Algorithms II, CS 502

Algorithms Basics

Ugur Dogrusoz

Computer Eng Dept, Bilkent Univ

What is an Algorithm?

- Procedure that always halts with a correct solution to the problem at hand

Why study Algorithms?

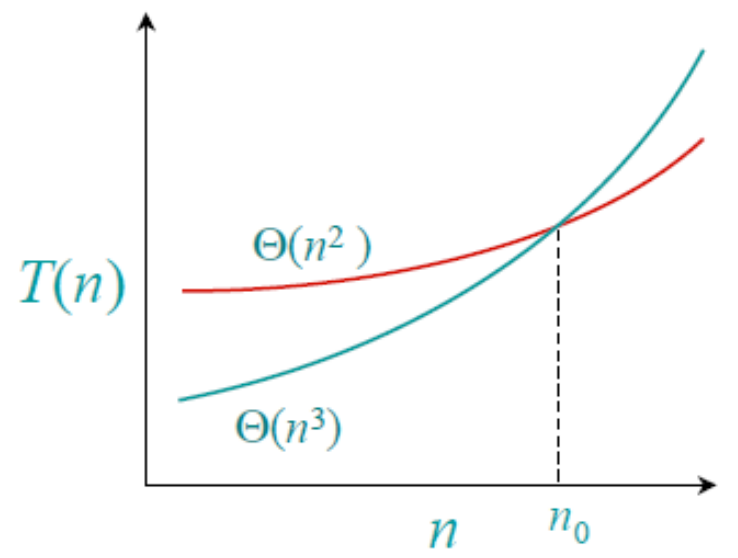
- Analyze performance to determine “feasible vs. not”
- Algorithmic mathematics (e.g. big-O notation) allows comparing performance of two algorithms for the same problem
- Build a repertoire of algorithms for future use
- Learn various algorithm design paradigms and apply to new problems

Kinds of analyses

- Worst case (usually):
  - T(n) = maximum time it takes for an algorithm for any input of size n
- Average case (sometimes):
  - T(n) = expected time of algorithm over all inputs of size n
  - Need to know statistical distribution of inputs
  - Harder
- Best case (rarely):
  - Can always cheat with a slow algorithm that works fast on some input

Asymptotic notation

- Use for running time or memory requirement analysis
- Ignore machine-dependent constants, look at growth in T(n) as n goes to infinity
- When input size gets large enough, a quadratic algorithm always beats a cubic one



O-notation

- Formally

$$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le f(n) \le cg(n) \text{ for all } n \ge n_0\} .$$

- Informally
  - drop low order terms, ignore leading constants to form an upper bound

$$3n^3 + 90n^2 - 5n + 6046 = O(n^3)$$

Ω-notation

- Formally

$$\Omega(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le cg(n) \le f(n) \text{ for all } n \ge n_0\}.$$

- Informally
  - drop low order terms, ignore leading constants to form a lower bound

$$3n^3 + 90n^2 - 5n + 6046 = \Omega(n^3)$$

o- and ω-notation

- Strict versions of O and Ω

$$3n^3 + 90n^2 - 5n + 6046 = O(n^3)$$

$$3n^3 + 90n^2 - 5n + 6046 \neq o(n^3)$$

$$3n^3 + 90n^2 - 5n + 6046 = o(n^{3.01})$$

$$3n^3 + 90n^2 - 5n + 6046 = \Omega(n^3)$$

$$3n^3 + 90n^2 - 5n + 6046 \neq \omega(n^3)$$

$$3n^3 + 90n^2 - 5n + 6046 = \omega(n^{2.99})$$

Θ-notation

- Formally

$$\Theta(g(n)) = \{f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \le c_1 g(n) \le f(n) \le c_2 g(n) \text{ for all } n \ge n_0\} .$$

- Informally
  - drop low order terms, ignore leading constants to form a tight (both lower and upper) bound

$$3n^3 + 90n^2 - 5n + 6046 = \Theta(n^3)$$

Algorithm design paradigms

- Divide-and-conquer
- Dynamic programming
- Greedy
- Branch-and-bound
- …

Methods for running time complexity

- Master Method
  - Applies to limited types of algorithms
- Substitution Method
  - Difficult to make the guess that works
  - Might not work (lead to induction that works)
- Recursive Tree Method
  - Difficult to get tight complexity

Example: Fibonacci numbers

- Calculate n^{th} Fibonnaci number
  - $F_0=0$, $F_1=1$, $F_i=F_{i-1}+F_{i-2}$ for $i \geq 2$
- Divide-and-conquer solution
  - Running time?
  - How to improve?