type can be selected.

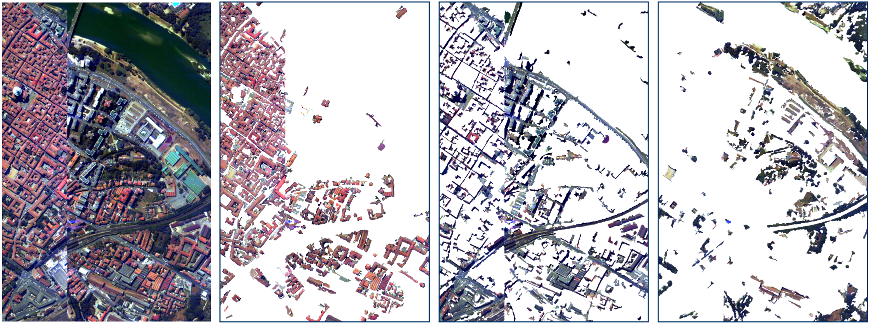
Object Detection



(a) Image (b) Buildings (c) Roads (d) Vegetation (e) Water

Figure 16: Examples of object detection.

Object Detection



(a) Image (b) Buildings (c) Roads (d) Vegetation

Figure 17: Examples of object detection.

Image Segmentation

- Z. Kato, T.-C. Pong, “A Markov random field image segmentation model for color textured images,” *Image and Vision Computing*, vol. 24, no. 10, pp. 1103–1114, October 2006.
- Markov random fields are used as a neighborhood model for image segmentation by classifying pixels into different pixel classes.

Image Segmentation

- The goal is to assign each pixel into a set of labels $w \in \Omega$.
- Pixels are modeled using color and texture features.
- Pixel features are modeled using multivariate Gaussians, $p(\mathbf{x}|w)$.
- A first-order neighborhood system is used as the prior for the labeling process.

Image Segmentation

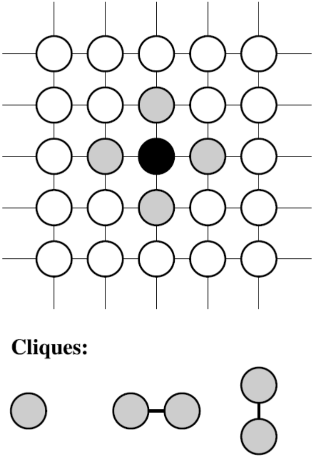


Figure 18: The Markov random field used as the first-order neighborhood model for the labeling process.

Image Segmentation

- The prior is modeled as

$$p(w) = \frac{1}{Z} \exp\left(- \sum_{c \in \mathcal{C}} V_c(w_c) \right)$$

  where V_c denotes the clique potential of clique $c \in \mathcal{C}$ having the label configuration w_c.
- Each clique corresponds to a pair of neighboring pixels.
- The potentials favor similar classes in neighboring pixels as

$$V_c = \delta(w_s, w_r) = \begin{cases} +1 & \text{if } w_s \neq w_r, \\ -1 & \text{otherwise.} \end{cases}$$

Image Segmentation

- The prior is proportional to the length of the region boundaries. Thus, homogeneous segmentations will get a higher probability.
- The final labeling for each pixel is done by maximizing the posterior probability

$$p(w|\mathbf{x}) \propto p(\mathbf{x}|w)p(w).$$

Image Segmentation

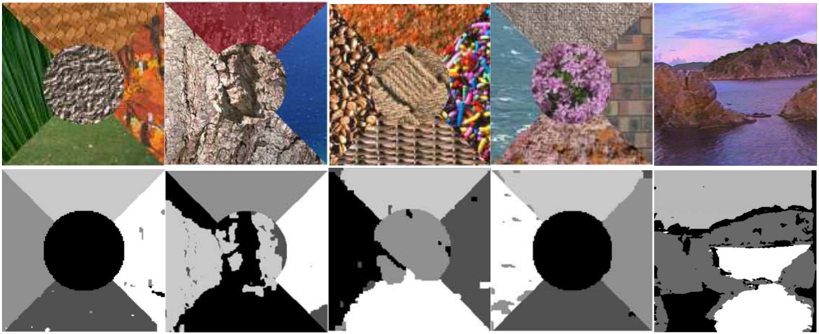


Figure 19: Example segmentation results.

