When does a graph $G$ contain a Hamilton cycle, a cycle that contains every vertex of $G$? Any graph $G$ that contains a Hamilton cycle is called Hamiltonian. Similarly, a path in $G$ containing every vertex is a Hamilton path.

To determine whether or not a given graph has a Hamilton cycle is much harder than deciding whether it is Eulerian, and no good characterization is known of the graphs that do.

## 1 Simple sufficient conditions

Purely global assumptions, like high edge density, will not be enough: we cannot do without the local property that every vertex has at least two neighbors. But neither is any large (but constant) minimum degree sufficient: it is easy to find graphs without a Hamilton cycle whose minimum degree exceeds any given constant bound.

**Theorem 1.1** [10.1.1][Dirac 1952] ♠ Every graph with $n \geq 3$ vertices and minimum degree at least $n/2$ has a Hamilton cycle.

A low independence number $\alpha(G)$ ensures $G$ has long cycles, though not necessarily a Hamilton cycle. Put together, high connectivity and low independence number, complement each other to produce a sufficient condition for Hamiltonicity:

**Proposition 1.2** [10.1.2] Every graph $G$ with $n \geq 3$ and $\kappa(G) \geq \alpha(G)$ has a Hamilton cycle.

**Theorem 1.3** [10.1.4][Tutte 1956] Every 4-connected planar graph has a Hamilton cycle.