(1) If “heterological” is heterological, then it is not heterological. (Why not?) If “heterological” is not heterological, then it is heterological. (Why?) In any case, “heterological” is both heterological and not heterological. This leads us to a paradox. This formulation is due to Grelling (1908).

(2) This is known as a Contingent Liar sentence:

\[(\sigma) \quad \text{Saddam has weapons of mass destruction and this proposition is false.}\]

I think the most natural reading here is one in which “this proposition” refers to the proposition expressed by the whole sentence \(\sigma\). But clearly we can also consider it being used to refer to the proposition expressed by the second conjunct alone (in which case it refers to the ordinary liar proposition).

Let’s just analyze \(\sigma\) assuming the first reading. If Saddam does not have weapons of mass destruction, then the first conjunct is false. Then it would seem that this proposition is simply false. \((\text{false} \land \text{whatever} = \text{false})\) Well, the second conjunct also says that, so no paradox arises.

However, if Saddam has weapons of mass destruction then \(\sigma\) is simply reduced to the liar sentence and hence is paradoxical. \((\text{true} \land \text{whatever} = \text{whatever})\)

Notes:
(a) I leave it to you to understand why this is called a Contingent Liar. Learn the meaning of this word within the realm of logic!
(b) Nothing much changes if we adopt the second reading mentioned above.

(3) This, in fact, is the more difficult question of the homework and is known as Løb’s Paradox, after M.H. Løb’s 1955 paper.

Consider

\[(B) \quad \text{If this sentence (proposition) is true, then } A.\]

So \(B\) asserts: “If \(B\) is true, then \(A\).” Now, assume the antecedent of \(B\), viz. that the proposition expressed by \(B\) is true.

But then, we have both

- the proposition expressed by \(B\) and
- its antecedent

By modus ponens \((\rightarrow \text{elim})\), it follows that \(A\). Thus we have shown that if the antecedent of \(B\) is true, then \(A\). In other words, we have established the truth of \(B\). (We've simply done \(\rightarrow \text{intro}\).) Therefore, by \(B\), since \(B\) is true, \(A\) is true. (One last application of \(MP\).)

General Remark: All of the paradoxes (in parts 1, 2 and 3) are genuine in the sense that they have no clear logical flaws.