Binary Image Analysis

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Outline

- Introduction to binary image analysis
- Thresholding
- Mathematical morphology
- Pixels and neighborhoods
- Connected components analysis
Binary image analysis

- Binary image analysis consists of a set of operations that are used to produce or process binary images, usually images of 0’s and 1’s where
  - 0 represents the background,
  - 1 represents the foreground.

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00010010001000
00011110001000
00010010001000
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Some application areas

- Document analysis
- Industrial inspection
- Surveillance

Adapted from Shapiro and Stockman; Cheung and Kamath
Example: red blood cell image

- Many blood cells are separate objects.
- Many touch each other → bad!
- Salt and pepper noise is present.
- How useful is this data?
- 63 separate objects are detected.
- Single cells have area of about 50 pixels.

Adapted from Linda Shapiro, U of Washington
Thresholding

- Binary images can be obtained by thresholding.
- Assumptions for thresholding:
  - Object region of interest has intensity distribution different from background.
  - Object pixels likely to be identified by intensity alone:
    - intensity > a
    - intensity < b
    - a < intensity < b
- Works OK with flat-shaded scenes or engineered scenes.
- Does not work well with natural scenes.
Use of histograms for thresholding

- Background is black.
- Healthy cherry is bright.
- Bruise is medium dark.
- Histogram shows two cherry regions (black background has been removed).

Adapted from Shapiro and Stockman
Automatic thresholding

- How can we use a histogram to separate an image into 2 (or several) different regions?

Is there a single clear threshold? 2? 3?

Two distinct modes

Overlapped modes

Adapted from Shapiro and Stockman
Automatic thresholding: Otsu’s method

- Assumption: the histogram is bimodal.
- Method: find the threshold $t$ that minimizes the weighted sum of within-group variances for the two groups that result from separating the gray levels at value $t$.
- The best threshold $t$ can be determined by a simple sequential search through all possible values of $t$.
- If the gray levels are strongly dependent on the location within the image, local or dynamic thresholds can also be used.
Automatic thresholding

A Pap smear image example: RGB image (left) and grayscale image (right).
Automatic thresholding

Histogram of the negative image (top-left), sum of within-group variances versus the threshold (bottom-left), resulting mask overlayed as red on the original image (top).
Otsu’s method

- [http://www.labbookpages.co.uk/software/imgProc/otsuThreshold.html](http://www.labbookpages.co.uk/software/imgProc/otsuThreshold.html)
The word **morphology** refers to form and structure.

In computer vision/image processing, it is used to refer to the shape of a region.

The language of mathematical morphology is set theory where sets represent objects in an image.

We will discuss morphological operations on binary images whose components are sets in the 2D integer space $\mathbb{Z}^2$. 
Mathematical morphology consists of two basic operations:
- dilation
- erosion

and several composite relations:
- opening
- closing
- conditional dilation
- ...
Dilation

- Dilation expands the connected sets of 1s of a binary image.
- It can be used for
  - growing features
  - filling holes and gaps

Adapted from Linda Shapiro, U of Washington
Erosion

- Erosion shrinks the connected sets of 1s of a binary image.
- It can be used for
  - shrinking features
  - removing bridges, branches and small protrusions

Adapted from Linda Shapiro, U of Washington
Basic concepts from set theory

• Let $A$ be a set in $\mathbb{Z}^2$. If $a = (a_1, a_2)$ is an element of $A$, we write $a \in A$; otherwise, we write $a \notin A$.

• Set $A$ being a subset of set $B$ is denoted by $A \subseteq B$.

• The union of two sets $A$ and $B$ is denoted by $A \cup B$.

• The intersection of two sets $A$ and $B$ is denoted by $A \cap B$.

