Probabilistic Graphical Models
Part III: Example Applications

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We will look at example uses of Bayesian networks and Markov networks for the following applications:

- Statistical text analysis — probabilistic latent semantic analysis
- Scene classification — probabilistic latent semantic analysis
- Object detection — probabilistic latent semantic analysis
- Image segmentation — Markov random fields
- Contextual classification — conditional random fields

The probabilistic latent semantic analysis (PLSA) algorithm has been originally developed for statistical text analysis to discover topics in a collection of documents that are represented using the frequencies of words from a vocabulary.
PLSA uses a graphical model for the joint probability of the documents and their words in terms of the probability of observing a word given a topic (aspect) and the probability of a topic given a document.

Suppose there are $N$ documents having content coming from a vocabulary with $M$ words.

The collection of documents is summarized in an $N$-by-$M$ co-occurrence table $n$ where $n(d_i, w_j)$ stores the number of occurrences of word $w_j$ in document $d_i$.

In addition, there is a latent topic variable $z_k$ associated with each observation, an observation being the occurrence of a word in a particular document.
Figure 1: The graphical model used by PLSA for modeling the joint probability $P(w_j, d_i, z_k)$. 
The generative model $P(d_i, w_j) = P(d_i)P(w_j|d_i)$ for word content of documents can be computed using the conditional probability

$$P(w_j|d_i) = \sum_{k=1}^{K} P(w_j|z_k)P(z_k|d_i).$$

$P(w_j|z_k)$ denotes the topic-conditional probability of word $w_j$ occurring in topic $z_k$.

$P(z_k|d_i)$ denotes the probability of topic $z_k$ observed in document $d_i$.

$K$ is the number of topics.
Then, the topic specific word distribution $P(w_j|z_k)$ and the document specific word distribution $P(w_j|d_i)$ can be used to determine similarities between topics and documents.

In PLSA, the goal is to identify the probabilities $P(w_j|z_k)$ and $P(z_k|d_i)$.

These probabilities are learned using the EM algorithm.
In the E-step, the posterior probability of the latent variables are computed based on the current estimates of the parameters as

\[
P(z_k|d_i, w_j) = \frac{P(w_j|z_k)P(z_k|d_i)}{\sum_{l=1}^{K} P(w_j|z_l)P(z_l|d_i)}.
\]

In the M-step, the parameters are updated to maximize the expected complete data log-likelihood as

\[
P(w_j|z_k) = \frac{\sum_{i=1}^{N} n(d_i, w_j)P(z_k|d_i, w_j)}{\sum_{m=1}^{M} \sum_{i=1}^{N} n(d_i, w_m)P(z_k|d_i, w_m)},
\]

\[
P(z_k|d_i) = \frac{\sum_{j=1}^{M} n(d_i, w_j)P(z_k|d_i, w_j)}{\sum_{j=1}^{M} n(d_i, w_j)}.
\]
Figure 2: Four aspects (topics) to most likely generate the word “segment”, derived from a $K = 128$ aspects model of a document collection consisting of abstracts of 1568 documents on clustering. The displayed word stems are the most probable words in the class-conditional distribution $P(w_j | z_k)$, from top to bottom in descending order.
Figure 3: Abstracts of four exemplary documents from the collection along with latent class posterior probabilities $P(z_k|d_i, W = \text{“segment”})$ and word probabilities $P(w = \text{“segment”}|d_i)$. 
Scene Classification


- The PLSA model is used for scene classification by modeling images using visual words (visterms).

- The topic (aspect) probabilities are used as features as an alternative representation to the word histograms.
Figure 4: Image representation as a collection of visual words (visterms).
Scene Classification

Figure 5: 10 most probable images from a data set consisting of city and landscape images for seven topics (aspects) out of 20.
Object Detection


- We used the PLSA technique for object detection to model the joint probability of the segments and their features in terms of the probability of observing a feature given an object and the probability of an object given the segment.
Figure 6: After image segmentation, each segment is modeled using the statistical summary of its pixel content (e.g., quantized spectral values).
Figure 7: (a) PLSA graphical model. The filled nodes indicate observed random variables whereas the unfilled node is unobserved. The red arrows show examples for the measurements represented at each node. (b) In PLSA, the object specific feature probability, \( P(x_j | t_k) \), and the segment specific object probability, \( P(t_k | s_i) \), are used to compute the segment specific feature probability, \( P(x_j | s_i) \).
After learning the parameters of the model, we want to find good segments belonging to each object type.

