PART a: The translation into PL is

\[
\begin{align*}
(A \land B) &\rightarrow (C \leftrightarrow \neg D) \\
(B \land C) &\rightarrow (A \leftrightarrow D) \\
(\neg A \land \neg B) &\rightarrow (\neg C \land \neg D) \\
(\neg C \land \neg D) &\rightarrow (\neg A \land \neg B)
\end{align*}
\]

\[\therefore \neg((A \land B) \land C) \land ((\neg A \land \neg B) \rightarrow \neg C)\]

See the instructor’s door for the truth table. The premises are true in lines 3, 5, 6, 7, 10, 11 and 16 (and no others) and for these lines the conclusion is also true. Thus the argument is valid.

[N.B.] $\phi \leftrightarrow \psi$ stands for $(\phi \rightarrow \psi) \land (\psi \rightarrow \phi)$.

PART b: We can translate the listed 13 statements into PL as follows:

1: $(A \land B) \land C \rightarrow D$
2: $\neg A \land M \rightarrow L$
3: $F \land E \rightarrow \neg D$
4: $G \land M \rightarrow C$
5: $\neg B \land F \rightarrow \neg H$
6: $(\neg D \land B) \land E \rightarrow G$
7: $M \land \neg I \rightarrow J$
8: $H \land M \rightarrow K$
9: $(K \land J) \land \neg L \rightarrow E$
10: $\neg H \land F \rightarrow L$
11: $M \land L \rightarrow \neg F$
12: $(K \land I) \land A \rightarrow E$
13: $\therefore M \rightarrow \neg F$

We may now write a program which constructs a truth table for these statements and checks the validity of the argument.

The table will require $2^{13}$ rows. We must consider a row only when all the premises evaluate to True in that row. If this is the case, then we further check whether the conclusion is also True in that row.

By the way, the argument is valid. Let’s illustrate this using a method due to Boole, i.e. the Logic of Classes (1847).

In this method, our letters denote classes and instead of negation, we think of complement. Consider the first statement above:

\[A \land B \land C \rightarrow D\]

We can rewrite this as follows (why?)

\[A \cap B \cap C \subseteq D\]

This means that (why?)

\[(A \cap B \cap C) \cap D = A \cap B \cap C\]
More compactly,

$$(A \cap B \cap C) \cap D' = 0$$

where $'$ denotes the complement and 0 denotes the empty class. We can even suppress $\cap$ for notational simplicity and get

$$ABCD' = 0$$

Proceeding in a similar fashion we obtain

1: $ABCD' = 0$
2: $A'ML' = 0$
3: $FED = 0$
4: $GMC' = 0$
5: $B'FH = 0$
6: $D'BEG' = 0$
7: $MI'J' = 0$
8: $HMK' = 0$
9: $KJL'E' = 0$
10: $H'FL' = 0$
11: $MLF = 0$
12: $KIAE' = 0$
13: $MF = 0$

Now the tree in the above figure gives the proof.

[N.B.] In the figure, a number attached to a circle tells us the number of a premise that guarantees the product is 0. For example, looking at the branch that terminates with the attached number 8, we see that
\[ MFL'AHK' = 0 \]

because premise 8 says that \( HMK' = 0 \), and

\[ MFL'AHK' = (HMK')FL'A = 0FL'A = 0 \]

Note that we used all the premises to prove the required result \((MF = 0)\). Try to understand this proof fully. It’s fun!