• The complement of a set $A$ is the set of elements not contained in $A$:

$$A^c = \{w | w \notin A\}.$$ 

• The difference of two sets $A$ and $B$, denoted by $A - B$, is defined as

$$A - B = \{w | w \in A, w \notin B\} = A \cap B^c.$$
Basic concepts from set theory

- The reflection of set $B$, denoted by $\tilde{B}$, is defined as
  $$\tilde{B} = \{w | w = -b, \forall b \in B\}.$$  

- The translation of set $A$ by point $z = (z_1, z_2)$, denoted by $A_z$, is defined as
  $$A_z = \{w | w = a + z, \forall a \in A\}.$$  

**FIGURE 9.1**
(a) A set, (b) its reflection, and (c) its translation by $z$.  

Adapted from Gonzales and Woods
Structuring elements

- Structuring elements are small binary images used as shape masks in basic morphological operations.
- They can be any shape and size that is digitally representable.
- One pixel of the structuring element is denoted as its origin.
- Origin is often the central pixel of a symmetric structuring element but may in principle be any chosen pixel.
Structuring elements

FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.
Dilation

- The *dilation* of binary image $A$ by structuring element $B$ is denoted by $A \oplus B$ and is defined by

$$A \oplus B = \{z | B_z \cap A \neq \emptyset\},$$

$$= \bigcup_{a \in A} B_a.$$

- First definition: The dilation is the set of all displacements $z$ such that $B_z$ and $A$ overlap by at least one element.
- Second definition: The structuring element is swept over the image. Each time the origin of the structuring element touches a binary 1-pixel, the entire translated structuring element is ORed to the output image, which was initialized to all zeros.
Dilation

Binary image A

Structuring element B

Dilation result

(1st definition)
Dilation

Binary image A

Structuring element B

Dilation result

(2nd definition)
Dilation

Pablo Picasso, *Pass with the Cape*, 1960

Adapted from John Goutsias, Johns Hopkins Univ.

Pablo Picasso, *Pass with the Cape*, 1960
Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

**Figure 9.5**
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Adapted from Gonzales and Woods
Erosion

- The *erosion* of binary image $A$ by structuring element $B$ is denoted by $A \ominus B$ and is defined by

$$A \ominus B = \{z | B_z \subseteq A\},$$

$$= \{a | a + b \in A, \forall b \in B\}.$$

- First definition: The erosion is the set of all points $z$ such that $B$, translated by $z$, is contained in $A$.
- Second definition: The structuring element is swept over the image. At each position where every 1-pixel of the structuring element covers a 1-pixel of the binary image, the binary image pixel corresponding to the origin of the structuring element is ORed to the output image.
Erosion

Binary image A

Structuring element B

Erosion result

(1st definition)
Erosion

- Binary image A
- Structuring element B
- Erosion result

(2nd definition)
Erosion

Pablo Picasso, *Pass with the Cape*, 1960

Adapted from John Goutsias, Johns Hopkins Univ.
Erosion

FIGURE 9.7  (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1’s, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Adapted from Gonzales and Woods
The *opening* of binary image $A$ by structuring element $B$ is denoted by $A \circ B$ and is defined by

$$A \circ B = (A \ominus B) \oplus B.$$
Opening

Binary image A

Opening result

Structuring element B
Opening

Pablo Picasso, Pass with the Cape, 1960

Adapted from John Goutsias, Johns Hopkins Univ.
The *closing* of binary image $A$ by structuring element $B$ is denoted by $A \bullet B$ and is defined by

$$A \bullet B = (A \oplus B) \ominus B.$$
Closing

Binary image A

Structuring element B

Closing result
Examples

Adapted from Gonzales and Woods

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of $A$. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)
Properties

- Dilation and erosion are duals of each other with respect to set complementation and reflection, i.e.,
  \[(A \ominus B)^c = A^c \oplus \tilde{B}.\]

- Opening and closing are duals of each other with respect to set complementation and reflection, i.e.,
  \[(A \bullet B)^c = A^c \circ \tilde{B}.\]
Properties

- Opening satisfies the following properties:
  - $A \circ B$ is a subset of $A$.
  - If $C$ is a subset of $D$, then $C \circ B$ is a subset of $D \circ B$.
  - $(A \circ B) \circ B = A \circ B$.