Image Segmentation

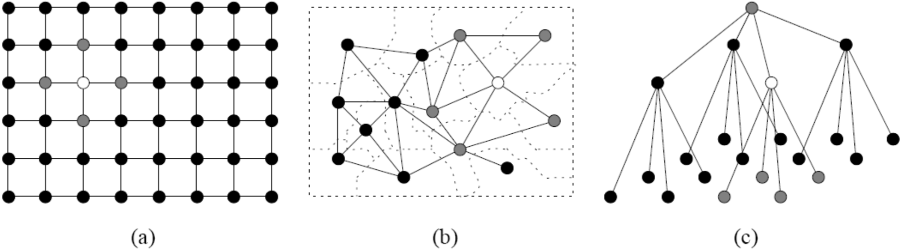


Figure 20: Example Markov random field models used in the literature. (a) First-order neighborhood system. (b) Non-regular planar graph associated to an image partition. (c) Quad-tree.

Contextual Classification

- A. Rabinovich, A. Vedaldi, C. Galleguillos, E. Wiewiora, S. Belongie, “Objects in Context,” *IEEE International Conference on Computer Vision*, 2007.
- Semantic context among objects is used for improving object categorization.

Contextual Classification

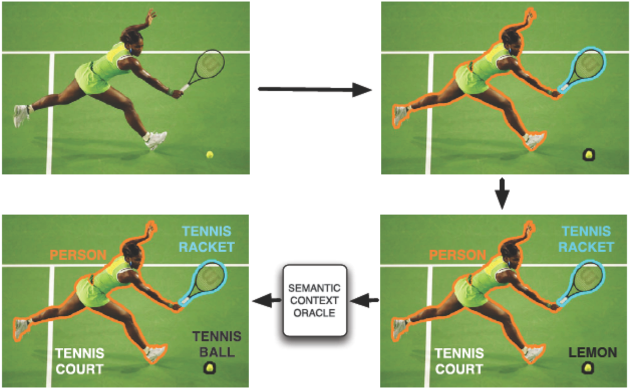


Figure 21: Idealized context based object categorization system: an original image is perfectly segmented into objects; each object is categorized; and object's labels are refined with respect to semantic context in the image.

Contextual Classification

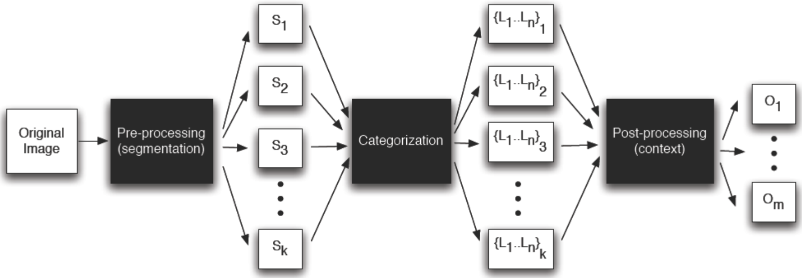


Figure 22: Object categorization framework: $S_1, \ldots, S_k$ is the set of k segments for an image; $L_1, \ldots, L_n$ is a ranked list of n labels for each segment; $O_1, \ldots, O_m$ is a set of m object categories in the image.

Contextual Classification

- A conditional random field (CRF) framework is used to incorporate semantic context into the object categorization.
- Given an image I and its segmentation $S_1, \ldots, S_k$, the goal is to find segment labels $c_1, \ldots, c_k$ such that they agree with the segment contents and are in contextual agreement with each other.

Contextual Classification

- This interaction is modeled as a probability distribution

$$p(c_1, \ldots, c_k | S_1, \ldots, S_k) = \frac{B(c_1, \ldots, c_k) \prod_{i=1}^{k} A(i)}{Z(\phi, S_1, \ldots, S_k)}$$

  with

$$A(i) = p(c_i | S_i) \text{ and } B(c_1, \ldots, c_k) = \exp\left(\sum_{i,j=1}^{k} \phi(c_i, c_j) \right),$$

  where $Z(\cdot)$ is the partition function.
- The semantic context information is modeled using context matrices that are symmetric, nonnegative matrices that contain the co-occurrence frequency among object labels in the training set.