This is done by comparing the object specific feature distribution $P(x|t)$ and the segment specific feature distribution $P(x|s)$.

The similarity between two distributions can be measured using the Kullback-Leibler (KL) divergence $D(p(x|s)||p(x|t))$.

Then, for each object type, the segments can be sorted according to their KL divergence scores, and the most representative ones for that object type can be selected.
Figure 8: Examples of object detection.
Figure 9: Examples of object detection.

Markov random fields are used as a neighborhood model for image segmentation by classifying pixels into different pixel classes.
The goal is to assign each pixel into a set of labels $w \in \Omega$.

Pixels are modeled using color and texture features.

Pixel features are modeled using multivariate Gaussians, $p(x|w)$.

A first-order neighborhood system is used as the prior for the labeling process.
Figure 10: The Markov random field used as the first-order neighborhood model for the labeling process.
The prior is modeled as

\[ p(w) = \frac{1}{Z} \exp \left( - \sum_{c \in C} V_c(w_c) \right) \]

where \( V_c \) denotes the clique potential of clique \( c \in C \) having the label configuration \( w_c \).

Each clique corresponds to a pair of neighboring pixels.

The potentials favor similar classes in neighboring pixels as

\[ V_c = \delta(w_s, w_r) = \begin{cases} +1 & \text{if } w_s \neq w_r, \\ -1 & \text{otherwise.} \end{cases} \]
The prior is proportional to the length of the region boundaries. Thus, homogeneous segmentations will get a higher probability.

The final labeling for each pixel is done by maximizing the posterior probability

\[ p(w|x) \propto p(x|w)p(w). \]
Figure 11: Example segmentation results.
Figure 12: Example Markov random field models used in the literature. (a) First-order neighborhood system. (b) Non-regular planar graph associated to an image partition. (c) Quad-tree.

Semantic context among objects is used for improving object categorization.
Figure 13: Idealized context based object categorization system: an original image is perfectly segmented into objects; each object is categorized; and object’s labels are refined with respect to semantic context in the image.
Contextual Classification

Figure 14: Object categorization framework: $S_1, \ldots, S_k$ is the set of $k$ segments for an image; $L_1, \ldots, L_n$ is a ranked list of $n$ labels for each segment; $O_1, \ldots, O_m$ is a set of $m$ object categories in the image.
A conditional random field (CRF) framework is used to incorporate semantic context into the object categorization. Given an image $I$ and its segmentation $S_1, \ldots, S_k$, the goal is to find segment labels $c_1, \ldots, c_k$ such that they agree with the segment contents and are in contextual agreement with each other.
This interaction is modeled as a probability distribution

\[ p(c_1, \ldots, c_k | S_1, \ldots, S_k) = \frac{B(c_1, \ldots, c_k) \prod_{i=1}^{k} A(i)}{Z(\phi, S_1, \ldots, S_k)} \]

with

\[ A(i) = p(c_i | S_i) \quad \text{and} \quad B(c_1, \ldots, c_k) = \exp \left( \sum_{i,j=1}^{k} \phi(c_i, c_j) \right), \]

where \( Z(\cdot) \) is the partition function.

The semantic context information is modeled using context matrices that are symmetric, nonnegative matrices that contain the co-occurrence frequency among object labels in the training set.
Figure 15: An example conditional random field. Squares indicate feature functions and circles indicate variable nodes. Arrows represent single node potentials due to feature functions, and undirected edges represent pairwise potentials. Global context is represented by $h$. 
Contextual Classification

Figure 16: An example context matrix.
Figure 17: Example results where context improved the categorization accuracy. Left to right: original segmentation, categorization w/o contextual constraints, categorization w/ contextual constraints, ground truth.
Figure 18: Example results where context reduced the categorization accuracy. Left to right: original segmentation, categorization w/o contextual constraints, categorization w/ contextual constraints, ground truth.