- Closing satisfies the following properties:
  - $A$ is a subset of $A \bullet B$.
  - If $C$ is a subset of $D$, then $C \bullet B$ is a subset of $D \bullet B$.
  - $(A \bullet B) \bullet B = A \bullet B$. 
Boundary extraction

- The *boundary* of a set $A$ can be obtained by first eroding $A$ by $B$ and then performing the set difference between $A$ and its erosion, i.e.,

$$\text{boundary}(A) = A - (A \ominus B)$$

where $B$ is a suitable structuring element.

**FIGURE 9.14**
(a) A simple binary image, with 1’s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).
Conditional dilation

- Given an original binary image $B$, a processed binary image $C$, and a structuring element $A$, let $C_0 = C$ and $C_n = (C_{n-1} \oplus A) \cap B$. The *conditional dilation* of $C$ by $A$ with respect to $B$ is defined by

$$C \oplus |_B A = C_m$$

where the index $m$ is the smallest index satisfying $C_m = C_{m-1}$.

- Given a structuring element that when applied to a binary image
  - removes the components that do not satisfy certain shape and size constraints, and
  - leaves a few 1-pixels of those components that do satisfy the constraints, conditional dilation recovers the latter components using what remains of them after the erosion.
Region filling

- Given set $A$ containing the boundary points of a region, and a point $p$ inside the boundary, the following procedure fills the region with 1’s:

$$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots$$

where $X_0 = p$ and $B$ is the cross structuring element. The procedure terminates at iteration step $k$ if $X_k = X_{k-1}$.

- The set union of $X_k$ and $A$ contains the filled set and its boundary.
Examples

Detecting runways in satellite airport imagery


Note: These links do not work anymore, but you can use the Wayback Machine for older versions.
Examples

I - INTRODUÇÃO
A Terapia de Linguagem com crianças permeada por dificuldades relativas a fatores patologia. Uma vez que, nela estão envolvidas a da desestruturação do ego do paciente.

Muitas vezes, existem resistências ao contato psicóticos de defesa, frente aos quais o ter impotente.

Segmenting letters, words and paragraphs
Examples

Extracting the lateral ventricle from an MRI image of the brain
Examples

Detecting defects in a microelectronic circuit

Examples

Grading potato quality by shape and skin spots
Examples

Traffic scene  Temporal average  Average of differences

Lane detection example

Adapted from CMM/ENSMP/ARMINES
Examples

Threshold and dilation to detect lane markers

White line detection (top hat)

Detected lanes

Lane detection example

Adapted from CMM/ENSMP/ARMINES
Pixels and neighborhoods

- In many algorithms, not only the value of a particular pixel, but also the values of its neighbors are used when processing that pixel.
- The two most common definitions for neighbors are the 4-neighbors and the 8-neighbors of a pixel.

\[
\begin{array}{c|c|c|c}
\text{W} & \ast & \text{E} \\
\text{S} & & \\
\end{array}
\quad
\begin{array}{c|c|c|c|c}
\text{NW} & \text{N} & \text{NE} \\
\text{W} & \ast & \text{E} \\
\text{SW} & \text{S} & \text{SE} \\
\end{array}
\]

a) four-neighborhood $N_4$  

b) eight-neighborhood $N_8$

Figure 3.2: The two most common neighborhoods of a pixel.
Connected components analysis

- Once you have a binary image, you can identify and then analyze each connected set of pixels.
- The connected components operation takes in a binary image and produces a labeled image in which each pixel has the integer label of either the background (0) or a component.
Connected components analysis

Methods for connected components analysis:

- Recursive tracking (almost never used)
- Parallel growing (needs parallel hardware)
- Row-by-row (most common)
  - Classical algorithm
  - Run-length algorithm (see Shapiro-Stockman)

![Binary Image and Connected Components Labeling](image)

Adapted from Shapiro and Stockman
Connected components analysis

- **Row-by-row labeling algorithm:**
  1. The first pass propagates a pixel’s label to its neighbors to the right and below it. (Whenever two different labels can propagate to the same pixel, these labels are recorded as an equivalence class.)
  2. The second pass performs a translation, assigning to each pixel the label of its equivalence class.

- A union-find data structure is used for efficient construction and manipulation of equivalence classes represented by tree structures.
Connected components analysis
Connected components analysis