Contextual Classification

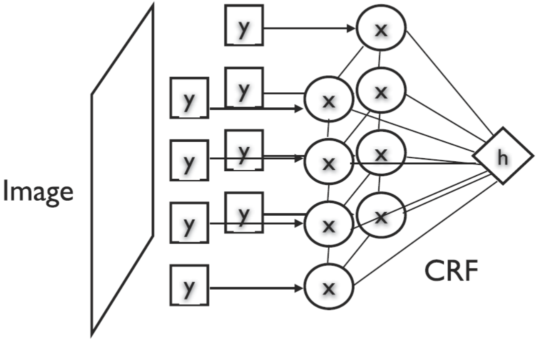


Figure 23: An example conditional random field. Squares indicate feature functions and circles indicate variable nodes. Arrows represent single node potentials due to feature functions, and undirected edges represent pairwise potentials. Global context is represented by h.

Contextual Classification

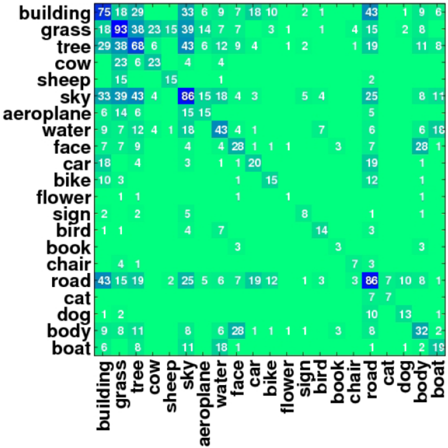


Figure 24: An example context matrix.

Contextual Classification

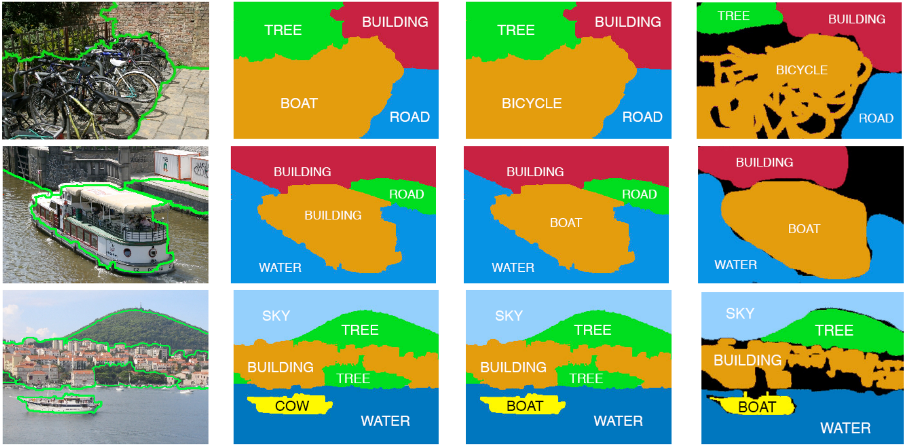


Figure 25: Example results where context improved the categorization accuracy. Left to right: original segmentation, categorization w/o contextual constraints, categorization w/ contextual constraints, ground truth.

Contextual Classification

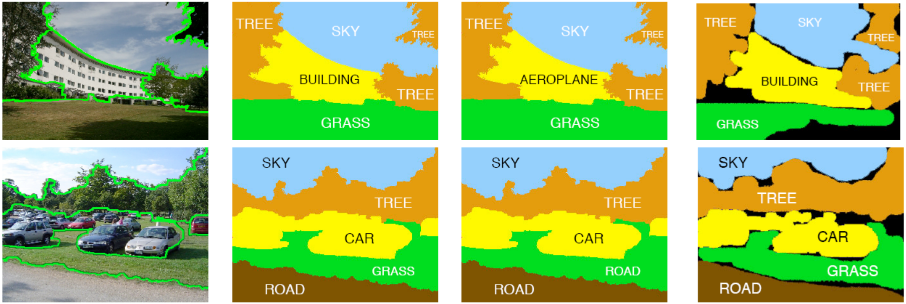


Figure 26: Example results where context reduced the categorization accuracy. Left to right: original segmentation, categorization w/o contextual constraints, categorization w/ contextual constraints, ground truth.