CS476: Automata Theory and Formal Languages

Homework 3

Assigned: 23/12/2013

Due: 31/12/2013, 17:00

Questions

1. (20pts) State whether the following statements are true or not. You must give a BRIEF explanation or show a counter example to receive full credit.

   (a) If a language L is recognizable then the language $\bar{L}$ is decidable.

   (b) If languages L_1 and $L_1 \cup L_2$ are decidable then L_2 must also be decidable.

   (c) If a language L is not in P then it must be undecidable.

   (d) Given a language L in NP-c, there exist a polynomial-time reduction from L to Boolean Satisfiability Problem (SAT).

2. (20pts) Give a Turing Machine that decides language $L = \{w \in \{0,1\}^* : n_0(w) \neq 2n_1(w)\}$.

3. (20pts) A multiple-head Turing machine has m heads reading cells of one tape. A move of this TM depends on the state and on the symbol scanned by each head. In one move, the TM can change state, write a new symbol on the cell scanned by each head, and can move each head left, right, or keep it stationary. Since several heads may be scanning the same cell, we assume the heads are numbered 1 through m, and the symbol written by the highest numbered head scanning a given cell is the one that actually gets written there. Show that this variant of TM model is equivalent to ordinary TM model.

4. (20pts) Disprove (by reduction) or prove that the following languages are decidable.

   (a) $L = \{\langle A \rangle : A \text{ is a DFA and } L(A) \text{ is infinite}\}$

   (b) $L = \{\langle M, M', k \rangle : M \text{ and } M' \text{ are TMs and } |L(M) \cup L(M')| \geq k\}$

5. (20pts) Given two languages L_A and L_B, language L_C is defined as follows:

$$L_C = \{w : w \in L_A \text{ and } w^R \in L_B\}$$

   (a) Let A and B be deciders for languages L_A and L_B, respectively. Construct a decider C for language L_C.

   (b) Let language L_B be decidable. Prove that L_A is decidable if L_C is decidable.